ADVANCE ORGANIZERS IN LEARNING
ABSTRACT MATHEMATICS

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The advent of modern mathematics, with its emphasis on the more abstract aspects of the subject, has forced the mathematics teacher to re-evaluate his methods of presentation. In teaching abstract mathematical material, one important variable is how the new material is introduced. Frequently, in the mathematics textbook or lesson, abstract material is introduced with a few paragraphs of history about the subject or its inventor, presumably to increase motivation. A well known algebra textbook, for example, introduces a section on group theory by relating the fact that Evariste Galois, an early mathematician instrumental in the invention of group theory, was killed in a duel in 1832. Another way of introducing such material is with concrete examples, models (e.g., Scandura, 1966c) or embodiments (e.g., Dienes, 1964) of the abstraction to be learned. The use of such illustrative materials is commonplace in mathematics instruction, but at the college level its introduction typically follows, rather than precedes, more formal presentation in terms of definitions and theorems.

In studying a related problem, Ausubel (1960) found that learning meaningful verbal material could be enhanced by using, what he termed, an advance organizer. Organizers were defined as introductory material at a high level of abstraction, generality, and inclusiveness. In this (Ausubel, 1960) study, the organizer provided an overview, in familiar terms, of a topic on metallurgy. The criterion material consisted of more detailed information about the topic presented in more technical language. By advance organizer, then, what Ausubel seems to be referring to is a general non-technical overview or outline in which the non-essentials of the to-be-learned material are ignored. Such introductory material has typically proved easier to learn than the more technical material that follows.

1 Historical comments, of course, are often made simply to increase appreciation for the subject and its founders. This study is concerned solely with the effects of introductory material on the learning of abstract mathematics.
Presenting mathematical abstractions, on the other hand, normally involves the use of words or symbols having no referential meaning for the naive student (Scandura, 1964, 1966a, 1966b, in press). For this reason, *descriptions* of concrete models of abstract mathematical ideas, although the models themselves involve extraneous features, may be more readily interpretable than formal presentations of the corresponding abstract ideas in terms of the underlying definitions and axioms.

In this study, historical and model introductions to formally described abstract mathematical content were compared for their effects on learning efficiency with pre-service elementary school teachers. It was hypothesized that learning would be enhanced more by the model introductions which would, in effect, serve as *advance organizers*.

**Method**

*Material and Subjects.* The material to be learned consisted of two specially written passages, one dealing with abstract mathematical groups and the other with combinatorial topology. The group (G) material (approximately 1000 words) included a brief introduction to sets, the definition of a binary operation, and the abstract definition of a mathematical group in terms of the five group axioms. The topology (T) material (approximately 1000 words) included a brief introduction to topology, the definition of a network, the definition of a closed network, defining what is meant by traversing a network, and Euler's four rules for traversing a network in a single journey. The two sets of materials were prepared to compare as closely as possible in level of difficulty and abstractness. No attempt was made in either lesson to explain any rule or definition by means of examples or illustrations.

The historical introductions (H), used as controls, also were about 1000 words in length and dealt with men instrumental in the early development of each subject. The historical group (HG) introduction consisted of short accounts of the lives of Lagrange, Cauchy, Galois, and Gauss. These accounts contained such information as where and when each man was born, something of their early lives, some of their mathematical contributions, and how each died. The historical-topology (HT) introduction included a similar discussion of Euler and Riemann. In each case, an attempt was made to trace the development through the years of the topic in question.

The introductory organizers (O) were presented in the form of mathematical games. The OG introduction consisted of a game, called "fol-
allowed-by;” in which the structure of a mathematical group was presented in language familiar to the Ss. The game had three participants—commander, receiver of commands or receiver, and scorekeeper. The commander had four commands at his disposal: (1) As you Were, (2) Right face, (3) Left face, and (4) About face. These four commands were abbreviated W, R, L, A. The commander gave the receiver a sequence of commands, one at a time. The receiver carried out these commands and the scorekeeper kept a record as to the final position of the receiver. For example, the commander might have said, R, then A. In the organizer, this was abbreviated R * A, where * was an abbreviation for “followed-by.” Of course, the receiver, after making these two commands, would have been in the Left face (L) position. Hence, the scorer would have noted that R * A = L.

It was pointed out that the set (W, R, L, A) together with the binary operation (*), had certain properties; these properties correspond to the axioms of a mathematical group. At no time, however, was any mention made of groups per se.

The OT introduction consisted of a game, called “play like,” which required the subjects to pretend that they were ants. The basic idea was to present certain topological facts about lines, curves, arcs, and networks in a simple and understandable manner. An attempt was made in this game to illustrate each of Euler’s conclusions about traversing a network. As with the group game, no mention of topology, network, or Euler’s conclusions was made.

Knowledge of the group and topology learning material was tested by two nine-item tests, one on groups and one on topology. Each test was divided into three sections. Part I contained three true-false statements (i.e. theorems), about the structures taught. In each case, S was required to justify his choice by referring to the presented mathematical definitions, axioms, and theorems. The three questions in part II required the Ss to apply these definitions, axioms, and theorems in new concrete situations. Both the H and O introductions were designed so as not to include information which would result in a direct advantage on parts I and II. Part III of the tests contained three questions based specifically on the O material.

The group and topology tests were designed to be as similar as possible in form and in degree of difficulty. The maximum scores on parts I, II, and III, in both tests, were 6, 5, and 4, respectively. All of the materials were reproduced by mimeograph and were combined into organizer, material, and test booklets.

The Ss were 104 elementary education majors enrolled in three sections of a mathematics education course at the Florida State University. There were four men and 100 women.

