THE ALGORITHMIC APPROACH TO CURRICULUM CONSTRUCTION: A FIELD TEST IN MATHEMATICS

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This study constitutes a field test of an algorithmic approach to curriculum construction. First the method was used successfully to characterize the content inherent in a mathematics textbook in terms of rules (algorithms). The use of higher-order rules allowed a 40% reduction in total rules. Two rule-based curricula were compared experimentally. The discrete (D) rules curriculum consisted of all 303 (lower-order) rules. The higher-order (H) curriculum included 169 of these rules plus 5 higher-order rules. The Curriculum H subjects performed as well with the eliminated rules as did the Curriculum D subjects who learned the rules directly, and they performed significantly better on new tasks beyond the scope of either curriculum. In short, Curriculum H subjects were taught less but learned more.

Curriculum construction has traditionally been an artistic endeavor. Even today, the vast majority of texts and new curricula are developed almost exclusively on the basis of the curriculum constructor's subject-matter knowledge and professional know-how.

During the 1960s, a strong technological counterforce developed under the leadership of behavioral scientists. The basic position taken was that objectives must be stated in behavioral (operational) terms so as to make it possible to determine, through testing, whether learners have achieved individual objectives. As a result, a healthy debate developed between proponents of behavioral objectives (e.g., Gagné, 1970; Lipson, 1967; Mager, 1962; Popham, 1969; Tyler, 1964) and others who raised cautions concerning their use (e.g., Atkin, 1968; Ebel, 1970; Eisner, 1967).

In recent publications, the positions taken have become increasingly more flexible (e.g., Glaser, 1973; Resnick, 1972; MacDonald-Ross, 1973). A statement by Scandura (1971) summarizes much of the current view: "It is felt that complete reliance on operationally defined objectives has led some to fragmented curricula, curricula based on discrete bits of knowledge [p. 4]."

Elaborating on this view, Scandura (1972) has identified two basic inadequacies of the behavioral objectives approach considered in its simplest form: (a) The approach deals only with observable behavior and says little about how that behavior is to be generated, and (b) it provides no systematic way of dealing with interrelationships among the identified objectives, or equivalently, of building transfer into a curriculum.

Regarding Point a, the distinction between the behavior of a subject and the knowledge (rule) that makes that behavior possible is fundamental. It can easily be proven mathematically that if just one rule exists for generating a class of behaviors, then there is an infinite number of other rules that will do the same thing (e.g., see Rogers, 1967). This fact is important in
curriculum planning because in practice there is almost always more than one viable way of approaching a task. The subtraction methods of borrowing and equal additions, for example, are both widely used.

Regarding Point b, it is clearly an impossible task, with any but the most trivial curricula, to explicitly teach the learner all that the curriculum constructor wants him to know. The limitations imposed by time and the capacity of the learner to absorb and retain information make this impractical. Some attention to interrelationships would seem almost essential.

One approach to this problem is based on learning hierarchies (e.g., Gagné, 1970; Resnick, Wang, & Kaplan, 1970). As is well known, this approach makes use of task analysis (e.g., Miller, 1962) as a means of determining subordinate tasks. Subordinate tasks are prerequisite to so-called Higher Order 1 tasks in the sense that transfer to Higher Order 1 tasks frequently occurs once all of the prerequisites are learned. [It may be noted parenthetically that implicit in any specific task analysis is a specific underlying rule (which is not necessarily, and often is not, explicit in the analyst’s mind). Since different rules may underlie the same task (as indicated above) it follows that any given task, theoretically speaking, may be task analyzed in any number of different ways (e.g., consider task analyses of subtraction based on borrowing and equal additions).]

In the present study, we have adopted a second (algorithmic) approach to the problem of transfer (Scandura, 1972). This approach bears some relationship to learning hierarchies, but it includes an important conceptual generalization that has often gone undetected because of the common use of the descriptor “higher order.”

