Reprint from:

Hans Spada/Wilhelm F. Kempf (Eds.)

Structural Models of Thinking and Learning

Hans Huber Publishers Bern Stuttgart Vienna 1977
A Deterministic Theory of Learning and Teaching*

Joseph M. Scandura

In presentations at the 1970 AERA and Structural Learning Conferences, I proposed that deterministic theorizing in behavioral science might actually be an easier and more productive first step than stochastic theorizing (SCANDURA, 1971a). As justification, in view of the then all-pervasive influence of stochastic theorizing, I described the nature of deterministic partial theories, the idealized conditions under which they must be tested and the general nature of their relationships to stochastic theories. I then went on to introduce the rudiments of a comprehensive partial theory of structural learning, along with preliminary tests of various aspects of the theory and some applications to education. This theory has since been elaborated, refined and partly formalized (SCANDURA, 1973a), extended (SCANDURA, 1974, 1976), and applied to a variety of educational problems (e.g., DURNIN & SCANDURA, 1973; EHRENPREIS & SCANDURA, 1974; LOWERRE & SCANDURA, 1973; SCANDURA, 1972, 1973b; SCANDURA, DURNIN & WULFECK, 1974; WULFECK, this volume).

This chapter is organized into sections as follows:
1. Deterministic and probabilistic theorizing,
2. A deterministic theory of teaching and learning,
3. Curriculum analysis (theoretical foundations, structural approach to curriculum construction),
4. The learner (individual differences measurement, cognitive constraints-control and processing capacity),

* Preparation of this report was supported by the IPN at the University of Kiel and N.I.H. Grant HD09185-01 to the author.
5. Optimizing instruction,
6. A brief summary of directly related research.

1. Deterministic and Probabilistic Theorizing

In this section, I shall restrict myself to a few basic points concerning the relationships between deterministic and probabilistic theorizing in behavioral science. (Direct quotations below are all taken from SCANDURA, 1971a. For further discussion, see SCANDURA, 1976a, 1976b, Chapter 1; HILKE, KEMPF, & SCANDURA, this volume).

"Consider the paradigm most typically used in testing behavioral theories. First assumptions are made about how individuals learn or behave .... (Typically such assumptions are stated in terms of probabilities.) Second, inferences are drawn from these assumptions yielding predictions about group statistics. Third, on the basis of obtained results, inferences are made about the basic assumptions (i.e., individuals) .... What needs to be made clear is that what any given subject does on a given occasion may have little to do with the particular assumptions made. For example, in stochastic models of paired-associate learning it is usually assumed that each subject has the same probability of learning on each trial. Even the most superficial analysis of relevant data, however, indicates clearly that the probability of success for different subjects may vary greatly."

Although individual differences play a central role in more recent research in mathematical psychology, such as that reported in this volume, a purely stochastic approach as a cumulative enterprise has certain important and inherent limitations. In order to account for more and more variance (e.g., more of the underlying psychological processes) stochastic models necessarily become more and more complex. As a consequence, literally hundreds of subjects may be required to estimate pa-
rameters in order to obtain sufficient stability.

In contrast to most stochastic theories, contemporary deterministic theories in behavioral science (e.g., NEWELL & SIMON, 1972; SCANDURA, 1971, 1973) deal directly with individual processes. They provide a basis for explaining and predicting what individuals will do in specific situations. On the other hand, deterministic theories are intrinsically idealizations and, hence, deal necessarily only with portions of reality. They may aptly be referred to as partial theories (cf. SCANDURA, 1971).

Unfortunately, these differences have not adequately been taken into account in testing current information processing (deterministic) theories in psychology. As in testing stochastic theories, the effective goal of most experimentation has been to determine which (of one or more) processes (deterministic theories) best account for average, group behavior. In view of the above, there is a serious problem with this approach to theory verification. Not only does average, group behavior fail to say anything definitive about individual behavior in specific situations, but the very fact of collecting it denies the very nature of deterministic theories.

To make the point more poignantly, consider an example from physics: Galileo's famous experiments at the Leaning Tower of Pisa. Suppose instead of dropping two iron balls of different weights, that Galileo had dropped an iron ball and a feather. How different his results would have been. Moreover, if counseled by a twentieth century behavioral scientist - one committed to a stochastic approach to science, Galileo might have further perverted such findings (to insure reliability) by comparing the average rates of fall of 100 iron balls and 100 feathers. (Note parenthetically that the results of such an experiment in another age might well have led to an experimental rush to uncover the scientific laws governing the rates of fall of various types of droppings.)

Although such findings conceivably could have immediate in-
terest (and possibly even practical value), it is unlikely that they would have inspired Newton in his attempts at theoretical synthesis. Of much more direct interest were Galileo's actual experiments, which were more directly analogous to what one might expect to happen under idealized conditions — say, where a feather and an iron ball are dropped in a perfect vacuum. The point is that the much simpler results associated with Galileo's actual experiments, where gravitational force was essentially uninfluenced by other forces, had far greater and longer-lived generality — far greater and longer-lived than alternative scientific laws concerning droppings.

One cannot help but wonder in contemporary cognitive psychology whether specific laws (rules/information processing theories) introduced to account for unconstrained average behavior (e.g., droppings in the atmosphere) could in the long run have far less generality than simpler deterministic laws (determined under idealized behavior conditions).

The problem, of course, is to determine just what these idealized conditions might be. EBBINGHAUS, for example, thought that nonsense syllables might provide a promising way to study human learning processes. After years of debate and research, it is increasingly recognized that this was a false hope. The years of prior learning and experience, which each subject brings into any learning situation, affect learning in fundamental ways, even with respect to supposedly unfamiliar material.

Another set of idealized conditions, one which we will return to later, involves human behavior in situations where the information processor (the subject) effectively has a perfect memory for relevant information and an unlimited capacity for processing information.

"In constructing a deterministic partial theory, it is critically important that the theorist do so in a way which is compatible with the broader reality (he seeks ultimately to explain (p. 26)). For example, the ultimate aim of curriculum designers/
structural analysts is not just to analyze content - that in itself might be attempted in any number of different ways. What is needed is a theory of content analysis which is likely to be compatible with a more encompassing theory of instruction once one is developed.

To do this, it will generally be in the theorist's interest to know just what aspects of reality his present theory does not consider. Stated differently, he must know what boundary conditions must be satisfied in order for his partial theory to apply.

