Research in Mathematics and Science Education

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The purpose of this paper is not to review all or even much of the past or current research in science education; this has been done elsewhere. Monographs covering research in science education up to about 1960, for example, are available from the U. S. Department of Health, Education, and Welfare. The Journal of Research in Science Teaching also provides a good source.

We would like here to put such research into a perspective and to indicate some of the strengths and weaknesses of various approaches to educational research generally. Some of the examples given will pertain specifically to our area of mathematics education, but their general relevance can also be shown in science education.

BACKGROUND

To make this task practicable, we shall limit our discussion to research on teaching and learning, the area with which we are most familiar. As you well know, the traditional methods paradigm for research on teaching and learning has been designed to assess the relative effectiveness of two or more instructional methods. A major difficulty with this sort of research is that too often, both method and content are varied simultaneously, but, not independently. Such an approach allows one to say nothing about either separately. Thus, for example, if a modern mathematics or science program is taught by the discovery method and a traditional program by traditional methods, then, the results on any criterion variable such as problem solving ability, attitude, and the like may be due to methods differences, content differences, or both in some unknown combination.

In other studies, the concomitant variations in content and/or method are more suitable. The variations in content, to which we refer, are those differences that can be attributed to the presentation (with resultant learning and practice) of different, but unspecified, aspects of the same material. Thus, when teaching by exposition, the instructor may simply state Newton's Laws of Motion, whereas when using the discovery method, the students might actively engage in experiments involving moving objects. Often, fundamental differences in method also go undetected. Expository instruction, for example, typically continues until the student can perform the required task, whereas discovery instruction normally proceeds until the student "catches on." In such situations as these, the possibility of detecting differences due to instructional methods depends criti-
cally on what criterion measures are used. By way of illustration, two methods of instruction may influence the amount learned to about the same degree, whereas the differences in attitude engendered by the two methods may be highly reliable. Even more critical, from the standpoint of interpretability, is the fact that in studies of this kind, it is impossible to specify the basis for whatever differences are found. Some of our research (Scandura, 1964), for example, has demonstrated that relatively minor variations within exposition and/or discovery methods may alter the experimental outcomes drastically.

Until recently, the pattern of methods research had not changed appreciably since the 1920's and 1930's, a pattern which was largely established during the Thorndikean Era. Throughout much of this period, researchers in psychology in and education were almost indistinguishable. The rise of behaviorism in America, an approach which gave psychology a means of obtaining greater respectability in the scientific community, and the crossing of the Atlantic of the Gestalt brand of psychology, which educators found more palatable, led to an ever widening gap between research in psychology and in education. Only recently have there been signs of some rapprochement.

This lack of understanding between educators and psychologists has had a profound influence on educational research. Educators, desiring results of immediate applicability, have often dealt with problems and questions with little transfer potential. Much concern has been given to such questions as, ‘Is it better to teach biology via the ‘green,’ ‘blue,’ or ‘yellow’ books?’ The fact that the results of research based on questions of this sort, had little generality, and seemed to lead nowhere as to the formulation of any theory of instruction, was disheartening. Although a few energetic souls remained active in research throughout their careers, a result of this situation was that most educational researchers quit with the dissertation and most teachers and teacher educators lost faith in educational research findings.

**Types of Research**

Hopefully for the future, educational research is presently in the process of rebirth. There are several new approaches which appear promising—we shall outline four.

*Action Research.* The first is based on the philosophy that known learning principles are inadequate to explain real school learning, and that it would be better to start over from scratch rather than to try to patch up existing theory. This philosophy, originally championed by subject matter people—particularly the revolutionary brand of mathematics and science educator—has resulted in a large amount
of sponsored action classroom research in the late 1950's and early 1960's. By such research, we refer to the sort of thing for which the University of Illinois arithmetic project and the PSSC are famous. They seek to find out what makes for good teaching by trying out various methods and seeing what "works." The approach is that of the inventor as opposed to that of the scientist—like the Edison who creates the light bulb rather than the scientist who searches for the underlying principles on which the light bulb is based. There are at least two major limitations of action research. One limitation reflects the fact that invention is, by nature, quite arbitrary. Whatever teaching method is proposed, as a result of action research, necessarily exemplifies only one way of reaching the educational objective in question. The results of action research do not, in themselves, lead to better understanding of the teaching-learning process and, yet, more refined teaching technologies will depend on such understanding. In fact, even the educational objectives, themselves, often remain the private property of the innovator. Action research, at best, can create sound methodologies for present day classrooms, but, it cannot uncover principles on which future instructional inventions may be based. Another limitation of action research is the difficulty of transmitting knowledge about what has been learned about teaching except in terms of situation-bound specifics or vague generalities. Two methods have been widely used to disseminate information acquired in this manner. One approach has been to bring teachers in to the project center for intensive training. For the most part, this has proved a satisfactory, although expensive, method. Another approach has been to develop films and tape recordings. The use of these media is based on the assumption that teaching is an art, and that artistry is best learned by emulation. The difficulty here, of course, is to know what to emulate. Is the teacher doing a good job because the children are raising their hands or because they are jumping up and down in their seats? Is it because the teacher doesn't seem to be saying too much; because the students determine the pace at which the instruction proceeds; or is it because they are talking to one another?