This approach is based on a recent theory of structural learning (Scandura, 1973) in which Higher Order 2 rules may operate on other rules (e.g., subordinate ones) to generate what in task analysis are Higher Order 1 rules. [Mathematically speaking, the distinction is precisely that between a function (Higher Order 2) and a function value or output (Higher Order 1).] Unlike the “directions,” which are sometimes necessary in moving from one level in a hierarchy to another, Higher Order 2 rules operate on classes of rules and are not limited to rules in particular hierarchies. Higher Order 2 rules may operate between various levels of any one of a class of hierarchies, including hierarchies that appear superficially quite different. Consider, for example, the following two, simple Higher Order 1 tasks: (a) Given a certain number of yards, find the equivalent number of inches; and (b) Given an airplane (on the ground), get it up in the air and then back down (safely).

One way to analyze these tasks is to break each task into component parts: (a) one subordinate task for converting yards into feet and another for feet into inches and (b) subordinate tasks for “taking off” and “landing.” Although they involve quite different rules, the hierarchies have a common structure and, in particular, the Higher Order 1 tasks (more exactly, the rules for solving the indicated Higher Order 1 tasks) can be generated from the respective subordinate task (rules) by applying a Higher Order 2 composition rule (for details, see Scandura, 1973b, pp. 213–218). This Higher Order 2 composition rule would be reflected in typical learning hierarchies by parallel but different “instructions.” In effect, a single, Higher Order 2 rule can take the place of a potentially infinite class of separate “instructions.”

Further, Higher Order 2 rules are not limited in their application to traditional hierarchies. For example, given a rule for converting inches to centimeters (1 inch = 2.54 centimeters), it is a simple matter to envision a Higher Order 2 inverse rule which applies to such rules and generates their inverses. In this case, the output would be a rule for converting centimeters to inches (1 centimeter = inch/2.54). For a discussion of these and other differences between Higher Order 1 and Higher Order 2 rules, see Scandura (1973a). In this article, all subsequent references to “higher order” are of Type 2.

The algorithmic approach to curriculum construction is based on these ideas and dis-
tinctions. Specifically, the approach builds on the notion that each behavioral objective corresponds directly to a class of tasks that can be computed (solved) by applying a rule or algorithm. It is further assumed that curricula (i.e., what is to be learned) can be represented in terms of finite sets of rules, including higher-order rules that operate on rules. In effect, the task of curriculum construction may be viewed as one of identifying a finite set of rules that provides an efficient account of the desired behaviors. The algorithmic approach is basically a method for devising curricula based on behavioral objectives and characterized in terms of rules and higher-order rules.

The first step in this method is to select text materials to analyze. This, of course, involves making value judgments concerning the type of material to be considered. All of the tasks implicit in the text material are then identified and stated as behavioral objectives. Next, rules are written for solving each of the tasks, and parallels among these rules are identified. Such parallels are indicative of common structure and provide a basis for devising higher-order rules. Finally, those rules that are derivable by application of the higher-order rules to other rules in the characterizing set may be eliminated.

To summarize, the algorithmic approach provides a potential basis for overcoming the two, aforementioned major limitations of the simple behavioral objectives approach. First, it makes specific what the subject must learn in order to demonstrate mastery on behavioral objectives and, thus, might provide a viable basis for instruction. Second, it makes explicit provision for the inclusion of higher-order relationships among objectives. Indeed, it provides a systematic way for possibly building transfer potential into a curriculum.

Two studies are reported. The first study was strictly analytical in nature and was designed to determine the general feasibility of the algorithmic approach. Specifically, we wanted to determine the practicality of characterizing the knowledge inherent in a given mathematics text in terms of a finite set of rules, including higher-order rules.

The second study was contingent upon the success of Study 1. The purpose of this study was to determine (a) whether making rules explicit provides a viable basis for instruction in the classroom, and (b) whether the introduction of higher-order rules provides an adequate basis for improving the ability of students to transfer.

**Study 1**

*Feasibility of the Algorithmic Approach to Curriculum Construction*

In order to judge the feasibility of the approach, the following two criteria were established:

1. A subjective appraisal of (a) the ease with which the tasks (behavioral objectives) inherent in the given text material could be identified, (b) the ease with which rules associated with each of the respective tasks could be written, and (c) the extent to which the tasks and rules identified were compatible with the approach taken in the text.

2. The extent and ease with which the higher-order rules inherent in the text could be (a) identified and (b) used to eliminate those rules (and their corresponding tasks) that were derivable by application of the higher-order rules to other identified rules.