"Given (an adequate, comprehensive, and deterministic) theory ..., probability should enter only where one wants to make exact predictions in real world situations where the experimenter practically speaking cannot, or does not wish to, find out everything he would need to know and to specify. In effect, a truly adequate deterministic theory would make it possible to generate any number of stochastic theories by loosening one or another of various conditions which must be satisfied in order for that theory to apply. (p. 24)." (Although I did not state it explicitly at the time, what I had in mind were complementary, stochastic observation theories - see below, also HILKE, KEMPF, & SCANDURA, this volume; REULECKE, this volume).

A deterministic theory might generate predictions concerning data collected under non-idealized conditions in several ways. One way would be simply to convert deterministic statements about individuals directly into analogous probabilistic statements by adding a randomly distributed error term. In effect, group data would be compared with transformed but nevertheless deterministically derived predictions using standard statistical methods. This type of conversion is implicit in most deterministically formulated theories in the behavioral sciences, particularly where the theories have been formulated informally. A study by EHRENPREIS & SCANDURA (1974) is also of this type but it is based on a deterministic theory which has under idealized conditions been shown to predict in near deterministic
fashion (SCANDURA, 1974a). In this case, presumably, deviations from assumed idealized conditions would be reflected in degree by obtained deviations from deterministic predictions.

A second perhaps more interesting approach is to introduce random parameters and, thereby, to narrow the gap with stochastic theories of mathematical psychology. This might be accomplished in various ways, one of which might be direct analogy. That is, convert deterministic assumptions about behavior into corresponding probabilistic assumptions. The SUPPES & MORNINGSTAR (1972) model (see SPADA, this volume), for example, can be generated from the deterministic method for assessing behavior potential in the structural learning theory by substituting probabilities for atomic (0 or 1) assumptions.

This approach, however, does not distinguish between data which is totally consistent with the deterministic theory and data which is not. Taking into account such requirements as specific objectivity (see SCHEIBLECHNER, this volume), HILKE, KEMPF, & SCANDURA (this volume) proposed a class of stochastic models which is consistent with a weakened form of the deterministic theory and which, thereby, makes such a distinction possible. The individual parameters in this class of models reflect confoundings of (internal, external, and structural) deviations from the ideal and characteristics of items and individuals. In this sense, they correspond to parameters in observation theories dealing with individuals which complement the deterministic structural learning theory (by dealing with deviations from the ideal).

As noted previously, the application of such stochastic models requires large numbers of subjects. In applications where such numbers cannot be obtained, probably most cases, it is important to have simpler observation theories in which the parameters are related to some defined population of subjects (in which individual differences are neglected) and correspond to just those idealized conditions, which are allowed to vary. More specifically, we want parameters which reflect the degree
to which specific idealized conditions are allowed to vary in empirical test situations.

Suppose, for example, that an experiment ideally should be run under memory-free conditions but that the experimental conditions nevertheless require recall of previously learned information. Then, parameters could be introduced to allow explicitly for this. Moreover, it might be possible to devise independent, analytically (rather than empirically) determined metrics (e.g., number of learning trials; number and magnitude of distraction during testing) which reflect the degree to which specific idealized conditions vary in given empirical situations. If so, this would make it possible to analytically derive corresponding values of needed parameters in new situations—for example, via interpolation or extrapolation from given metric values, which reflect deviations from the ideal, and known values of observation theory parameters (determined in other situations). Derived parameter values might reasonably be expected to have predictive value in new situations to the extent that the metrics accurately reflect deviations from the ideal and that the deterministic theory itself is adequate.

One stochastic theory, which was formulated with some of the above considerations in mind, is proposed in SCANDURA (1973a). In this model, the chunking and distraction parameters, α and β, refer directly to the influence of such deviations. In preliminary tests of the model (VOORHIES & SCANDURA, 1976), the values of α and β were found to depend directly on the degree to which the empirical situations deviated from the ideal. In effect, the model is a theory about observables, one which complements the structural learning theory. The model proposed by REULECKE (this volume) to deal with "unsharpness" is also of this type. In this case all deviations from the ideal are summarized in terms of a single parameter.

It has sometimes been assumed implicitly that one can compare deterministic and stochastic theories directly.

For example, one might be tempted to propose that a stochas-
tic theory provides a better account of certain data than a deterministic theory, if the stochastic theory does a better job of predicting average behavior (which it is designed to do), better than a deterministic theory predicts the non-idealized behavior of individuals in specific situations (which it is not designed to do).

Clearly, such an argument would miss the point. Any reasonable comparison of theories must be with respect to the same standards. The problem is not to determine whether one type of theory is better - a completely deterministic account of the same phenomena would obviously provide more complete understanding. On the other hand, stochastic assumptions would appear essential to fill the inherent gap in most situations between idealizations and empirical reality. The problem is to find deterministic theories that work - that is, theories which approach deterministic predictions under (near) idealized conditions but which also are sufficiently robust to allow strong (probabilistic) predictions, stochastic generalization with respect to real world phenomena, and/or complementary theories of observation.

2. A deterministic Theory of Teaching and Learning

Any theory of teaching and learning must include as a minimum:
(a) an underlying theory of learning,
(b) a way of measuring student progress,
(c) the assignment of values to educational objectives and costs (e.g., time/money) to instruction, and
(d) some way to maximize value per unit cost (e.g., ATKINSON, 1972).

Although necessary, however, these ingredients are not sufficient. For example, in contrast to most formal models of teaching and learning, which are very limited in scope, a truly viable and comprehensive theory must deal with instructional do-
mains of arbitrary generality.

In principle, such a theory must allow for the full variety of possible educational goals: facts, rules, content structure, communication and other processing skills, problem-solving strategies, logical reasoning, aptitudes, motivation/attitudes, and values. Because many of the most important educational goals cannot easily (if at all) be stated explicitly (e.g., a "feeling" for music), this will necessitate, I think, the capacity for implicit (but potentially operational) characterization of educational goals and not just finite lists of specific objectives (e.g., the ability to add pairs of numbers from 0-9).

A completely adequate theory of teaching and learning also will have to provide a suitable way to characterize relevant characteristics of the learner. As a minimum, I do not see how this can be accomplished:

(a) without direct and explicit reference to underlying cognitive processes and to the constraints which influence cognition and

(b) without an approach to individual differences measurement which recognizes that individuals do not enter educational situations as a tabula rasa.

Among other things, the former implies a cognitive learning theory which deals with how available knowledge interacts in producing behavior and how it is modified. The latter implies some way to determine and measure all of the knowledge that is both relevant in an educational setting and available to individual learners.

To insure consistency with the above requirements for comprehensiveness, such a theory should provide for the assignment of values to broad educational goals and not just to specific objectives of behaviors. In addition, general methods must be developed for assigning costs to various kinds of instruction, and for optimizing instruction.

