The Basic Unit in Meaningful Learning—Association or Principle? (Set-Function Language)¹

Theoretical development in educational psychology has been extremely slow. One major reason has been the typically imprecise definition of independent and dependent variables in research on meaningful learning and teaching. Such research can bear only an ambiguous relationship to theory. Similarities and essential differences often go undetected. As McDonald has put it,² “Conceptual clarity means (a) specification, stated in terms as nearly operational as possible, of the behavior involved in a task or method; (b) some delineation of the range of phenomena included and excluded; and (c) precise description of the appropriate tests.”

Stating research objectives and defining variables in unambiguous terms, however, is not sufficient. The teaching-learning process has all too frequently been studied in terms of administrative variables, such as class size, grade level, and teaching experience. The variables chosen must have broad explanatory potential, not be merely symptomatic of and inextricably related to the question at hand. Theory development depends on much more than mere fact finding.

To provide a substantive base for their research, educational psychologists have frequently resorted to the languages, paradigms, and theories of the mother science of psychology. Medializational elaborations and operant conditioning paradigms of the stimulus-response (S-R) language and more general, but less well-specified, cognitive theories have been popular.

Each approach has important limitations. From one point of view,
parsimony suggests that the properties of overt S-R associations should also be attributed to mediational links. Yet, practice has shown that mediational interpretations become increasingly cumbersome and less precise as situations become more complex. Similar difficulties have plagued researchers who have used operant techniques to study meaningful verbal learning. The results simply are nowhere near so clear in complex human learning as they are in the "Skinner Box." It is increasingly recognized, for example, that knowledge of results is not directly analogous to feeding a pigeon and that, in any case, other factors, such as subject-matter structure, are probably of greater importance in promoting efficient learning. A general limitation of cognitive theories is their relative imprecision. Typically, "cognitions" either are not clearly specified in observable terms or are only partially defined.

In short, the choice to date has been between a precise, but seemingly inappropriate, S-R language and presumably more relevant cognitive formulations which leave much to be desired insofar as scientific cohesiveness and rigor are concerned.

The purpose of this paper is to introduce what I feel are the basic ingredients for a new scientific language for formulating research questions on meaningful learning. This so-called Set-Function Language (SFL) is precise and seems particularly well suited for dealing with mathematics, my own area of concern, and science, but it undoubtedly can be used with other subject matters as well. Rather than try to detail the SFL or to summarize the related research that we have completed or have under way, let me simply try to convey the general idea. In the process, SFL and S-R formulations of several meaningful learning tasks will be contrasted.

The SFL is behavioristic, as is the S-R language, but, unlike the S-R language, the SFL denies the primacy of the S-R association. To illustrate some of the advantages of the SFL, consider the situation depicted in Figure 1. Suppose an experimental subject is required to learn to say the appropriate word when shown a learning stimu-
lus—for example, to say “black” when shown the large black triangle. After the original four S-R pairs are mastered so that the subject can reliably give the correct response to each stimulus, the question remains as to just what was learned. Did the subject learn four distinct pairs—four discrete associations—and notice no relationships between them? Or, did he learn the two principles, “If triangle, then color,” and “If circle, then size”?

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<thead>
<tr>
<th>LEARNING</th>
<th>TEST ONE</th>
<th>TEST TWO</th>
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<tbody>
<tr>
<td>△</td>
<td>BLACK</td>
<td>△</td>
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<tr>
<td>○</td>
<td>LARGE</td>
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<td>△</td>
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<td>○</td>
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Fig. 1.—Sample learning, assessment (test 1), and prediction (test 2) stimuli and learning responses.

This question first began to bother me during the summer of 1962. In a study designed by Greeno, we found, in a verbal concept learning situation, that essentially a subject either gives the correct response the first time he sees a transfer stimulus or the transfer item is learned as its control.

The thought later occurred to me that if transfer obtains on the first trial (if at all), then responses to additional transfer items, at least under certain conditions, should be contingent on the response given to the first transfer stimulus. In effect, a first transfer stimulus could serve as a test to determine what had been learned during the original learning, thereby making it possible to predict what response a subject would give to a second transfer stimulus. To test this assumption, I had a total of about fifteen (highly educated) subjects overlearn the list shown in Figure 1. Prior to learning the list, both the subjects and the experimenter agreed on the relevant
dimensions and values—size (large-small), color (black-white), and shape (circle-triangle). The subjects were told to learn the pairs as efficiently as they could, since this might make it possible for them to respond appropriately to the transfer stimuli. After learning, the test 1 stimuli were presented, and the subjects were instructed to respond on the basis of what they had just learned. They were told they were correct no matter what the response. Then, the test 2 stimuli were presented in the same manner.

