NEW DIRECTIONS FOR THEORY AND RESEARCH ON RULE LEARNING

I. A SET-FUNCTION LANGUAGE

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The search for a suitable scientific language in psychology has had a long history. Unfortunately, as with theories, there is no a priori basis for deciding between alternatives. Which will prove most useful can be determined only after a period of use. Nonetheless, certain characteristics appear desirable. One of these is precision. The primary requirement, however, is that the language accurately represent the important characteristics of the phenomena in question. Without such fidelity the language can have no real value— it is the sine qua non.

In order to construct a precise descriptive language, which adequately reflects meaningful learning, a basic behavior unit must be selected. The history of science has shown that the hypothesis-generating and predictive value of any theory or scientific language is determined in large part by the appropriateness of its basic building blocks.

Many theorists have been primarily concerned with extending S–R formulations to account for complex phenomena (e.g., BERLYNE, 1965; KENDLER and KENDLER, 1962; MALTZMAN, 1955; GOSS, 1961; OSGOOD, 1953; STAATS and STAATS, 1964). Although it has been repeatedly emphasized that the S–R approach is simply a way of working, of baring essentials, the neo-associationist implicitly believes that the association provides the most precise and efficient unit with which to describe behavior.

The fundamental, and perhaps most questionable, assumption underlying neo-associationism is that mediating links in an S–R chain have the same properties as overt S–R associations (BERLYNE, 1965, 17–19). In view of the success achieved in viewing animal and simple human learning in terms of associations, parsimony would seem to call for such a principle. Yet, practice has shown that mediation interpreta-

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tions become increasingly cumbersome and less precise as situations become more complex. More important, Anderson (1964) and Fodor (1965) have recently argued convincingly that multi-stage explanations only give the appearance of greater explanatory power. Single stage formulations can always be devised which are equivalent.

Largely for these reasons, other theorists and highly reputable writers (e.g., Ausubel, 1963; Bartlett, 1932, 1958; DiNes, 1963; Gagne, 1962, 1965; Mandler, 1962, 1965; Miller, Galanter and Pribram, 1960; Piaget, as described in Flavell, 1963; Polya, 1962, 1965; Newell, Shaw and Simon, 1958) feel that the S–R language does not capture the essence of meaningful learning. Typically, they find the idea of an association or network of associations to be incapable of reflecting all that a human does when confronted with a problem situation. Constructs are needed to enable Ss to think (Mandler, 1965, 325). Thus, Bartlett (1932, 1958) speaks of organization and rules, Gagne (1962, 1965) of knowledge and principles (and learning sets), Mandler (1962, 1965) of structure, Miller, Galanter and Pribram (1960) of TOE units (and heuristics), Piaget (Flavell, 1963) of schemas, Polya (1962, 1965) and Newell, Shaw, and Simon (1958) of heuristics, and Tolman (see Hilgard, 1956, 191) of cognitive maps and sign-significate relations.

Several of these writers (e.g., Miller et al., 1960; Polya, 1962, 1965; Newell et al., 1958) have attempted to deal with problem solving in its full complexity. Emphasizing the role of heuristics, they have been either of the opinion that problem solving should be treated as an art (Polya, 1962, 1965) or that the computer (Miller et al., 1960; Newell et al., 1958) provides the only really effective means for dealing with the complexities involved. The former view, of course, is antithetical to science. Computers, on the other hand, although they provide a valuable tool and possibly a viable model, do not alleviate the scientist of the responsibility for identifying basic behavioral units and for stripping theory of non-essentials. Computer simulation, due to the technical complexities and practical problems involved, may be as much a hindrance as a help in theory construction.

The present practice of repeatedly extending associationistic schemas to account for new facts, particularly as regards meaningful learning, is highly reminiscent of pre-Copernican astronomy. At that time too, emerging facts were incorporated into what we know now to be an unnecessarily complex (geocentric) theory.
Others (e.g., Ausubel, 1963; Bartlett, 1930, 1958; Dienes, 1963; Piaget, as described in Flavell, 1963) have also offered appealing analyses of meaningful learning and problem solving, but they have been forced to gloss over many subtleties. There has been no sufficiently precise language available for formulating their ideas. Although Piaget has made considerable use of logic in his theoretical work, it has served primarily to describe internal capabilities. In tying these capabilities to observables, Piaget has simply used the French language, sometimes in rather abstruse fashion.