Consideration was also given to the sheer numbers of tasks and rules involved and particularly to the extent these numbers could be reduced by the introduction of higher-order rules.

**Method**

Part 3 of *Mathematics: Concrete Behavioral Foundations* (Scandura, 1971) was chosen for analysis. (This book was later analyzed in its entirety and was published as a workbook, Scandura, Durnin, Ehrenpreis, Luger, 1971.)

The first step was to identify the individual tasks (behavioral objectives) inherent in the text. This was accomplished by going through the text paragraph by paragraph and asking what performance capabilities might reasonably be expected of a student who had studied the material. For example, the following tasks were identified in *Mathematics: Concrete Behavioral Foundations* on pages 182 and 191, respectively.
Task A. Given a whole number \( m \), determine whether or not \( x \) is an additive identity for \( m \).

Task B. Given a whole number \( m \), determine whether or not \( y \) is a multiplicative identity for \( m \).

The second step was to identify and eliminate redundancies in the tasks identified. Very few (less than 8) such redundancies were found in the text aside from prerequisites to other tasks. Prerequisites tasks were not eliminated because it was felt desirable to maintain the original sequencing of ideas in the text.

Third, one efficient rule was constructed for each task. The rules were written so as to be compatible with the text material. The rules written for the illustrative Tasks A and B were as follows:

**Rule A.** Find the sum \( m + x \) and then the sum \( x + m \). If \( m + x = x + m = m \), then \( x \) is an additive identity for \( m \); if \( m + x \neq m \) or \( x + m \neq m \), then \( x \) is not an additive identity for \( m \).

**Rule B.** Find the product \( m \times y \) and then the product \( y \times m \). If \( m \times y = y \times m = m \), then \( y \) is a multiplicative identity for \( m \); if \( m \times y \neq m \) or \( y \times m \neq m \), then \( y \) is not a multiplicative identity for \( m \).

The fourth step was to look for higher-order relationships among the rules. These relationships were stated as tasks to be performed, and higher-order rules underlying these (higher order) tasks were then constructed. For example, Rules A and B are obviously related. The nature of this relationship can be illustrated by the following task and its underlying higher-order rule:

**Task H.** Give a rule for demonstrating that a given set of numbers provides an instance of a property (e.g., commutativity) under some operation, generate a corresponding rule involving another operation.

**Rule H.** In the given rule, replace the original operation by the new operation and any “special” element (e.g., the identity) by its counterpart.

Fifth, the higher-order rules identified in Step 4 were used to eliminate those tasks and corresponding rules that are derivable by application of the higher-order rules to other rules identified. Thus, for example, Rule B was eliminated inasmuch as it could be generated by applying Rule H to Rule A.

Results and Discussion

The chapters analyzed lent themselves very naturally to a task-rule type of analysis. The major requirements found necessary for such analysis were a thorough familiarity with the subject matter and a good working knowledge of the algorithmic approach.

Upon completion of Step 3, a list of 303 tasks and their corresponding rules (one rule for each task) had been identified. A list containing 174 rules was obtained upon completion of Step 5. Five of these were higher-order rules acting on rules concerned with (a) ordered sets (ordinal numbers) and unordered sets (cardinal numbers), (b) well-defined operations, (c) properties of number systems (e.g., commutativity), (d) inverse operations, and (e) particularization (i.e., assigning values to variables). The first higher-order rule made it possible to eliminate 4 rules; the second, 17; the third, 84; the fourth, 7; and the fifth, 22.

Although the process of identifying the “tasks” inherent in the text was time-consuming, it was felt that the list of 303 tasks gave almost complete coverage of the material. Once the tasks were identified, there was little difficulty encountered in writing rules compatible with the approach taken in the text. This was partly due to the fact that the narrative and illustrative examples provided adequate guidance. The identification of the higher-order rules and the subsequent reduction of the list of 303 rules to the final list of 174 rules was the most difficult step. Some of the higher-order rules (e.g., ordered sets and unordered sets) were easier to identify than others (e.g., particularization), and the analysis was pursued just far enough to demonstrate the feasibility of the approach. Doubtless, a more intensive analysis would have resulted in a larger number and variety of higher-order rules. Overall, based on the criteria established, it was concluded that the algorithmic approach was feasible.