The results were clear-cut. All but three of these subjects gave the responses “black” and “large,” respectively, to the test 1 stimuli, and also responded with “white” and “small” to the test 2 stimuli. It would appear that when a subject thinks he is right and the new situation remains relevant, he will continue to respond in a similar manner.

On what basis could this happen? It was surely not a simple case of stimulus generalization; the responses were distinct and, in any case, did not depend solely on common stimulus properties. The first test 1 stimulus, for example, is as much like the fourth learning stimulus as the first—and yet, “black” was invariably given as the response rather than “small.” Perhaps the simplest interpretation of the obtained results is that most of the subjects discovered the two underlying principles while learning the original list, and later applied them to the test stimuli. In effect, the relationships between the S-R pairs themselves, combined with a response consistency hypothesis, provide a basis for assessing what was learned.

Before introducing the SFL, let us ask how the S-R theorist might represent what was learned in the preceding exercise. In S-R psychology, the basic building block is the association, which was originally viewed as a learned connection between an observable stimulus—light, nonsense syllable, or mathematical problem—and an observable consequence or response—salivation, another nonsense syllable, or solution. A connection or association is said to have been formed if the corresponding response appears with a positive probability whenever the stimulus is presented. Learning a concept,
presumably a more complex form of learning, involves the ability to give a common response to any one of a set of stimuli. To say that a subject has acquired the concept of red, for example, implies that the subject is able to say some common response when shown any red object but will not give this response to any non-red stimulus. In short, whereas an association pairs one stimulus with one response, a concept is a many-one relationship.

Since the association is felt to be basic, the S-R theorist has felt obliged to represent the many-one concept relationship as a composite of one-one relationships. This has been made possible by the postulation of mediating links or associations. Thus, the many-one relationship may be represented,

\[ S_1 \xrightarrow{M_r} R \xleftarrow{S_3} \]

where the stimuli \( S_1 \), \( S_2 \), and \( S_3 \) are connected to the mediating response, \( M_r \), whose stimulus properties, in turn, elicit the observable response, \( R \). In the case of the concept red, \( M_r \) might be an internalized representation of the label “red.”

But the original task we were confronted with involved principles—and principles, from the standpoint of S-R associationism are more complex than either associations or concepts. It is probably because of this felt complexity that most psychologists, particularly experimental psychologists, have simply not been much concerned with principles. Since the notion did not readily fit into their scheme of things, they did what any thinking man would do—ignored the idea.

Nonetheless, an increasing number of pedagogically oriented psychologists have come to recognize the central role of the principle in meaningful learning, and at least two association-based representations of the principle have been proposed. Rather than delve into the possible limitations of these formulations, I shall propose an S-R formulation of my own—and then attack it.
First, consider the representation,

\[ S_1 \rightarrow \text{Triangle} \rightarrow \text{Color} \rightarrow R_1 \]
\[ S_2 \rightarrow \text{Triangle} \rightarrow \text{Color} \rightarrow R_2. \]

This representation has been chosen to reflect the principle, "If triangle, then color," as it relates to the situation depicted in Figure 1. In this case, the overt stimuli, \( S_1 \) and \( S_2 \), are presumed to elicit the mediator "triangle," which, in turn, elicits the mediator "color." "Color," then, is presumed to elicit \( R_1 \) and \( R_2 \)—with equal probability. Of course, we know that this is not what happens from the study just described; \( S_1 \) goes with \( R_1 \), and \( S_2 \) with \( R_2 \).

To overcome this inadequacy, we may postulate a second pair of connecting chains,

\[ S_1 \rightarrow \text{Black} \rightarrow R_1 \]
\[ \text{Triangle} \rightarrow \text{Color} \rightarrow R_2 \]
\[ S_2 \rightarrow \text{White} \rightarrow R_3. \]

According to this interpretation, \( S_1 \) elicits \( R_1 \) and not \( R_2 \) because \( S_1 \) elicits "black" as well as "triangle" and has no direct relationship to "white." Of course, \( S_2 \) elicits "white" for the same (implied) reason. With this representation in hand, the S-R associationist is now able to predict the results of the experiment described. "Black" is given as the response to the first test stimulus, a small black triangle, because of the prelearned association between the stimulus and "black" and the learned mediating association, "triangle elicits color." In short, two associative connections are better than one.