In short, the choice, to date, has been between a precise, but seemingly inappropriate S–R language, and presumably more relevant cognitive formulations which leave much to be desired insofar as scientific cohesiveness and rigor are concerned.

No serious investigator in the area today really questions that meaningful behavior involves the ability to 'perform successfully on an entire class of specific tasks, rather than simply on one member of the class (Gagné, 1962, 355)' For example, most psychologists, cognitive and noncognitive alike, believe that knowing how to add means that the learner is able to give the correct response to any addition problem, not just one. The issue seems to be whether it is more feasible to view the underlying knowledge as consisting of networks of associations or whether it would be better to adopt a new basic behavior unit. Thus, Gagné (1964, 1965) has shown how many higher forms of learning depend on simpler forms, such as the association. The unfortunate fact of the matter, however, is that no one has been able to devise a completely satisfactory way to represent rules in terms of associations (cf. Scandura, 1966a, 1967). For example, Gagné's (1964, 1965) representations of the rule do not use the S–R language but involve such constructs as chains of concepts and, more recently (1966) 'action' concepts as well. As Tracy Kendler (1964) suggested in reacting to Gagné's (1964) original paper on the subject, new properties may emerge at the rule level. It would seem that in the study of meaningful learning it may well be more desirable to adopt as basic a higher form of learning.

This paper is the first of three which together constitute a monograph

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3 Nonetheless, many neo-associationists, particularly in the United States, would prefer to invoke a mechanism of S–R generalization (e.g. Hull, 1943; Berlyne, 1965, 168–171) to explain how Ss can give new responses to new stimuli. More is said below about the relative merits of this position.
on 'New Directions for Theory and Research on Rule Learning'.
The purpose of this paper is: (1) to introduce a new scientific language,
formulated in terms of the mathematical notions of sets and functions,
and (2) to contrast the S-R language with this set-function language
(SFL). In the SFL, the rule or principle \(^4\) is taken as the basic unit of
behavior. This adoption turns out to be highly parsimonious from a
mathematical point of view. The second paper in the series reports
some of the related empirical research my students and I have con-
ducted. The emphasis is on the guidance provided by the SFL in
formulating the research and on modifications and extensions of the
SFL suggested by the obtained results. The last paper is concerned with
SFL based analyses in such problem areas as reversal and non-reversal
shifts, Piagetian conservation tasks, and symbolic and concrete learning.
Theoretical direction is also given.

THE RULE – BASIC UNIT IN THE SET-FUNCTION LANGUAGE (SFL)

As suggested above, psychologists seem to be split down the middle
on the issue of whether the association or the rule (or some equivalent)
is the appropriate unit of analysis in meaningful learning. While the
issue cannot be resolved on purely logical grounds, mathematical
parsimony is a major factor to consider as it is in all science. In this
section, I shall introduce a formal characterization of meaningful
knowledge and show why this characterization favors the adoption of
the rule, rather than the association, as the basic unit of behavior.

To provide motivation for this characterization, consider the nature
of the knowledge underlying three closely related behavior potentials:
(1) the ability to say 'four' given a particular conglomeration of four
dots, (2) the ability to say 'four' given any conglomeration of four
objects, and (3) the ability to say the number of elements in an arbitrary
conglomeration of objects. The 'knowledge' underlying the first men-
tioned behavior potential would appear to be a clear instance of what
neuro-associationists call an association – a direct connection between
one stimulus and one response. The second involves the ability to give
a common response to a class of stimuli and would, thus, qualify as a
concept. The third potential is rule-like and involves the ability to give

\(^4\) Although the terms rule and principle are used interchangeably in
ordinary discourse, it appears desirable in certain experimental situations to
make a distinction between them. For example, see the experiment on rule
generality which will appear in a subsequent issue.
the appropriate response in a class of responses to any stimulus in a class of stimuli.