In many ways most important, however, is the general approach. Whereas almost all existing theories of teaching and learning
are based on the assumption that human behavior is essentially random, I do not believe that it will be practical to devise stochastic theories, which satisfy the above conditions, until we first achieve deterministic understanding (under idealized conditions). In effect, we need a deterministic theory which makes it possible, in principle, and under idealized conditions, to explain, predict, and control (subject to boundary conditions) the learning of individual subjects with respect to specific content domains.

Further, and in accord with the levels of empiricism notion on which the structural learning theory is based (SCANDURA, 1971a, 1973a), the practical utility of such a theory would be increased were it to consist of a series of deterministic partial theories, hierarchically related according to increasing structure (and, correspondingly, according to decreasing restrictiveness of the boundary conditions which must be met to insure the necessary idealized conditions for testing). In educational applications, the inherent gap between idealizations and reality is filled by "professional know-how" (SCANDURA, 1972, 1973b).

While it would be presumptuous to imply that the to-be-proposed theory satisfies all of the above requirements, it is thought to be a step in this direction. In some respects, the theory has achieved a degree of rigor and empirical support that would convince most potential critics. In other respects, however, the theory represents hardly more than a sketchy map and a hunting license with the promise of plentiful game.

Discussion of the theory is divided into three sections. The first deals with educational curricula/content analysis; the second section, with the learner (i.e., individual differences measurement and a cognitive learning theory); and the third, with problems of optimization. The first section begins with some preliminaries upon which the rest of the theory rests. Directly related empirical research is summarized in the concluding section.
3. Curriculum Analysis

3.1. Theoretical Foundations of Educational Curricula

In my view, any scientifically defensible approach to educational objectives and to content analysis must satisfy certain basic conditions.

It must be possible, in principle, to determine through observation whether or not any given educational goal has been achieved. We assume, for example, that the stimulus situations, which effectively influence relevant behavior, can be represented externally (i.e., made observable) for purposes of testing.

The stimulus situations and the behaviors associated with curricula may be rather complex. (Notice in this regard that no commitment need be made as to the sufficiency of paper and pencil tests.) One such "stimulus", for example, might be a complex diagram representing the circulation system in man, such as in one of the IPN biology units (EULEFELD, KATTMANN & SCHAEFER, 1975), even though this system itself might take some time to acquire. More generally, the stimulus situations and behaviors may consist of complexes of arbitrary entities and relations, which in traditional content analysis are often referred to as relational nets (cf. SCANDURA, 1975, 1976). Internally, such complexes consist of elements (degenerate rules), which correspond to external entities; rules, which correspond to relations; and higher order rules, which correspond to relations between relations; all of which collectively act as psychological units or "chunks" (e.g., see SCANDURA, 1975, p. 509; 1976).

Generally speaking, some educational goals can only be loosely formulated (e.g., students should "understand" arithmetic). We shall require only that it be possible to specify criteria which make it possible to determine whether or not a given task is an instance of an educational goal. In this case, it is rea-
sonable to assume that any curriculum may be represented as a finite set of goals.

Since time, capacity of the human mind, and hence instruction are necessarily finite, we assume that all educational objectives (in contrast to goals) are necessarily computable.* More exactly, given any educational objective, there is a rule/procedure/algorithm, or equivalently a set of procedures, which accounts for that objective. (This corresponds to Pat SUPPES' (this volume) comments concerning register machines.) It must be emphasized, however, that any complete account of an objective requires not only a set of procedures but some "laws of combination" which tell how the individual procedures may interact in accounting for specific instances. The latter correspond to control mechanisms in computer science.

It is a mathematical fact that if one procedure exists which accounts for an objective, then there is an infinite number of others which do the same thing (e.g., see ROGERS, 1967).

Since it would be impossible to teach all possible rule sets (there is an infinite number), the "teacher" must select for instruction those that he wants the students to know (i.e., those that are consistent with his values and which satisfy his sensibilities regarding the internal structure of the discipline).

Each curriculum must be accompanied by a set of basic assumptions which characterize the entering capabilities of all students in the target population at the beginning of instruction. Some entering capabilities are assumed to be available to all

* Notice the implied distinction between educational objectives and the more general notion of educational goal. Although learnable knowledge is assumed to be algorithmic, (i.e., all knowledge is assumed to be representable in terms of procedures), one can easily conceive of educational goals which are not computable and, hence, which can never be completely mastered. Thus, for example, it is well known in mathematics, that no procedure even exists for deciding whether or not an arbitrary formula in number theory is a theorem. Where educational goals are not computable, the educator's main concern is to determine one (or more) rule account(s) which adequately serves his purposes.
students (e.g., for readers of the SPADA-KEMPF book the ability to read standard scientific English). Other entering capabilities, which, while not uniformly available, are of a sufficient-ly elemental character that they can be assumed to be either uniformly available to individuals in the population or not at all (e.g., the ability to perform simple carrying in subtraction, knowledge of particular words). I have referred to the latter as atomic rules.

For some educational objectives, it is possible to identify single procedures which satisfy the above requirements. Such educational goals correspond directly to what have been called "behavioral/operational objectives".

But not all educational objectives have single solution rules that are educationally viable (e.g., it may be difficult or impractical to identify one explicitly). In this case, it may still be possible to identify a finite set of rules, which collectively accounts for (all or most of) the objective.

Moreover, the number of viable possibilities is limited further by our concern for (compatibility with) a broader reality (which includes basic cognitive processes). In particular, in accounting for educational objectives/goals, we assume that rules are allowed to operate on other rules and that the derived rules, in turn, may be used to generate appropriate behaviors to given stimulus situations. This mode of interaction is assumed to be compatible with human cognition, a promisory note that I will try to discharge later on.

Let me illustrate some of the kinds of higher order rules that may underlie educational curricula and try to make the assumed laws of interaction more precise.

(1) \[ \{ xB \rightarrow By \mid x = \text{string of } a's, y = \text{binary numeral} \} \]

representing no. a's

Two sample instances are aaaaaB → B101 and aaaB → B11.