Design and Procedures. Two variables were independently manipulated; organizer (H or O) and material (G or T). The Ss in each section
were randomly assigned to one of the four treatment groups so that all groups were of equal size. The small nonsignificant differences due to course section were ignored in reporting the results. Originally, a $2 \times 2$ factorial analysis was planned. For the reasons mentioned in the results section, however, separate analyses of the G and T data seemed more appropriate.

The experiment was performed during regular class meetings. All treatments were equally represented at each session. The Ss were simply told that they were taking part in an experiment designed to determine how people learn mathematical materials. They were urged to do their best and were told that the results would be made available to them after the data were analyzed. The materials were presented, one booklet at a time, to prevent looking back. This procedure also made it possible to record the time taken on each of the three booklets. The entire experiment was completed during one class period.

Results and Discussion

With combined scores on parts I and II and combined scores per minute as the measures, overall performance in the O groups was superior to that in the H groups ($F (1,100) = 4.13, 6.30, p < .05, .02$). The differences due to materials were also significant but the interactions between type of introduction and material were not. Separate analyses of the G and T data are reported below for the following reasons: (1) the organizer effect seemed to be stronger with the topology material, (2) the interaction did not reflect this effect, (3) differences in scale sensitivity in different ranges may have as great an effect on interactions as real effects when there are significant main effects.

Group OT performed better than group HT or parts I and II of the test, both separately ($t (50) = 2.01, 2.11$, respectively; $p < .025$) and together ($t (50) = 3.30, p < .005$). In effect, the OT organizer increased ability to use the topology principles taught to verify or refute certain abstract conjectures and to apply these principles in new situations. The corresponding group theory effects were in the same direction, but were not reliable ($t < 1$).

Neither difference probably would have been reduced if time had been held constant. The corresponding H and O groups failed to differ significantly in mean time spent on the organizers, lesson, and tests. The

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This has been found to be good standard practice in experimental psychology. Whatever is to be gained in reducing the error sum of squares by including unimportant nonsignificant variables (e.g., class section) in an analysis is likely to be lost in reducing the corresponding degrees of freedom.

In view of the experimental hypothesis advanced above and the considerable loss in power inherent in using separate analyses, one-tailed tests have been reported where they are appropriate.
<table>
<thead>
<tr>
<th>No. Correct</th>
<th>Time</th>
<th>Organizer</th>
<th>Lesson</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
<td>I and II</td>
<td>III</td>
<td>I and II</td>
</tr>
<tr>
<td>Historical</td>
<td>4.12 (1.98)</td>
<td>6.77 (2.66)</td>
<td>8.04 (1.59)</td>
<td>7.64 (1.82)</td>
</tr>
<tr>
<td>Groups</td>
<td>4.92 (1.26)</td>
<td>2.65 (1.81)</td>
<td>3.04 (1.22)</td>
<td>7.64 (2.66)</td>
</tr>
<tr>
<td>Topology</td>
<td>3.23 (1.59)</td>
<td>2.65 (1.21)</td>
<td>7.58 (2.59)</td>
<td>2.65 (1.55)</td>
</tr>
<tr>
<td>Game</td>
<td>4.19 (1.67)</td>
<td>3.48 (1.67)</td>
<td>3.77 (2.55)</td>
<td>3.77 (2.55)</td>
</tr>
<tr>
<td>Groups</td>
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<td>8.00 (2.11)</td>
<td>3.31 (1.09)</td>
<td>8.76 (1.30)</td>
</tr>
<tr>
<td>Topology</td>
<td>4.12 (1.03)</td>
<td>8.00 (2.11)</td>
<td>3.31 (1.09)</td>
<td>8.76 (1.30)</td>
</tr>
</tbody>
</table>
H groups, in fact, took an average of over 12 percent more time on the lessons.

To determine learning efficiency, combined test scores (parts I and II) per minute spent on the lesson were analyzed. On this measure, the O-H difference was significant with topology but only reached a suspect level with groups \( t(50) = 1.96, 1.62; p < .05, .10, \) respectively).

Whether number correct or score per minute is the more appropriate measure is a moot question. Certainly both absolute performance and rate of acquiring performance are both relevant. The results, however, are much the same. The game organizers were generally facilitating, but more so with the topology materials.

It is possible that the Ss were more familiar with materials related to groups than to topology. Because of their similarities with group structures, familiarity with the syntactical constraints evident in arithmetic and elementary algebra may have served to facilitate learning and/or as a basis for solving some of the test problems. The effectiveness of model organizers may decrease with increasing familiarity with similar models (e.g., arithmetic). Using different materials and conceptually different organizers, Ausubel (1960) has made a similar interpretation.

The mechanisms by which the organizers improved test performance are equivocable. They may have improved learning by providing appropriate subsuming concepts (e.g., Ausubel, 1960, 1963; Scandura & Roughead, in press). That is, the organizer may have made it easier to interpret the abstract statements given in the lesson by providing concrete referents. On the other hand, the organizers may have provided a direct basis for positive transfer on the test problems. Since the game organizers were models of the abstract mathematics taught, it may have been possible to abstract, from the organizer alone, some of those elements necessary for successful test performance (on parts I and II).

The tests, of course, were constructed so as to minimize the possibility of direct transfer from organizers to tests. To make definitive statements in this regard, however, would require the introduction of an additional control involving only an organizer. In addition, a study in which no advance material is given would help clear the air insofar as possible negative transfer is concerned. That such controls have not typically been used in advance organizer studies (e.g. see Ausubel, 1963) is somewhat surprising. The results of a study using organizer-only controls would appear to have both theoretical and practical implications. In future studies it would also be desirable to determine the effects of presentation order (e.g., Scandura, 1966b). The question of how learning efficiency depends on the order of presenting abstractions and concrete models of these abstractions is becoming increasingly important to curriculum reformers in mathematics and should be studied experimentally. In short, more attention needs to be given to explicating the underlying mechanisms.
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