On deeper analysis, it turns out that the first behavior potential does not involve a connection between a particular nominal (overt) stimulus and a unique overt response. In describing this behavior potential, the intention clearly is to indicate the ability to say 'four' when shown any set of four dots arranged in a particular way. Similarly, any number of variations in the way 'four' is spoken would be allowed — all would be judged equivalent as long as they all mean four. The connection would, therefore, seem to be better thought of as being between properties which are common to the set of nominal stimuli, and properties common to the set of equivalent ways of saying four.5

Analogous consideration of the second behavior potential suggests that a similar connection is involved — this time with a more inclusive set of nominal stimuli and, concomitantly, fewer properties in common among the nominal stimuli. There does appear, however, to be a fundamental difference in the nature of the constructs underlying what neo-associationists call associations and what they call concepts. The nominal stimuli associated with a concept are not all equivalent, and certainly are not before the concept is learned. The connection, therefore, would seem to be between a class of distinctive properties,6 each defining a set of equivalent nominal stimuli, and a single set of equivalent overt responses — in short, a many-to-one relation. In the example above, each distinctive way of arranging four objects would constitute an element on the stimulus side. The response class would again consist of one element, the set of equivalent ways of saying four.7

The third behavior potential involves even more. Not only are the

5 Perhaps this fact is responsible for Jacobovits' (1966) recent conclusion that all S–R formulations involve mediation of the form S→r_m~s_m→R. More is said about this in the next section.

6 Actually, the elements of this class may consist of n-tuples of properties (e.g., large red triangle) rather than 'single' properties (e.g., red.). Each element, in turn, defines a set of nominal stimuli which are equivalent insofar as these properties are concerned. Thus, to talk about an n-tuple of properties is essentially the same as talking about a class of overt stimuli equivalent on these properties.

7 Exactly where the dividing line between a concept and an association is, of course, will always depend on prior learning. Even learning to equate different presentations of the same physical stimulus might be viewed as a concept where time (of presentation) is the crucial dimension to be abstracted over.
nominal stimuli functionally distinct but so are the overt responses. The stimulus and response classes consist, respectively, of (equivalent) conglomerations of objects and of 'sound' equivalents which refer to some (any) number. Furthermore, particular kinds of sounds can only be associated with certain kinds of objects. Making a sound for the number six, when shown a conglomeration of three objects, simply wouldn't do. Thus, the underlying knowledge connects a set of *functionally distinct* stimulus properties (those specifying the number of objects) and a set of *functionally distinct* response properties (those specifying a number) so that each functional stimulus is associated with one functional response. Since each set of properties denotes a class of stimuli or responses, the underlying knowledge may also be viewed as a relation between a *class of classes* of overt stimuli and a *class of classes* of overt responses.

While the differences among the three illustrative behavior potentials seem obvious enough, it may not be so clear that there also are important similarities. In each case, for example, the functional stimuli are quite distinct from the nominal stimuli – in fact, each functional stimulus is common to a class of (equivalent) nominal stimuli. Each functional stimulus, in turn, corresponds to a functional (internal) response, common to a class of (equivalent) overt responses. Thus, the primary difference in the 'knowledge' underlying the three illustrative behavior potentials resides in the number of functional stimuli and responses involved. The first behavior potential involves one functional stimulus, one functional response and the connection between them. The second involves a many-to-one connection, and the third, a large (infinite) number of distinct functional stimuli and responses.

It might appear, then, that the knowledge underlying the third behavior potential may be viewed as a collection of discrete associations but such an interpretation would in no way account for the close interrelationships among the constituent pairs. A similar comment would apply to the second potential. The 'associations' belong together in a fundamental behavioral sense. The very description of the third behavior potential, for example, implies the ability to give the appropriate response in the associated class of responses to any stimulus in the associated class of stimuli.

* A formal characterization

Depending on one's convictions, there would seem to be at least two
ways of characterizing the knowledge underlying the three behavior potentials. First, the association might be maintained as the basic unit of analysis and a relationship between stimulus-response pairs postulated according to whether or not they ‘belong’ together. Where only one (functional) stimulus-response pair is involved, of course, the relationship would be a behavioral tautology. Second, the rule may be taken as basic and the concept and association viewed as special cases. The concept would be a rule having a common (functional) response. The association would be further restricted and would consist of only one instance. In either case, it would be possible to account for all three kinds of behavior potential. There would seem to be little basis for selecting one or the other on strictly behavioral grounds. Any such choice will have to be made for other reasons.