Given the space available, let us consider the simple symbol goal in (1). While hardly a viable objective itself, real educational objectives share the same abstract structure. Rule set (2) provides an adequate account of curriculum (1).
(2) \[ A = \{r_1, r_2, \ast\} \]

where \( r_1 = xxBy \rightarrow xBOy, \ r_2 = xxaBy \rightarrow xBly, \)

and \( \ast \) is a higher order composition rule. ¹

In particular, for each task in (1) there is either a rule in \( A \)
which generates the corresponding behavior, or such a rule may
be derived by application of rules in \( A \) to other rules in \( A \).
More specifically, a rule set \( A \) is said to account for a task
instance if for some finite number \( n \) there is a rule in \( A^n \)
which "solves" the task

\[ A = \{r_1, r_2, \ast\} \]
\[ A^2 = A(A) = \{r_1, r_2, \ast, r_1 r_1, r_1 r_2, r_2 r_1, r_2 r_2, \ast\ast\} \]
\[ A^3 = A^2(A) = \{r_1, r_2, \ast, r_1 r_1, \ldots, r_2 r_2 r_2, \ast\ast\ast\} \]
\[ \ldots \]
\[ A^n = A^{n-1}(A) \]

For example, consider the task instance aaaaAB \( \rightarrow \) B101. In this
case we see that \( \ast \) applied to itself (twice) gives \( \ast (\ast, \ast) = \ast\ast \).
In turn, double composition \( \ast\ast \) applied to \( r_1 \) and \( r_2 \) (twice)
gives \( \ast\ast(r_2, r_1, r_2) = r_2 r_1 r_2 \). Finally, \( r_2 r_1 r_2 \) applied to aaaaaB
gives

\[
\begin{array}{ccccccc}
  x & x & r_2 & x & x & r_1 & r_2 \\
  aa & aa & aB & a & a & B1 & aBO1 & \rightarrow & B101
\end{array}
\]

Although this formulation illustrates the general approach, I
should caution that it is somewhat restrictive. Specifically,
newly generated rules in \( A^n \) may be applied only to rules in \( A \);
hence the notation \( A^n(A) \). As is shown in SCANDURA (1973, 1976),
the following slightly more general form of interaction is re-
quired in dealing with cognition.

¹ As used in this example, \( \ast \) is both slightly more general and slightly
more restrictive than composition as normally defined in mathematics.
Such details will not concern us here, cf. SCANDURA, 1973, Chapters 5
and 9.
(4) $A^N(A^M)$

Furthermore, it is neither reasonable to assume that composition (or any other psychologically relevant rule) is necessarily universal in its applicability, or that composition is the only type of higher order rule that might be included in a rule set.

For example, consider the higher order rule

(5) $r_a \Rightarrow r_b$

Rule (5) operates on rules involving $a$'s and converts them into corresponding rules involving $b$'s. For example, this rule operates on $r_1$ and generates $r_a \Rightarrow r_b(r_1) \Rightarrow x'x'By \Rightarrow x'BOy$ where $x'$ is a string of $b$'s. The addition of just this one rule to set $A$ would double its generating power to include an equivalent set of tasks involving $b$'s instead of $a$'s. Further, every time a new rule involving $a$'s is added to the rule set, overall generating power is increased to include not only the corresponding tasks involving $a$'s but a parallel set of tasks involving $b$'s. (For concrete examples, please see EHRENPREIS & SCANDURA, 1974; SCANDURA, DURNIN, & WULFECK, 1974; SCANDURA & DURNIN, 1976).

3.2. Structural Approach to Curriculum Construction

In order to identify underlying competence (i.e., rule sets underlying given curricula), the curriculum constructor must be thoroughly familiar with both the content and the relevant "culture" and developmental level of the intended student population. For complex curricula, even given such familiarity, it will generally be impossible to identify any rule set which is completely adequate (just as in linguistics there is no perfect grammar).

Nonetheless, the above constraints on competence make it possible to proceed in a quasi-systematic manner. Furthermore, experience in applying this method suggests that it is at least
partially self-correcting.

For example, consider "symbol curriculum" (6) which consists of four educational objectives/goals.

\[
\begin{align*}
&B - \text{By} | x = \text{string a's, y = binary numeral} \\
&B' - \text{By} | x' = \text{string b's, y = binary numeral} \\
&T - \text{z} | \text{no. a's in } w = \text{no. b's in } z
\end{align*}
\]

X (complementary, incompletely defined ("fuzzy") set of tasks involving a's and b's)

The first step in analyzing a curriculum is to select a representative (finite) sample of tasks from the curriculum. The sample need not be exhaustive, although in practice it is desirable to include all tasks which the curriculum constructor deems both important and qualitatively different in some sense from other tasks. In the present case, suppose we select

\[
\begin{align*}
\text{aaaaB - B101} \\
\text{aab - B10} \\
\text{bbbbB - B11} \\
\text{bbbbB - B100} \\
\text{aaT - Tbb}
\end{align*}
\]

The next step is to devise solution rules for the sampled tasks which are consistent with how the curriculum constructor feels that a knowledgeable student should or might be expected to solve them. The theory is relatively neutral as to how this is done: via teacher fiat, subject matter intuition, informal observation of typical subjects, or through more systematic observation methods. The one major constraint is that all component operations and decisions must be atomic relative to the student population. (In addition, certain capabilities will ordinarily be assumed of all students.) Since the inclusiveness of such units varies directly with population sophistication, the complexity of procedures cannot be determined in the abstract. Thus, for example, whereas it might be difficult to detail the competence underlying a particular job analysis in sufficient detail for an unselected group of individuals, it might be a relatively straightforward undertaking for an ade-
quately prepared group of trainees. Hopefully, as we identify more and more constraints and subsequently incorporate them into our method of analysis, progressively less intuitive judgement will be involved.

Given the five instances above, of course, the analysis is trivial. Let

$$K = \{ r_2r_1r_2, r_1r_2, r_2'r_2', r_1'r_2', r_{ab} \}$$

be the corresponding set of rules, where $r_1$, $r_2$, and * are as above, $r_1'$ and $r_2'$ parallel $r_1$ and $r_2$ with a's replaced by b's and $r_{ab}$ replaces each a to the left of T in the input with one b to the right of T in the output.

Clearly, K accounts for some of the task instances in (6), but it does not account for all of them — for example, bbbbbB-B101. In this case, we can either add a new solution rule or search for a more basic rule set from which both the original and the new solution rules may be derived. In general, the decision as to which direction to go will depend on such criteria as atomicity, assumed entering capabilities, and scope of the intended curriculum. For example, if a curriculum appears to be limited to the original and a small number of newly sampled tasks, it would most likely be sufficient to simply add the corresponding, new solution rules. In any case, the possibility of checking the adequacy of a given rule set at each stage, makes the analysis to some extent self-correcting.

In attempting to identify a more basic rule set from which the initial set (K) can be generated, the main things to look for are parallels among the initial rules.