Let there be no doubt that inventive action research has served a valuable function in improving mathematics and science education throughout the land. Nonetheless, this has only been a first step.

Now that the writing and rewriting of the materials phase of the new mathematics and science curricula is nearing the stage of completion, at least on the gigantic scale to which we have become accustomed, there is an ever increasing pressure to evaluate these new programs. Although there are some large scale evaluative studies in progress which are quite reminiscent of early curricular research, there is
also an increasing recognition that more sophisticated and intensive research is needed. Attempts to get at the rudiments of teaching and learning generally, as well as in mathematics and science, are underway at a number of centers throughout the country. Research is beginning or going on at Michigan, Wisconsin, Maryland, Illinois, Texas, Minnesota, Stanford, and Pennsylvania which emphasizes the teaching and learning of mathematics and science and mathematical type materials. Other institutions are planning research centers along these lines.

Our opinion is that such research probably will not soon improve on the master teacher, at least in the classroom situation. It may, however, provide the master teacher with a better understanding of why what he does is good. Probably more important, it may make it possible to improve teacher education by providing sound information about teaching and learning which is capable of efficient transmission.

Another point is that the sort of research that is needed will require competencies which are different from those possessed by most mathematics and science educators.

Technological Research. A second modern approach to research on teaching and learning is based on the opinion that teaching should be treated as a technology based on a science of learning. Thus, many investigators, typically those with a background in experimental psychology, have been attacking the educational problem with the tools of their trade—such psychological notions as contiguity, reinforcement, and mediation. Most of these scientists are under no delusions that variables already identified in the laboratory will be sufficient in the educational situation; they are willing to embellish, but they are not willing to disregard what is already known about learning.

There is no question that many learning variables operate in the classroom, but there is a question as to whether the technological approach, based on a relatively molecular science, is the most efficient means of understanding the educational process. There are many ways of explaining the same phenomena. Thus, the pressure on an enclosed gas resulting from changes in its volume may be explained in terms of kinetic theory. But, explanation may be more easily accomplished and the necessary measurements more readily made by referring to the familiar gas law: pressure $\times$ volume = constant.

Similarly, the motions within our solar system may be just as accurately described when the earth is taken as center of the solar system as in terms of the Copernican Theory in which the sun plays the central role. The essential difference is in the efficiency of explanation.
There is no doubt that this sort of technological research should be encouraged—it is the means by which a bridge can be built between psychology of learning and the instructional process. Nonetheless, there is no more need to forego the direct study of teaching (and in this we include the teaching of mathematics and science), than there was for the early chemist to ignore chemistry and expend all his energy first trying to explain chemical phenomena in terms of the more established science of physics. One more example, this by Goodman[2] (1964), will crystallize the point. Suppose you wished to synthesize a chair with molecules. "If you were to get even a quarter of an inch up one of the legs, by that time you would be in such a state of indeterminacy that you would give up in complete and utter disgust. It would be better to employ a carpenter." The question is whether it would be any more feasible to construct a theory of carpentry.

Another limitation, or advantage, of the technological approach is that, whatever the findings, implementation will require a considerable degree of instrumentation. With this in mind, there are several significant attempts underway to utilize computer technology in instruction. Perhaps the furthest developed, from a theoretical point of view, is the Socrates System (Stolurow, 1961) formerly at the University of Illinois. Socrates is designed for individualized instruction. Not only does it provide a high degree of flexibility in sequencing within programs, but it allows the computer to select the most appropriate program—from those available—for achieving a particular outcome on the basis of the individual student’s response history. Consideration is given to both the number of correct responses and the time it takes the student to answer.

The Plato System, also at Illinois, is designed to teach students how to prove mathematical theorems. In an improved version of this system the computer will be capable of determining whether the next step in the student’s proof follows from any one or two previous steps. The student, however, must justify each step in his proof by stating the underlying axiom or theorem in detail so as to be unambiguous to the computer.

IBM’s new coursewriter system will operate out of Yorktown Heights, New York, and make it possible to both explore what can be done with computerized instruction and to conduct more refined empirical research at several locations throughout the country. One of these stations has been installed at Florida State University.

*Many-Variable Research.* Still a third philosophy underlying modern educational research is that the educational process is extremely complicated—that many variables are involved. Thus, some investigators, mostly educational and psychological research method-
ologists who naturally have emphasized those things they know best, have argued that more sophisticated research designs are needed.