Perhaps a more formal characterization of the knowledge underlying a rule may be suggestive. In line with the discussion above, a rule consists of a set of stimulus properties (D) which determine responses, a set of response properties (R), and an operation (O) between them such that each element in the first set is associated with exactly one element in the second. Thus, to distinguish one rule from another, three things must be specified: (1) a set of determining (D) stimulus properties (e.g., the set consisting of the numbers of objects in particular conglomerations of objects) from which the response properties are derived, (2) a class of response (R) properties (e.g., the set consisting of properties associated with ‘saying a number’), and (3) the operation (O) by which R is derived uniquely from D. In short, a rule may be characterized as an ordered triple, (D, O, R), where D, O, and R are defined as above.

8 The results of several experiments, which will be reported in Part II of this monograph, suggest that success on one test instance of a rule (i.e., giving the appropriate response to one test stimulus) typically implies success on other instances of the rule. To the extent that this hypothesis is valid, stimulus-response connections are related in an all-or-none fashion depending on whether or not they are instances of a common rule.

What I have called functional stimuli and internal responses are viewed by neo-associationists as mediating responses and response produced stimuli, respectively.

9 This combining operation might alternatively be called a transformation or mapping.

10 Equivalently, a rule may also be characterized in terms of description spaces of the sort used by Hunt (1962) to define concepts. A description space
This definition formally parallels that of the mathematical notion of a function; for it too can be characterized by a triple consisting of two sets (a domain and a range) and a transformation between them such that each element of the first set is paired with one element in the second. Hence, by definition, a rule is a function.

Since the characterizing sets are of arbitrary size, and, indeed, may be of infinite size, mathematical parsimony would seem to suggest that the rule be viewed as basic. The concept and association, then, would be viewed as special cases. A concept would be a constant-valued function and an association would be a function whose characterizing sets each contain only one element.

For certain purposes in what follows it is desirable to make a distinction between a rule statement and the construct itself. A rule statement, of course, must faithfully reflect the corresponding rule. To provide an adequate representation, it should make reference to the (unobservable) elements $D_1$, $O_1$, and $R_1$ characterizing a particular rule. This may be accomplished by a statement of the form $R'_1 = O'_i(D'_i)$ where $D'_1$, $O'_i$, and $R'_1$ are symbolic (observable) representations of $D_1$, $O_i$, and $R_1$, respectively. Primes are not used in what follows, except where necessary for clarity.

To prove useful, a new scientific language must in some substantive way represent an improvement over existing formulations. Hopefully, the foregoing discussion suggests that the SFL may provide a more

is simply a product space in which the one dimensional component spaces are stimulus dimensions. Each dimension is partitioned into values. For example, consider a description space consisting of the three dimensions, size, color, and shape, with the values large and small; black, white, and green; and triangle, respectively. Any stimulus object with some combination of these values may be placed in one of the resulting six ($2 \times 3 \times 1$) categories.

A derived description space is simply another description space derived from the first by mapping points (categories or descriptions) in the first space into a second space. The derived dimensions and values may be identical or different from those in the original space. The set of dimensional properties, $D$, corresponds to a description space; $R$ corresponds to a derived space; and $O$ corresponds to the mapping from one to the other.

This characterization, in terms of description spaces, has certain advantages; it is precise, can be represented graphically, and emphasizes the distinctions and relationships between values and dimensions. It has the disadvantage, however, of requiring somewhat more mathematical background than is necessary for present purposes. For this reason and because of space limitations, no further use of these notions is made in what follows.
precise and parsimonious basis for describing meaningful behavior than does the S–R language. In the following sections, SFL and neo-associationistic (S–R) formulations are compared in more detail and theoretical direction is given. The question of whether the SFL may lead to new and important questions and/or provide guidelines for formulating existing questions in researchable form will be considered in Parts II and III of this series.  

**FURTHER COMPARISON OF THE SFL AND S–R LANGUAGE**

Although it also is tied inextricably to observables, and is therefore behavioristic, the SFL is not ‘associationistic’, nor even ‘neo-associationistic’, in the traditional sense of these terms (e.g., Berlyne, 1965). In the SFL, a clear distinction has been made between observables which may result in the acquisition of rules (e.g., rule statements) and those (stimulus) inputs on which these constructs are presumed to operate. To be sure, the organism is thought to operate on the stimuli; the stimuli do not operate on the organism (although, of course, they do provide the occasion for making particular responses).