Such parallels may involve identical components, analogies, greater/lesser generality, and/or relationships between rules, including their component operations, domains/decisions, and/or overall structures. For example, observing the central rule played by composition and noting that all of the solution rules may be accounted for in terms of a set of simpler rules, we get

$$K' = \{ *, r_1, r_2, r_1', r_2', r_{ab} \}$$
where $K'$ is an innate basis (SCANDURA, 1973) for $K$ since $K = (K')^n$ for some $n$. $((K')^n$ contains all allowable $n$-fold composites of $r_1$, $r_2$, $r_1'$, and $r_2'$ - e.g., $r_2 r_1 r_2$.) $K'$ is clearly a far more powerful rule set than $K$. But, it still has important inadequacies. For example, it does not account for a task as simple as bbT-Taa.

Recognizing this, we can further increase potential generating power by introducing $r_a \Rightarrow r_b$, and eliminating $r_1$ and $r_2$ as redundant. This gives the second generation innate basis.

$$K'' = \{ *, r_1, r_2, r_{ab}, r_a \Rightarrow r_b \}$$

Not only does $K''$ account for everything that $K'$ does, but it accounts for all observables of the form zT-Tw (where z is a string of b's and w is an equal string of a's). Further, the addition of any new rule involving a's is tantamount to also adding a corresponding rule involving b's.

Along with the increase in potential computing power obtained in this manner, it is important to notice that greater simplicity of the individual rules has also been achieved. Whereas the individual rules (in $K$) are composed of several atomic (unitary) rules, those in $K''$ are all atomic. (The appropriate level of analysis for a given student population is one at which all rules are either atomic or uniformly available to members of the population.)

There is more to be said about such things as semantics and perception (SCANDURA 1973, 1976), but discussion here would detract from the main message. I only wish to caution that the syntactic character of our example should not mislead one into thinking that the theory applies only to highly structured content. Indeed, I should like to close this part with a rather strong conjecture: Anything which can be taught at all, can be formulated in structural terms - moreover, as a result of such analysis, one can do a better/more efficient job of instruction.
4. The Learner

On that note let me turn to the learner. The competence associated with a given curriculum and student population may be thought of as representing the potential knowledge had by an idealized student on entering into a curriculum.

What a particular learner does and is able to do in a given situation, however, depends not on some ideal but on what he or she knows as an individual. More specifically, in the present theory, individual behavior is assumed to depend on:
(a) that specific and available knowledge which is relevant to the situations in question and
(b) constraints on the use of such knowledge which stem from the basic nature of man as an information processor.

4.1. Assessing Behavioral Potential (Individual Differences Measurement)

It is one thing to say that behavior depends on the relevant knowledge available to a learner, and quite another thing to measure such knowledge. Because of its sheer magnitude, it is obviously impossible to measure all of any individual's knowledge. It is equally impossible to measure all possible individual variations, even with respect to a given modest-sized class of tasks.

A basic assumption in the structural learning theory is that this is not necessary; it is only necessary to measure knowledge with enough precision to distinguish the observables of interest. Such measurement is necessarily relative and depends on the adequacy of our assumptions concerning: atomicity of component rules, entering capabilities, and level of analysis. To see how these assumptions enter, let us review the structural approach to assessing behavior potential and then show how
the approach can be extended to provide a basis for determining entering abilities with respect to entire curricula.

Recall first that any rule can be represented in terms of atomic rules. The critical assumption of purposes of measurement is that the level of refinement is appropriate to the population.

In general, a path through a rule acts in atomic fashion if and only if each component rule acts in atomic fashion. The chain, in effect, is only as strong as its weakest link. Given the assumption of atomicity, there are only a finite number of paths through any given rule.

For illustrative purposes recall the class of tasks (1) and consider solution rule (11)

![Diagram]

where each of the component operations (1 and 2 corresponding to rules \( r_1 \) and \( r_2 \), respectively) and decisions act in atomic fashion. In this case, there are three distinct paths through rule (11), represented by

(11a)

(11b)

(11c)

Paths (11a) and (11b) are obviously atomic since they each involve only a single (atomic) operation. Path (11c) is also atomic since each of its components is.

364
Figure 1: The large circle represents the class of tasks (1). Taken as a whole, this corresponds to superordinate path (11c). Equivalence classes (subsets) a and b correspond to paths (11a) and (11b), respectively. Equivalence class c then consists of those tasks in Class (1) which can only be solved via superordinate path (11c).

Each path makes it possible to generate responses to a uniquely specified equivalence class of stimulus items and to no others (see Fig. 1). The "intersection" of these classes, therefore, effectively partitions the domain. For example, path (11a) (basically) applies to all strings which contain a number of a's equal to some power of two (e.g., aaaaaaaaB) and path (11b) applies to all strings which contain a number of a's equal to $2^n-1$, where $n=1,2,3,...$ (e.g., aaaaB, aaaaaaaB). Path (11c) is superordinate to both and applies to these as well as to all other strings.

The fact that the paths of a rule partition its domain makes it possible to pinpoint through a finite testing procedure exactly what it is that each person knows relative to that rule.

According to the atomicity assumption, it is sufficient to test each person on one item selected randomly from each mutually exclusive (nonoverlapping) equivalence class. Success on any one item, implies success on any other item drawn for the
same equivalence class, and similarly for failure. In effect, individual knowledge is represented in terms of rules, specifically in terms of sub-rules of given rules of competence.

Although this example is a bit too simple to illustrate the point, it is important to note that the level of refinement of a rule into components is critical in determining test efficiency. If a rule is refined too much, testing will be unnecessarily inefficient. On the other hand, if the components are too molar (e.g., assuming rule (11) itself to be atomic), the assumption of atomicity will not hold and testing will correspondingly be imprecise.

In testing (and constructing sub-rules to characterize individual knowledge), it also is important to keep in mind hierarchical relationships among the various paths. In particular, success on a path which is superordinate to other paths necessarily implies success on the subordinate paths. (Note: One path is superordinate to another if the equivalence class associated with the former includes the class associated with the subordinate path.)

Hence, testing on a subordinate path is necessary only where a person fails on all paths superordinate to it. In effect, the number of test items required to measure knowledge may be reduced accordingly through sequential testing.

Let me emphasize that the knowledge attributed to different individuals may vary even where only one rule of competence is involved. The idea is directly comparable to measuring different distances with the same ruler.

The competence underlying any realistic curriculum, of course, will involve more than just one rule. To determine the associated knowledge, it might appear sufficient on first thought to simply test for each rule separately. If one takes into account the different levels of structural analysis that are possible, however, the situation becomes more complex.