For many years, educational researchers have included many criterion variables in their experimentation. Thus, they may not only measure the amount learned, but retention, transfer, attitudes, and interests. In spite of this concern with large numbers of dependent variables, the number of independent variables in research on teaching and learning has typically been severely limited. Proponents of the "complex design" are quick to point out that the availability of data processing equipment makes large-scale experimentation with many factors (independent variables) highly practical.

Some of these advocates also have argued that interactions between variables may be more crucial than overall main effects—and, that interactions can only be assessed when all of the interacting variables are included in the same experimental design. The large-scale research of McKeachie, Isaacson, and Milholland on the teaching of psychology is based largely on this assumption. In mathematics and science education, possible interactions between method and content may be at least as important as any effects due to variations in either content or method alone.

Perhaps the major limitation of the complex design approach is the very ability to deal with large numbers of independent variables. Thus, there may be a tendency for the experimenter to add variables to his design almost indiscriminately. The availability of a large variety of standardized tests by which to assess personality and other characteristics of teachers and learners further increases this temptation and makes choice a critical factor. Such choices are necessarily subjective.

This approach also poses a difficulty for any attempt at theory formulation. By dealing with available tests, which sometimes measure rather ill-specified outcomes, and educational variables, such as class size, use of television, amount of teacher preparation, and the like, which are probably of secondary importance, the generation of theory may be unnecessarily complicated. A more profitable approach may be to first identify the crucial variables underlying the teaching-learning process. We need to invent preliminary rationales, if not theories, which will have the effect of making educational research more general and cumulative, not specific and fragmentary.

To be sure, theory development is the result of invention rather than discovery. Since theory must relate to empirical findings, however, it is different from technological invention in which something is created to serve some practical function. Theory development, in education, will be no easy job, but a behavioral language which lends itself to
precise discourse on teaching and learning will surely help. A related need is to develop taxonomies by which subject matters may be classified so that empirical findings in one context may be generalized to others. It is our personal opinion that mechanical techniques, such as factor analysis, will not identify the critical variables. Such work may, in fact, actually hinder more refined analysis by "clouding" the basic issues. However, far be it from us to discourage any sort of research in which an investigator has faith.

Naturalistic Research. There is also a growing body of psychological types who are impressed with the first mentioned approach—that of action research—but, who wish to carry such research to its natural conclusion. The philosophy underlying this fourth approach to educational research is based on the thesis that the variables studied should be abstracted directly from the situation to be understood. Thus, people like Bruner, Dienes, Easley, Suppes, Piaget, Gagné, Blackwood, Watson, and Novak hope to obtain a better understanding of the whys and wherefores of the preceding action research by isolating and studying systematically those variables which appear to be of critical importance in the mathematics and science classrooms.

At Florida State University we have completed a number of studies during the past year involving the effects of such variables as prior learning, presentation order, and prerequisite practice on problem solving (Scandura, 1965 b; c, d). We found that problem solving performance was improved: (1) by the pre-experimental availability of prerequisite material, (2) when the prerequisite material was presented prior to the criterion material, and (3) by prerequisite practice only when the prerequisite material came first. Perhaps even more important, the repeated reintroduction of the criterion materials, coupled with problem solving practice, failed to overcome the prerequisite inadequacies. Although the study was conducted with college students, this result is reminiscent of Piaget's research in which, for example, young students cannot be taught to measure volume meaningfully before they have learned to conserve volume—e.g., recognize that the volume of liquid does not change with the shape of its containers.

We also began the development of a research-oriented doctoral program in mathematics education.

We are presently involved in a study concerned with the teaching of learning ability, the editing of a special publication of the National Council of Teachers of Mathematics (NCTM), and the development of a theoretical paper utilizing a mathematical language for formulating research problems on the teaching of mathematical and scientific materials.
MORE ON THE PYTHAGOREAN THEOREM AND ITS CONVERSE

I would like to comment on the article appearing in the April, 1966 issue of School Science and Mathematics entitled, “Pythagoras’ Theorem? Or Its Converse? Which Precedes the Other?” authored by B. Rameswar Rao. Mr. Rao makes the following statement. "So far, no proof for the converse Pythagoras’ theorem is known to have been occurring without the application of Pythagoras’ theorem. Thus everybody believes that Pythagoras’ theorem precedes its converse. . . . It is unfortunate that Mr. Rao did not investigate previous literature before making this statement.

In the December, 1961 issue of The Mathematics Teacher, I authored an article entitled, “Pythagorean Converse,” in which I presented six different proofs of the Pythagorean Converse, pointing out that all of the proofs did make use of the Pythagorean theorem. However, I did point out that in the October, 1951 issue of The Mathematics Teacher Mr. Victor Thebault authored an article entitled, “A Second Note On The Pythagorean Theorem” in which he gave a proof of the converse of the Pythagorean theorem which is independent of the Pythagorean theorem—in complete contradiction of Mr. Rao’s statement.

Respectfully,

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