In order to consider meaningful behavior and the important concomitant role of internal stimulation, S–R analysts have chosen to extend empirically determined properties of overt stimuli and responses

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11 In an earlier version of the SFL (Scandura, 1966a, 1967; Scandura et al, 1967), the principle has been characterized as a four-tuple (I,D,O,R) where I denotes the set of stimulus properties which determine when the rule (D,O,R) is to be applied. This characterization is compatible with the way Gagne (1965), for example, has defined a principle — that is, as a competency which may be described by a statement of the form, 'If A, then B,' where A and B are concepts. The four-tuple characterization translates as a statement of the form, 'If I' then O' (D') = R''.

A distinction was also made between the denotation of a rule — presumably, the associated set of observable S–R pairs — and the rule construct itself. More formally, the denotation of a rule was represented: \( (S_i, R_i) \mid i \in I \) where I is some index set and \( (S_k, R_a) \) and \( (S_k, R_b) \) implies \( R_a = R_b \). (This definition requires that there be one S–R pair for each stimulus, \( S_i \).) Since a function can equally well be defined as a set of ordered pairs, as in terms of a domain, range, and transformation, it was the denotation of a rule that was viewed as a function — a set of ordered pairs, \( (x, y) \), in which each value of the first variable, \( x \), is paired with only one value of the second, \( y \).

Much of the research reported in Parts II and III of this monograph is based on earlier versions of the SFL.
to mediating processes. In effect, the constructs are assumed to have the same properties as the observable phenomena they are seeking to explain. It may be this sort of circularity that has led some writers (e.g., Anderson, 1964; Fodor, 1965) to severely question the explanatory power of intervening stimuli and responses.

In short, the crucial difference between the SFL and S–R approaches appears to be the nature of the construct used to tie observables together. In the former, the rule is basic; in the latter, the association is the fundamental building block. In this section, additional comparison is made of the SFL and S–R language.

**THE RULE VERSUS THE MEDIATING S–R ASSOCIATION**

Both rules and mediating S–R associations are constructs which may be used to represent connections between one set of observables (stimuli) and another (responses). Mediating S–R associations, however, are often combined into chains of indeterminate length and various interconnections between different chains are often postulated for explanatory purposes. In the SFL, although a counterpart to chaining exists (see below), only one-stage connections are necessary. Of perhaps more concern, S–R associations refer to only one type of connection – one-to-one associations between stimuli and responses, either overt or mediating. On the other hand, the transformation, O, is quite arbitrary and applies to an indeterminate number of stimuli and responses. It operates between a set of functional stimuli (D) and a set of functional responses. I would propose that this distinction is a nontrivial one, and may have important empirical as well as theoretical implications.

The almost exclusive use of the S–R language, in formulating research on both animal and human learning, has not greatly hindered neoassociationists since general rules and transformations have played no important part in their research. Even research on concept learning has been largely limited to problems which have lent themselves to S–R mediation arguments. It has only been very recently that the rule learning aspects of concept learning, for which no ready neoassociationistic representation is available, have been dealt with explicitly (Haygood and Bourne, 1965). These authors have pointed out that concept learning may involve both (stimulus) attribute identification and rule learning. Their use of the term 'rule' parallels that used here
and refers to a means of 'computing' (i.e., determining exemplars of) a concept (Hunt, Marin and Stone, 1966).

Another difference between the SFL and S–R languages involves the distinction between a state (e.g., stimulus, response, functional stimulus) and an operator (e.g., operation, map, association, transformation). In the S–R language, stimuli and responses (either functional or overt) are always viewed as states. Associations are always viewed as operators which transform one state (e.g., nominal stimulus) into another (e.g., mediating response). They are never viewed, for example, as responses to (other) stimuli. In the SFL, however, the application of one (higher order) rule may produce other rules. When applied to particular arithmetic number series (i.e., series with a common difference between successive terms), for example, the rule, 'substitute the particular values of the first term (A) and the last term (L) into the formula \( [(A + L)/2] N \) where \( N \) is the number of terms in an arithmetic number series', results in rules (formulas in N) for summing specific types of number series which differ only in the number of terms. 12

Perhaps the major difference between rule and association constructs resides in the way they account for transfer from one S–R pair to another. Cognitivists are inclined to equate the question of transfer with 'what (rule) is learned'. If an S–R pair is within the scope of a learned rule, transfer is to be expected but not otherwise. Thus, Smedslund (1953) has argued that transfer, as viewed by neoassociationists, is a pseudo-concept resulting from the notion that 'what is learned' can be inferred from a (relatively) constant situation and a proximal-peripheral classification of learning and transfer stimuli.