Specifically, recall that a given level of structural analysis for a given curriculum is appropriate for a population only
if the component rules are essentially atomic (whereas at preliminary levels of analysis the rules are more complex). If one tests only on rules associated with a preliminary level of analysis, then failure on a particular path or rule may be due to failure on just one of the component atomic rules or on all of them. Thus, a student might fail on items associated with complex paths but because of the molar level of analysis, it would be impossible to determine the sources of such failure: no distinctions among unsuccessful testees would be possible. As noted below, such differences can be shown to have an important effect on transfer.

Nonetheless, testing with respect to preliminary levels of analysis may serve to increase the efficiency of testing, much as with superordinate paths of individual rules. For example, if a preliminary rule set is otherwise sufficient (i.e., if it accounts for the given curriculum), then success on a complex rule in the set eliminates the need to determine the availability of more basic source rules from which the complex rule may be derived. Any information obtained by further testing would be immaterial insofar as success on the curriculum itself is concerned.

On the other hand, the finer distinctions made possible by testing source rules may be important for other purposes. Thus, in assessing transfer potential, it may be important to know whether a learner has only a complex capability (rule) relative to the given curriculum or whether he has the potential for acquiring parallel capabilities as well. In our basic illustration, for example, it makes a difference whether a person uses the rule \( r_2r_1r_2 \) to solve problem aaaaaB-B101 directly, or whether the rule must be derived from more basic source rules (e.g., \( r_1, r_2, * \) and perhaps \( r_a \Rightarrow r_b \)). The latter case implies a potential for transfer (e.g., to aaB-B10 or perhaps bbbbbB-B101) that the former does not. Such distinctions can be taken explicitly into account by extending the curriculum to include relevant transfer tasks. (Fine distinctions among rules

367
may also be important in determining latency effects. For a compatible discussion see SCANDURA, 1973, Chapter 8).

4.2. Cognitive Constraints

As we have seen, the appropriateness of underlying rule sets depends partly on the scope of a curriculum. Thus, two different rule sets may account equally well for a given curriculum and differ only on extra-curriculum tasks. In this case the only invariants with respect to structural analysis are the rules of combination.

Such rules of combination, recall, were assumed to be compatible with the way in which human information processors use their available knowledge. In effect, although the knowledge attributed to different individuals may vary greatly, the use of all knowledge is assumed to be governed by the same cognitive control mechanism. It is critical, therefore, to know whether there are realistic control mechanisms which parallel the assumed rules of combination and, even more important, whether such mechanisms are consistent with human behavior.

(In view of the introductory discussion on deterministic theorizing, note that such mechanisms must be tested under appropriate idealized conditions. A natural requirement would be to parallel the conditions assumed in checking the adequacy of an obtained rule set. That is, all rules of knowledge should be uniformly available to the learner, the learner's processing capacity should not be overtaxed, and the learner should have all of the time necessary for responding.)

Of course, the influence of processing capacity, time, and so on may be important in real teaching and learning. Hence, any complete characterization of the learner will necessarily involve more than just available knowledge and an universal control mechanism. (Perhaps surprisingly, these limitations are not as restrictive in educational applications as one might ex-
pect - please see concluding section.)

To date, the only other major constraint specifically considered within the theory involves the generally recognized, strict limitations on human information processing capacity. Among the constraints that might but have not yet been considered are inborn instincts, processing speed, general arousal level (e.g., FARLEY, 1974), and encoding and decoding latencies (e.g. NEWELL & SIMON, 1972).

4.3. Control

Consider first the question of control. The basic assumption is that human beings are goal-directed information processors and that control shifts among various higher and lower level goals automatically in a fixed, predetermined manner.

For our purposes, the mechanism may be thought of, informally, as follows: Given a task for which the learner has a solution rule immediately available (i.e., a rule whose range includes the goal), the learner will apply the rule. (Although this statement appears almost tautological, it is an assumption. Rule use does not follow logically from availability.) When no such rule is available, control is assumed to automatically switch to a higher level goal satisfied by rules which do apply (i.e., rules whose ranges contain rules whose ranges include the original goal). With a higher level goal in force, the learner presumably selects from among available and relevant higher order rules in the same way as he would with any other goal: If the learner has an applicable higher order rule available (i.e., a rule whose range contains rules whose ranges include the original goal), then he will use it. Where no such higher order rules are available, the theory assumes that control moves to still higher level goals. Conversely, once a higher level goal has been satisfied, by application of some rule, control is assumed to revert to the next lower level.
This mechanism is deceptive. While extremely simple, it is potentially quite powerful and, when suitably generalized, provides a basis for explaining a wide variety of behavior, ranging from simple insight learning to breaking problems into subproblems, to motivation, to memory. The mechanism is discussed in detail and formalized in SCANDURA (1973). An updated, more general treatment is given in my forthcoming book on Problem Solving (SCANDURA 1976).


In discussing the control mechanism, no limits were imposed on the number of rules and elements that might be immediately available to the learner during cognition. It was implicitly assumed that human capacity for processing information is essentially unbounded. One need hardly refer to the massive literature on information processing, however, to conclude that the ability of humans to cope with problems depends in substantial part on the memory load imposed. Man is unquestionably a limited capacity information process.

In the present context, what we need to know is how processing capacity interacts with the proposed control mechanism in generating behavior. If the memory load imposed by a task is too great, the task may exceed the subject's capability, even where the subject "knows how" to solve the task. But what constitutes memory load, and how is it determined?

This is a problem on which my students and I have made some progress but, again, space limitations require that I refer you to SCANDURA (1973a) and the chapter by VOORHIES and SCANDURA in my forthcoming book (1976). For present purposes, we can just assume that some limits exist on the amount of goal shifting that is possible.
5. Optimizing Instruction

Optimizing instruction requires building on what we have done so far.

The first step in applying the structural learning theory to instruction is to analyze the given curriculum. The method of analysis involves: sampling a wide variety of tasks; identifying a set of solution rules (R) for solving the tasks; constructing more basic rule sets which consist of higher order and other source rules, and correspondingly eliminating solution rules which can be derived via the source rules; testing and refining the resulting rule set on new problems; and extending it where necessary so that it accounts for both familiar and novel tasks in the curriculum.

This method is reapplied to \( R = R_1 \) as many times as necessary until the individual rules are atomic (or correspond to assumed entering competencies). The resulting rule set \( R_m \) is more basic than the others \( R_1, R_2 \ldots \) (from which it is derived) in two senses: the individual rules are simpler and the rule set as a whole has greater generating power (i.e., it provides a potential basis for solving a greater variety of tasks, SCANDURA, 1973, pp. 114ff).