**Study 2**

**Evaluation of the Algorithmic Curriculum**

The second study, in contrast to the first, was experimental and involved a comparison of two rule-based curricula. The first curriculum (D) was characterized in terms of a list of discrete tasks and rules for solving these tasks, one rule for each task. The second curriculum (H) included the higher-
order rules and all Curriculum D tasks and rules except those derivable by application of the higher-order rules to other rules in Curriculum D (as described in Step 5, Study 1).

The degree of learning evidenced by students trained in Curriculum D provided evidence concerning the hypothesis that making rules explicit provides a viable basis for classroom instruction. This hypothesis was tested directly in terms of mastery rather than by comparison with some arbitrarily defined control.

Comparative performance of the students trained in Curricula D and H pertained to the question of transfer. We hypothesized that students trained in the higher-order rules (Curriculum H) would perform on the Curriculum D tasks that had been eliminated from Curriculum H as well as the subjects who had been trained on the Curriculum D tasks. This was expected (according to the theory) because these tasks could be solved by using higher-order rules to derive solution rules for these tasks. Furthermore, we predicted that the Curriculum H students would perform significantly better on tasks not in Curriculum D that could be solved in the same way (i.e., by use of the higher-order rules). No difference in performance was expected on tasks included in both curricula.

Method

Materials. The experimental materials were based directly on the analyses reported in Study 1. The discrete rules curriculum (D) resulted upon application of Steps 1-3 of the algorithmic approach. The higher-order rules curriculum (H) included Steps 4-5 as well. Curriculum D consisted of 303 tasks and rules, and Curriculum H of 174 task and rules including 5 higher-order tasks and rules. Although experimental materials were prepared for Chapters 5-9 of Mathematics: Concrete Behavioral Foundations, only those materials pertaining to Chapters 5, 6, and 7 were actually used in the experiment.

The two curricula were reproduced in the form of workbooks with the following format: First, a statement of the task. Second, a rule statement in simple terms. Next, from three to five worked examples (depending on the experimenter's judgment as to task difficulty), and finally, 10 exercises. In addition, a set of task exercises for review purposes was selected on the basis of the apparent difficulty subjects encountered during learning. All higher-order tasks were included in the Curriculum H review.

Pre- and posttests were also constructed. The pretest consisted of a sample of 40 exercises (stated exactly as they appeared in the workbooks) which (a) tested applications of rules found in both treatments, and (b) in the judgment of the experimenter, were most likely to discriminate among subjects. The posttest consisted of a stratified random sample of 32 task exercises from each of the following categories: (a) tasks found in Curriculum D and H, (b) tasks found only in Curriculum D, (c) higher-order tasks found only in Curriculum H, and (d) tasks that were found in neither treatment but which theoretically could be derived from rules found in Curriculum H. The exercises were selected randomly from each of these categories—10 each from Categories (a) and (b), and 6 each from Categories (c) and (d). (Kuder-Richardson Formula 20 was used to obtain reliability coefficients of .86 for both the pretest and the posttest. High curricular (content) validity (Cureton, 1951) was assured by the method of construction used.)

Subjects and design. The subjects were 48 (16 male, 32 female) Trenton State College summer school students enrolled in two sections of a course on teaching modern mathematics in the elementary grades. The first author was the instructor for both sections of the course.

Pretest scores were used to assign 24 subjects to a high group and 24 subjects to a low group. Within each group, 12 subjects were randomly assigned to each curriculum.

Procedure. During the first class meeting of each section, the subjects were told that the course was experimental and that, with the exception of unrelated outside reading, all of their work would be done individually and during class time. Then, the pretest was administered. The subjects had all the time they needed to complete the test. The highest score on the pretest was 24 exercises correct out of the possible 40. The median was 10 and the mean was 11.2. Forty-five of the 48 subjects had pretest scores below 20, and 26 had scores of 10 or below.

Each subject purchased a notebook at the beginning of the six-week term; all experimental work was done in these notebooks. The instructional workbooks and student notebooks were distributed at the beginning of each two-hour, five-minute class.