Rather than go into the SFL representation of principle learning at this point, let me first add more fuel to the fire by presenting another example. Suffice it to say at this point that the essence of a principle is captured reasonably well by a statement of the form, "If A, then B." Consider the S-R pairs,

\[ (7,1,6,4) \rightarrow 9 \]
\[ (4,3,9,3) \rightarrow 10 \]
\[ (6,5,3,9) \rightarrow 5 \]
\[ (9,7,3,6) \rightarrow (5,1,8,3) \rightarrow * \]
Suppose the learner is posed with the task of determining, from the instances on the left, that principle which will allow him to predict the appropriate responses to the stimuli on the right. He might find that a difficult task. But, if he is told that the responses (integers) can be derived uniquely from the integers in the first, third, and fourth positions of the stimuli, it might be easier. It would undoubtedly be still easier if he is told that the operations of addition and subtraction are involved.

More is involved in this example than in the first. The learner is required not only to discover the underlying principle, and a more complex principle at that, but hints have been given as to how he might accomplish the task.

How might these contingencies be formulated in terms of associations? An immediate thought is that position and operation cues might be viewed as mediating links,

![Diagram]

But, as a quick look will make clear, that simply does not work. Further elaboration would be necessary to indicate why \( S_1 \) goes with \( R_1 \), and \( S_2 \) with \( R_2 \). Of course, there undoubtedly are other alternatives, but I think you will agree that any S-R representation of these contingencies is likely to be extremely complex.

It is important to emphasize that this situation was not picked arbitrarily simply to embarrass S-R psychologists. Stimulus dimensions (e.g., color, position), which uniquely determine the responses and combining operations (e.g., name the color, addition and subtraction), by which the responses are derived from these stimulus properties, appear to be crucial aspects of all principles.

Fortunately, these characteristics play a central role in the SFL. In fact, it is assumed that four elements, \( I, D, O, R \), are needed to specify a principle. The stimulus properties in set \( I \) tell when the
principle is to be used; those in $D$ tell which properties determine the response; and the combining operation, $O$, tells how the response properties, $R$, may be derived from those in $D$.

Obviously, the attribute and operation cues, involved in the above situation, find natural counterparts in this characterization. Thus, $D$ involves the three dimensions corresponding to the three positions, and $O$ is a composite operation involving addition followed by subtraction. In addition, the response set $R$ involves the integers variable (i.e., set of integers responses), while those stimulus properties (in $I$) which determine when the principle is to be applied are those associated with being a "four-tuple" of integers. Notice that the properties in $I$ involve a first, third, and fourth position (those properties associated with $D$), so that $I$ necessarily involves $D$. In the first of the (two) task 1 principles, $I$ involved colored triangle, $D$ color, $O$ color naming, and $R$ the color names.

Since principles, as well as associations and concepts, can be represented in the S-R language, it is appropriate to ask whether these notions can also be represented in the SFL, in which the principle is taken to be basic. As can be seen in Table 1 this is indeed the case. Furthermore, unlike the S-R language, explicit distinctions are made between: (1) the observable S-R instances of a principle—the denotation; (2) the principle itself, that which underlies the behavior and whose presence can be inferred only indirectly; and (3) statements of the principle in symbolic form.

The denotation of a principle is simply taken to be equivalent to the mathematical notion of the function, a notion which may be defined as a set of ordered S-R pairs such that to each functionally distinct stimulus there is one corresponding (functionally distinct) response. The denotation of a concept is simply represented as a function in which there is one response common to all stimuli. To represent an ordinary association, the set is further restricted so as to include only one S-R pair.

The principle itself is characterized as an ordered four-tuple
(I, D, O, R), where I, D, O, and R are as previously defined. In the case of concepts and associations, there are certain restrictions placed on these elements, but they need not concern us here. Finally, notice that the statements in the column on the right are constructed from symbols, \( I', D', O', \) and \( R' \), representing the constructs \( I, D, O, \) and \( R \).