Once such a sequence of rule sets \( R_1, R_2, \ldots, R_m \) has been generated, the knowledge available to individual students can be determinated by the methods outlined above. In particular, sequential testing can be used with respect to the various hierarchically related rule sets, and paths in individual rules.

Learning may be assumed to take place as the learner interacts with the teaching environment according to the proposed control mechanism. Since this mechanism makes no provision for processing capacity, however, reasonable limits must be placed on goal-switching. Ideally, according to the theory, this should be done in accordance with computed memory loads (SCANDURA, 1973a, 1976), but other approximations may be more practical in real world application. One possibility is that used by WULFECK
(this volume), in which only a fixed number of levels of derivation is allowed.

What is learned at each stage depends both on what is presented to the learner and what he knows. The changes from stage to stage are cumulative. For example, given a basic rule set \( B \), which without loss of generality can be thought of as some student's entering knowledge (i.e., \( B = R_m \)), it is possible to determine by algorithmic means the rules that might possibly be learned and, correspondingly, tasks that might reasonably be presented to the learner at any given stage.

Let \( B^2 \) denote the rule set that might be learned at a given stage by a person who knows exactly the rules in \( B \), and in general \( B^n \), the rules immediately learnable given availability of the rules in \( B^{n-1} \). Each rule in \( B^n \) represents a unit of knowledge that might be acquired by a learner who, on entering a curriculum, knows only the rules in \( B \). In general, \( B^n \) will be a far more encompassing and powerful rule set than the initial rule set \( R \), from which \( B \), and ultimately \( B^n \), is derived. It is this feature which accounts for "creative" potential.

In order to talk about the optimization of instruction, useful and practical ways must be found to assign values to educational goals and costs to various types of testing and instruction. In the case of values, it would not be sufficient to simply assign values directly to educational goals as, in general, this would not allow for testing. The philosophically determined values assigned to educational goals must be converted into numerical values for rules associated with these goals. In particular, values must be assigned to those ultimately desired rules or types of rules that would be acquired by students who have already mastered the curriculum. (Compare this situation to that in structural analysis where the desired (atomic) rules correspond to entering capabilities which are tailored to the weakest students.)

In general, a rule will be valued just to the extent that it plays an "important" role in accounting for tasks associated
with the corresponding educational goal(s). A major problem for research will be to determine suitable ways of measuring "importance". Perhaps a minimal requirement for "importance" is that a rule enter directly into the solution of tasks associated with some educational goal. This would rule out, for example, subordinate or source rules that are only indirectly involved.

The cost (e.g., time, money) of testing and instruction, in general, will vary directly with task complexity relative to what the individual knows at the time. Hence, any really adequate measure will have to deal with relative complexity; complexity in the abstract can be no more than normative.

In general, given: an analyzed curriculum, the entering capabilities of a student population, constraints on student behavior (e.g., number of levels of allowed goal switching), and assigned values and costs, it is possible to determine the total cost and educational value associated with any sequence of instruction. Moreover, the number of possible instructional sequences will necessarily be finite - so, in principle, associated values and costs can all be determined and compared to determine sequences which are optimal in some desired sense (e.g., maximum value/unit cost, minimum cost/unit value).

As a very simple example, suppose $B = \{r_{ab}', r_{bc}', r_{cd}', *\}$ where $r_{ab}$ is a rule for converting from measure $a$ (e.g., yards) to measure $b$ (e.g., feet); $r_{bc}$ from $b$ to $c$; $r_{cd}$ from $c$ to $d$; and * is a higher order composition rule which operates on certain pairs of rules (e.g., $r_{ab}$ and $r_{bc}$) to generate their composites (i.e., $r_{ac}$ a rule which converts $a$ to $c$). If the learner is initially presented with the task of converting measure $a$ to $d$, and then the task of converting $a$ to $c$, the learner will fail, according to theory, on the first task. Its solution requires composing all three component rules, whereas the available higher order rule can compose only two at a time. The student would succeed, of course, on the second task and, in the process, acquire a rule for solving any $a$ to $c$ task. However,
if the learner is first presented with the a to c task, and then the a to d task, he will succeed on both and in the process learn one rule for converting from a to c and another for converting from a to d. (Once \( r_{ac} \) is learned, \( r_{ad} \) can be generated by applying * to \( r_{ac} \) and \( r_{cd} \).) If each learned rule is given a value of one and the time cost for each task is assumed to be five minutes, then, while the time required for each sequence would be ten minutes, the educational value of the second instructional sequence would be twice the first.

Although direct comparison of alternative sequences is possible in principle (where reasonable assumptions can be made as to the entering capabilities of learners or where the cost of testing is minimal), such comparison will rarely be feasible via "brute force" evaluation of all possible sequences. Practical considerations will ordinarily require the use of more efficient (only partly predictable) heuristic search methods.

(Note: In WULFECK's research (this volume), which was an important first step in this direction, the problem was limited to rearranging given sets of tasks into learnable sequences (i.e., sequences of tasks that can be solved in turn according to the postulated mechanisms). In this case, it is feasible to determine learnable sequences by strictly algorithmic means.)

In general, optimization will involve a far more complex balancing of gains versus costs. The case of testing versus instruction is particularly interesting because it poses qualitatively different problems. The theory provides a strictly deterministic account of what can be learned only where sufficient information is available (via testing) concerning the learner's available knowledge. On the other hand, expending the costs necessary to get sufficient information could be counterproductive. In some cases, the "teacher" might do better by proceeding on the basis of partial information. Thus, ideally, the automated "teacher" (it is hard to imagine a human being with this much flexibility) must continually decide between testing and instruction. Does the information gained (about the
learner's knowledge) via testing, and the subsequent prediction (via the theory) which this testing makes possible, justify the costs, when compared with the value that might be gained through instruction?

In the case of partial information, the "teacher" would necessarily have to content himself with "non-deterministic" judgments (cf. SCANDURA, 1973a, chapter 8), or with probabilistic approximations (e.g., VOORHIES & SCANDURA, 1976; HILKE, KEMPF, & SCANDURA, this volume).

To summarize the current situation briefly, the area of optimization poses a veritable gold mine of unanswered and, in many cases, unexplored problems. For example, a major open question of great theoretical and potential practical importance concerns the conditions under which local optimization will insure overall optimization. On the empirical side, WULFECK's research (this volume) provides a first attempt, under some rather stringent simplifying restrictions, to explore the viability of the approach and specifically to determine the behavioral relevance of theory generated sequences.