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<th>TABLE 1</th>
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<tr>
<td><strong>NUMBER OF TASKS IN CHAPTERS 5 THROUGH 9</strong></td>
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<tr>
<td><strong>Curriculum</strong></td>
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<td><strong>5</strong></td>
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<td>Discrete rules</td>
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period and collected at the end. The Curriculum D and Curriculum H subjects met in separate classrooms for three class periods per week. Each subject was permitted to go on to the next task as soon as he had demonstrated success on a number of consecutive exercises equal to the number of illustrative examples constructed for that task. A daily record of each subject's performance was kept, as were anecdotal records indicating particular difficulties or problems that arose.

During the first meeting of each week (beginning with Week 2), subjects were given a set of individualized review exercises; the exercises selected were those exercises completed during the preceding two weeks which had given the subject the greatest difficulty. The subjects were required to work these exercises without the use of the workbook or their notebooks. On the final class meeting of each week, the review exercises and solutions were returned to the subjects. They were told to check their solutions using their workbooks and notebooks and to correct any errors. As soon as this had been completed, the review materials were collected.

During the final three weeks, class time was set aside to enable subjects to review and study the rules they had learned up to that time (45 minutes during Weeks 4 and 5; 3 hours during Week 6). The amount of time spent working on the tasks and on review was the same for all subjects. In those few cases where a subject missed a class, he was required to make up the time missed.

At the end of the term, 14 of the 24 subjects in Curriculum D were working in Chapter 7, with the farthest advanced to Task 36. Two of the 24 Curriculum H subjects completed Chapter 7, and 16 others were working in Chapter 7. All Curriculum H subjects completed the five higher-order tasks.

The posttest was given during the next-to-last class meeting. The subjects were given all of the time they needed and they were encouraged to do their best.

Results and Discussion

Mastery. The Curriculum D subjects were successful on 339 of the 362 posttest tasks on which they as individuals had been trained, for a mastery level of 94%. (The number of test items on which each subject had been trained was determined by examination of his class workbook.) The Curriculum H subjects were successful on 183 of the 190 lower-order tasks (96%) on which they had been trained. It would appear that making rules explicit does provide a viable basis for instruction.

On the higher tasks, Curriculum H subjects performed as expected at a significantly higher level \( (F = 7.37, df = 1.44, p < .01) \) than the Curriculum D subjects. The overall means were 3.2 (Curriculum D) and 4.3 (Curriculum H) with a maximum score of 6. The data of the individual subjects showed that the Curriculum H subjects were successful on 103 out of the 144 posttest exercises (71.5%), whereas the Curriculum D subjects were successful on 77 out of 144 (53.4%) exercises. These proportions differed significantly (arc sine transformation, \( Z = 2.9, p < .01 \); cf. Baggaley, 1957).

It is of interest, nonetheless, that the Curriculum D subjects, although not trained on the higher-order tasks, did perform successfully 53.4% of the time. This suggests that some of the Curriculum D subjects may have known these (relatively simple) higher-order rules prior to the experiment, while others may have been able to induce them as they worked through Curriculum D. In any case, the performance level did not approach that of the Curriculum H subjects, and it can safely be concluded that training on higher-order rules had a positive effect. This gap undoubtedly would have been even greater had more sophisticated higher-order rules been introduced.

Transfer. The hypotheses pertaining to transfer were equally conclusive. As expected, Curriculum H subjects performed just about as well on tasks found only in Curriculum D as did the Curriculum D subjects who were trained on these tasks directly. The Curriculum D subjects were successful on 168 out of the 181 posttest tasks (93%) in this category, and the Curriculum H subjects were successful on 167 out of the corresponding 190 tasks (88%). The difference between these two proportions was not significant (arc sine transformation, \( 0 < Z < .5, p > .05 \)). The overall means were 8.3 (Curriculum D) and 8.0 (Curriculum H) with a maximum score of 10 \( (F = 1.02, df = 1.44, p > .05) \).