**Table 1**

<table>
<thead>
<tr>
<th>Principle</th>
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<tr>
<td><strong>Denotation (Observables)</strong></td>
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<tr>
<td>( {S, R} {s</td>
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**Concept**

\( \{S, R\} \{s|J\} \)

(\*Note: One response) \hspace{1cm} \( \{I, D, O, R\} \) \hspace{1cm} If \( I' \), then \( R' = O'(D') \)

**Association**

\( \{S, R\} \)

(\*Note: One pair) \hspace{1cm} \( \{I, D, O, R\} \) \hspace{1cm} If \( I' \), then \( R' = O'(D') \)

Due to space limitations, I will not go into Figure 2 in any detail. Let me simply point out that while there are common relationships between different pairs of the experimental paired-associate (PA) list, none exists in the control list. The principles involved in the experimental list might be stated, “If black, then shape,” and “If white, then size.” “If a small black circle, then circle,” is a candidate for one of the control list principles.

Space limitations also demand that I not attempt to detail my reasons for preferring the SFL to the S-R language. I would, however, like to mention three reasons why the SFL appears to me to
Fig. 2.—Nominal (actual), S-R mediation, and set-function language representations of paired-associate (PA) lists in which common relationships exist between pairs (experimental) and in which they do not (control).
be better suited for formulating research on meaningful learning, in general, and mathematics and science learning, in particular. The first major reason is that principles are so critical, even in the simplest forms of meaningful learning. And, as we have seen, it is much simpler to represent what is learned in the SFL than it is in terms of more cumbersome S-R representations. My second reason for preferring the SFL is that meaningful learning involves the ability to make the appropriate response in a class of responses to any one of a class of stimuli. Learning single principles may make such performance possible but not single associations. Lest there be some confusion, the stimuli and responses to which I am referring may be quite distinctive and not merely differ slightly along some physical dimension (i.e., the stimuli and responses may be functionally distinct). In simple learning, where the stimulus dimensions are continuous, the ability to give a new response to a new stimulus has traditionally been attributed to S-R generalization. Some of our recent data, however, suggest that such a postulate may be inappropriate in theories of meaningful learning. Meaningful transfer may more easily and accurately be accounted for in terms of what principle is learned. The third and perhaps most important reason is simply that there are many situations in meaningful learning—we have reviewed just two—which, while easily represented in SFL terms, are difficult, if not impossible, to formulate in the S-R language.

There are other situations which pose even more difficulty for the S-R language, while yielding to SFL analyses. During the past year or so we either have completed or have SFL-based research under way on the following topics: (1) the role of attribute (D) and operation (O) cuing in learning mathematical principles, (2) rule generality in mathematics learning, (3) the role of symbolism in mathematics learning, and (4) the expository presentation of what mathematical strategy is learned in discovery learning. In the future, we hope to get into even more complex cognitive tasks such as: (1) identifying (and representing) heuristics of use in mathematical
problem solving and proving mathematical theorems, (2) extending the SFL so as to provide a rigorous basis for such things as task analysis and Bloom’s taxonomy of educational objectives (cognitive domain), and (3) the construction of instructional sequences to meet multiple objectives. So far it appears that the SFL is capable of representing, if not all, then certainly many of the critical characteristics of such situations.

In the final analysis, the choice between scientific languages (and theories) involves efficiency and cohesiveness as well as the sheer ability to represent or account for observable phenomena. It is in this sense that the much heralded adaptive quality of the S-R language too often has led psychologists to overlook the fact that it is always possible to patch up an existing formulation to meet new situations. Parsimony does not simply refer to the maintenance of existing concepts but to the formulation of emerging structures in the simplest possible way. I might add, in conclusion, that what is presently being done with the S-R language (e.g., the incorporation of hierarchies and motivation factors) is quite analogous to what pre-Copernican astronomers were doing when they invented epicycles to represent planetary motions in an attempt to salvage geocentric theory.

Having made that emotion-laden comment, I rest my case.

NOTES

1. This article is based on a paper read at the American Educational Research Association convention in Chicago on February 19, 1966. A description of some of the recent empirical research, based in the set-function language, may be found in my "Precision in Research on Mathematics Learning: The Emerging Field of Psycho-Mathematics," Journal of Research in Science Teaching, IV (December, 1966), 253-74.


4. Typically, a distinction is made between a mediating response, \( M_r \), and its stimulus properties, \( M_s \), so that the chain \( S \rightarrow M_r \sim M_s \rightarrow R \) would be used rather than \( S \rightarrow M_{r,s} \rightarrow R \). The shorter form suffices for present purposes.

5. This representation was introduced by R. M. Gagné (e.g., *The Conditions of Learning* [New York: Holt, Rinehart & Winston, 1965]).

6. More recently, I have found it useful to make a distinction between rules and principles, a rule being completely characterized by \( D, O, \) and \( R \). The interested reader is referred to the references in footnotes 1 and 7.