6. A Brief Summary of Directly Related Research

In order not to leave the reader with the impression that the above theory is purely speculative, I should like to close with a brief summary of some of our more directly related empirical research. All of the chapters cited refer to my forthcoming book on human problem solving (SCANDURA, 1976).

The structural analyses which have been completed to date strongly suggest that the proposed approach is practicable and, in fact, the method of analysis and empirical work have developed hand in hand. Our first such attempt had a largely practical purpose, to determine the feasibility of identifying the rules and higher order rules underlying standard test material. In particular, my text Mathematics: Concrete Behavioral Founda-
tions (1971a) was systematically analyzed to determine, in turn, the implicitly defined tasks, corresponding rules, and higher order rules. As it turned out, this was not only possible, but the results were used as a basis for a commercial workbook paralleling the text (SCANDURA, DURNIN, EHRENPREIS, & LUGER, 1971).

At about the same time, a separate analysis was conducted of critical reading using an approximation method. In this case, paragraphs were categorized along various dimensions in a way that parallels more strictly rule based analyses. The material developed for this purpose were shown to provide a sound basis for both diagnosis and instruction (LOWERRE & SCANDURA, 1973, chapter 6), and they too have since been extended into a series of four school workbooks (SCANDURA, LOWERRE, & SCANDURA, 1974).

SCANDURA, DURNIN, and WULFECK (1974, chapter 3) undertook a more intensive analysis of geometry (straight-edge and compass) construction tasks. Among other things, this study demonstrated that heuristics (e.g., POLYA, 1962) can be made sufficiently precise, that they can be programmed on a computer and, moreover, so that it is possible to tell ahead of time in which situations any given heuristic will be useful and in which situations it will not be. WULFECK (this volume) later extended this analysis to provide a basis for his dissertation on instructional sequencing.

The above results and method of analysis used by SCANDURA et al (1974, chapter 3) were limited in three important ways: no attempt was made to include logical inference, all of the higher order rules had the effect of composing rules - no other kinds of higher order rules were considered, and no distinctions were made between rule derivation and breaking problems into subproblems. A subsequent study by SCANDURA and DURNIN (Chapter 4) dealt specifically with these limitations. In particular, a total of twenty-four lower order, derivation, and problem definition rules were shown to be adequate for proving over 130 theorems and proof exercises (along with an undeter-
mined number of others) in an experimental high school text on number systems.

To date, none of the structural analyses have taken motivation into account, nor has any serious attempt been made to analyze social or other complex domains. The furthest we have gone in this direction is a preliminary analysis of number conservation (SCANDURA, 1972b) and a rule-based extension of the LOWER-RE and SCANDURA study (chapter 6).

In the cognitive domain, studies have been conducted both in the laboratory under idealized conditions and as an adjunct to some of the structural analyses. For example, I (1974a, chapter 7) found that under idealized conditions the basic control mechanism, as it applies in rule derivation, can be assumed to be universally available to essentially all learners, at least from about the age of seven. The data from this study, however, are not consistent with other control mechanisms which have been proposed (e.g., that implicit in the production systems of NEWELL and SIMON, 1972) (see chapter 7).

In a later study (SCANDURA, chapter 3), the adequacy of an extended mechanism has been shown in the case of retrieval, rule selection, and breaking simple problems into subproblems. In studying more complex problem solving, however, it was more difficult to approach idealized conditions, and the influence of processing capacity appeared to play a somewhat bigger, although not yet critical, role.

Attempts have also been made to test a memory load model (SCANDURA, 1973a) under idealized conditions. Given constraints on time and resources, however, such conditions could only be approached. Correspondingly, while the results were consistent with the model in the standard (stochastic) sense, they deviated significantly from the deterministic ideal. A stochastic extension of the deterministic model (SCANDURA, 1973a), however, which explicitly allows for distractions ("unsharpness") and unwanted "chunking", provided a close fit with the data (VOORHIES & SCANDURA, chapter 9).
In educational applications, although deterministic predictions could not be expected, these results have held up surprisingly well. In a study by EHRENPREIS and myself (1974), for example, one group of students was taught a total of about 300 rules for solving a comparable number of tasks, one rule for each task. A second group was taught five higher order rules together with a reduced list of about 160 of the original rules. Interestingly enough, the second group did just as well as the first (in an absolute sense) on tasks on which only the first group had been trained. In addition, the second group performed (significantly) about 33% better on tasks for which neither group had been trained. In addition, the results of this study were relatively unaffected by memory, suggesting that the theory may be relatively robust concerning this type of deviation from the ideal.

I should also mention the study HAUSLLER (personal communication) is planning in physics education. This study should provide useful information concerning the effectiveness of teaching generalization rules in discovering functional relationships.

Among the more critical open questions at the present time are the following: Is the control mechanism innate? Or, does it develop slowly as the child matures? And, if the latter, what is the source of the developmental changes?

In the area of individual differences, there have been two major studies by DURNIN and myself. The first demonstrated the viability of the proposed approach to assessing behavioral potential. In this study, the underlying atomic rules were actually built into the subjects, and the testing itself was individualized and conducted under close to idealized conditions. Under these (near idealized) conditions, with testees ranging from elementary school students to Ph.D. candidates, it was possible to predict performance on new items, given performance on initially selected test items, with over 96% accuracy (SCANDURA & DURNIN, 1971; SCANDURA, 1973a, chapter 10).
In the second study, where testing took place under ordinary classroom conditions, and where judgements had to be made concerning the atomic rules, the predictions were accurate in about 84% of the cases (DURNIN & SCANDURA, 1973; chapter 11). In this study the structural approach also was compared with other forms of criterion referenced testing (e.g., item forms, HIVELEY et al, 1968). The structural approach not only provided superior prediction, but it accomplished this with fewer than half as many test items.

In my forthcoming book (1977) I also have considered the question of how to determine which of several alternative rules provides the best overall fit for a given set of (non-idealized) data (chapter 13). In addition, the validity of the structural approach to diagnostic testing (and remediation) has been demonstrated in the case of higher order rules in studies by SCANDURA (1974, chapter 7) and SCANDURA, DURNIN, WULFECK, and EHRENPREIS (1974, chapter 12).¹

¹ In a recent invited address at the Educational Testing Service SCANDURA has noted relationships between the structural and normative (ETS) approaches to testing, and has suggested some empirical comparisons that should be undertaken. That work, however, remains to be done. In SCANDURA (1976, in press), the structural approach is shown to provide a basis for resolving a number of outstanding instructional problems.
References:


380