In addition, the Curriculum H subjects, as predicted, performed at a significantly higher level \( (F = 30.03, df = 1.44, p < .001) \) than the Curriculum D subjects on tasks beyond the scope of either curriculum. On the six tasks where solution rules could be derived from given rules via the higher-
order rules, the obtained means were 3.1 (Curriculum D) and 4.1 (Curriculum H). The Curriculum H subjects were successful on 98 of the 144 transfer opportunities provided (68%), and the Curriculum D subjects were successful on 74 of the 144 (51.3%). These proportions differed significantly (arc sine transformation, $Z = 2.9$, $p < .01$).

Relation between transfer and mastery on higher- and lower-order rules. It is also worth noting that the difference in group performance of about 17%–18% on the higher-order tasks was directly reflected in performance on the six transfer tasks that were neither in Curriculum D nor Curriculum H. There too the difference was about 17%–18%. This observation suggests that subjects who had (directly or indirectly) learned a higher-order rule were able to apply it successfully to transfer tasks. In effect, it would appear that the availability of a suitable higher-order rule, together with appropriate lower-order rules, provides a sufficient basis for transfer to new tasks.

A more detailed analysis showed that (a) of the 235 cases where subjects were successful on a higher order task, they were successful on 166 of the corresponding transfer tasks, giving 71% correct prediction; and (b) of the 101 cases where subjects were unsuccessful on a higher-order task, they were unsuccessful on 72 of the corresponding transfer tasks, again giving 71% correct prediction. These findings are generally compatible with a number of related “laboratory” experiments (Scandura, 1973b). Although predictability did not approach the levels obtained there (86%–100% accuracy), these results demonstrate the robustness of the theory.

Other data showed that transfer was not affected by the direct presentation on the transfer tasks of the necessary lower-order rules. (The rules to which the higher-order rules applied were presented directly on four of the six transfer tasks.) The Curriculum D subjects were successful on 48 out of 96 transfer problems (50%) in which the needed lower-order rule was presented and 28 out of 48 transfer problems (54%) in which the subjects were trained on the needed lower-order rule but were not formally presented with it on the test. Correspondingly, the Curriculum H subjects were successful on 66 out of 96 transfer problems (69%) and 32 out of 48 transfer problems (67%). Neither pair of proportions differed significantly ($Z = .59$, $p > .05$ for the Curriculum D differences and $Z = .29$, $p > .05$ for the Curriculum H differences). This suggests that memory was not an essential factor, at least under the present conditions where the level of mastery on the lower-order rules was about 95%.

Summary and Implications

In summary, these results clearly show that rules provide a viable and explicit basis for instruction and transfer. The Curriculum H subjects not only had fewer rules to learn than did the Curriculum D subjects, but they were also able to solve tasks that the other subjects could not. The strong transfer effects were obtained even though the six-week course came to an end just as many of the Curriculum H subjects were reaching that portion of the workbook where the higher-order formulation had made it possible to eliminate large numbers of Curriculum D tasks (cf. Table 1). If time had permitted some of the subjects to complete Chapters 8 and 9, it seems quite possible that an even greater difference might have been obtained in favor of the Curriculum H subjects in amount of material covered per unit of time.

These results suggest that the algorithmic approach should be given serious consideration in planning future curriculum development. Curricula that are characterized in terms of rules and higher-order rules provide an explicit basis for instruction and, even more important, make specific provision for remote transfer, something which many subject-matter specialists feel is lacking in current curricula based on operational objectives.

The present study, of course, demonstrates the viability of the algorithmic approach only with respect to certain parts of mathematics. Further research is needed to determine the feasibility of the approach with other subject matters. A study on criti-
eral reading by Lowerre and Scandura (1973) is particularly relevant in that it was based on a short-cut approximation of the algorithmic approach to curriculum development called dimensional analysis.

It also should be emphasized that while knowledge can be rigorously characterized in terms of rules, this does not imply that knowledge must be imparted to students (e.g., young children) in this manner. As in this study, rules can be acquired by telling or by discovery from instances or, in other cases, by symbol juggling or concrete manipulation. The choice is up to the teacher and depends on factors other than the particular knowledge in question. Instructional formats other than workbooks, of course, should also be explored. The important point is that if we know precisely what (rule) it is that we want a child to learn, then we can facilitate learning far better than if we do not.

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