On conceptual foundations for Mathematics Education

by Joseph M. Scandura, University of Pennsylvania U.S.A.

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As a result of recent research findings showing a decline in computational and other mathematical skills, the public and a growing number of mathematics educators have become disenchanted with the so-called "new mathematics." Many apparently feel that the new curricula have been too abstract for the broad middle range of mathematics students. While this has not necessarily been a result of the new curricula themselves, but rather of the way they have been taught, mathematics educators have begun to give increasing attention to concrete, manipulative experiences in more open classroom settings. Implicit in such suggestions is the continuing belief that teaching is to be treated primarily as an art. Given this milieu, in their search for adequate conceptualization, it is natural that many mathematics educators have turned to Piagetian theory and the clinically based research on which it is based.

In line with Piaget's early and continued arguments against early S-R behaviorism, many mathematics educators have come to feel that any approach to curriculum which involves behavior must be diametrically opposed to current clinical, development-oriented approaches - and more important, that they must be inadequate. It is undoubtedly true that the behavioral objectives and hierarchical approaches to curriculum construction that are being used today have many limitations - as is true of clinical method I might add. This is not necessarily true of all current operational (behavioral) theories of complex human behavior, however. To the contrary, some new theoretical developments in structural learning provide a potential basis not only for extending and generalizing the behavioral objectives approach in mathematics education but of providing a rigorous integrated conceptual foundation within which to view all complex human behavior - including Piagetian research.

Among the ways in which behavioral objectives are felt to be inadequate for mathematics education are the following:

(1) Specifying only the behavior a child is capable of after learning leaves the "guts" out of the learning. For example, consider the child who has learned to place the decimal point in addition according to his own inadequate system.* He responds correctly,

*Bob Davis, personal communication.
say, with problems like \( .4 + .3 = .7 \) and \( .2 + .7 = .9 \). The system works fine here. But when we ask the child to add \( 3. + .2 \), he responds "5.".

(2) The learning which results from manipulating concrete objects (e.g., via concrete embodiments) cannot readily be formulated in behavioral terms. Consider, for example, conservation of number as typically treated by Piaget. Here, the concern is not just with what the child says when presented with a pair of sets to be compared as to number, but also with the basis for his responses.

(3) The more global, high level abilities and processes characteristic of mathematical thought cannot meaningfully be formulated in terms of behavior. Logical reasoning and the processes of discovery are often cited as two examples of high level abilities not easily handled in operational formulations.

(4) Real problem solving cannot sensibly be handled within the behavioral framework. Why is it that some people can solve problems when they know the requisite component skills whereas others cannot? The formulation of problems and the construction of subgoals is an even more complex process presumably which does not lent itself to behavioral formulation.

(5) Similar concerns have been voiced concerning motivation, attitudes, and social interaction.

Without attempting to detail my justification (for relevant but by no means exhaustive discussion, for example, see my new book on structural learning)** let me simply say that such statements today should be made only with extreme caution.

(1) Unlike the Piagetian formulation in which the link between theory and observables is often obscure, and unlike the operational objectives approach which is essentially devoid of theory (cognition), cognition and behavior are closely tied in newer structural approaches to psychology. In such theories, specifying behavior alone is not sufficient. The knowledge which makes that behavior possible must also be specified.

In this regard it is important to know that while the precise specification of such knowledge is usually accomplished by use of some formal algorithmic language, this does not imply that the knowledge must be imparted to children in the same manner. The same knowledge can frequently be acquired by telling or by selfdiscovery, by symbol juggling, or by concrete manipulation - the choice is up to the teacher and depends on factors other than the particular knowledge in question. For example, it may depend on whether the teacher during the course of learning wants the student to also gain experience in discovery.

There are two important points here: (a) if we know precisely what it is we want the child to learn, then we can facilitate learning far better than if we do not. (b) "Kno

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wing" the mathematical content involved is not equivalent to specifying the relevant knowledge. The former (in b) refers to an intuitive understanding and ability to use the content whereas the latter refers to the ability to describe in some suitable language such understanding and ability.

In the structural learning theory, and in computer science generally, knowledge is formulated in terms of procedures (rules). The difference here between knowing something and being able to describe that knowledge is quite analogous to the difference one might expect between a skilled mathematician and a skilled mathematics educator. Indeed, if there is any justification for a separate discipline of mathematics education and I believe that there is, then it is largely (if not precisely) because the mathematics educator must be highly skilled at specifying the knowledge underlying a wide range of behaviors associated with school mathematics.

The adoption of this point of view has a number of important corollaries relating to the other criticisms above.

(2) It becomes apparent that specifying the knowledge underlying concrete manipulation is essentially no different from symbol manipulation procedures such as computational algorithms. Rather than being contrary to Piagetian and other clinical approaches, this approach provides a potential basis for more rigorous analysis of such dynamic situations as those involved in a number conservation (e.g., see Scandura, 1972) and other student-teacher interchanges.

(3) While high level processes have traditionally been dealt with at a strictly intuitive level, there are strong indication that such processes are not beyond precise specification. In computer simulation and artificial intelligence, for example, many heuristics have been identified which could well parallel human processes. In research directly tailored to human abilities, high level heuristics involved in straightedge and compass geometry construction problems have been identified (Scandura, Durnin, & Wulfeck, 1972), knowledge underlying the ability to reason logically with written discourse has been both assessed and imparted in systematic fashion (Lowerre & Scandura 1972), some of the simpler high level rules underlying entire mathematical curricula have been identified and successfully field tested (Scandura, Durnin, Ehrenpreis, et al., 1971), and human-like rules have been devised for constructing a wide variety of sentences given the intended meanings of the speaker (Carroll, 1973).

(4) Regarding problem solving, a growing amount of research suggests that people who know all of the components of a problem solving procedure can solve the problem if and only if they know appropriate higher order rules (which operate on classes of given rules and generate new ones) (Scandura, 1973a, 1973b). Although details need to be worked out (Scandura, 1973a, p. 348), there is no reason to believe that the formulation of problems (e.g., the creation of subgoals) cannot be treated within the same theoretical framework.
(5) Less can be said about attitudes and the like but this appears to be a direct function of minimal research effort to date rather than in principle conceptual inadequacy.

In summary, although its development is still in the early stages, the newer structural learning approach to cognition provides a basis for curriculum and instruction development teacher education which is not only operational, and in that sense provides an extension of current behavioral approaches, but which is also quite compatible with the more clinically oriented Piagetian based "open" approaches to education. Because it provides a potentially more efficient, systematic basis for devising curricula and planning instruction more attention to and further development of such research should in the future pay handsome dividends to mathematics education.

References