To be sure, S–R theorists have found this sort of explanatory mechanism quite adequate for their purposes. In simple learning, which

12 In an earlier formulation of the SFL (see previous footnote), I in the four-tuple (I,D,O,R) characterization of a principle served to determine which rule, denoted (D,O,R) would be used in particular situations. It now appears that the process of rule selection might better be viewed as a higher order rule of some sort.

It should be noted that the number of properties (of a situation) necessary for a rule to be applicable determines the variety of stimuli to which the rule can appropriately be applied. The fewer conditions required, the more generally applicable will the rule be. The more conditions required, the more restricted it will be. This relationship, of course, follows directly from the fact that the more properties the elements of a set have in common, the smaller the set.
involves a continuous physical dimension, an increase in the distance between training and transfer stimuli typically results in a corresponding decrement in performance – what has been called a stimulus generalization gradient. Furthermore, although the data are much less clear, a similar sort of generalization gradient is presumed to exist on the response side. As early as 1943, Hull also proposed a third type of generalization in which the formation of an association between \( S_1 \) and \( R_a \) tends to result in the formation of an association between \( S_2 \) and \( R_b \). But, as Berlyne (1965, 169) has pointed out ‘Stimulus-response generalization, . . . , has scarcely been investigated at all’, hardly a desirable state of affairs if rule-like behavior, in which a number of (related) \( S-R \) pairs are involved, is as critically involved in meaningful behavior as has been proposed.\(^\text{13}\)

Presumably, \( S-R \) theorists feel that this sort of explanatory mechanism is more operational than rule interpretations (of transfer). With meaningful (rule-governed) behavior, however, the potential advantage of equating degree of transfer with ‘distance’ disappears. Transfer with meaningful tasks, in which the stimuli and responses may be quite distinct, is typically an all-or-none phenomenon (e.g., Greeno and Scandura, 1966; Scandura, 1966b) rather than incremental as generalization gradient data would seem to suggest.\(^\text{14}\)

At first glance, it might appear that this difficulty in accounting for meaningful transfer might be overcome by simply postulating an all-or-none relationship between different \( S-R \) pairs. The problem with this alternative is to define a meaningful metric on the nominal (overt) stimuli and responses. It would, of course, be possible to predict

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\(^{13}\) Response generalization may in reality be nothing more than stimulus-response generalization. Although maintaining Hull’s (1943) original trichotomy, Berlyne (1965) admits that presumed differences between response generalization and stimulus-response generalization may be primarily a matter of convention.

\(^{14}\) Even \( S-R \) associationists (e.g., Berlyne, 1965, 171–174) are generally agreed that all-or-none transfer provides indirect support for a rule interpretation. To the extent, then, that the variables involved in meaningful learning are discrete, a rule interpretation may prove more useful.

Furthermore, when the underlying stimulus dimension(s) are continuous, \( S-R \) theorists will need to consider the possibility that generalization gradients are artifacts of averaging individual differences in perceptual discrimination (and, hence, what rule is learned) over continuous dimensions.
all-or-none transfer according to whether or not a stimulus has one of a
specified set of properties. But, properties are not nominal stimuli and
any measure on them would appear to be equivalent to a more intu-
tively direct interpretation based on 'what (rule) is learned'.

While on the topic of generalization, I might add that the present
emphasis on discrete variables is not as limiting as might be expected.
First of all, the critical values of meaningful stimuli can often be readily
distinguished. The numeral '6' always represents the number six and
never five. Problems involving 'just noticeable differences (jnd)', as in
psychophysics, are rarely of primary concern with many subject matters
like algebra and probably others as well. In some ways, the analysis of
actual subject matters may be less difficult and more precise than the
analysis of nonsense syllables. Furthermore, generalization gradients,
almost always noted when training and transfer stimuli differ along a
continuous dimension, may be an artifact due to averaging over dif-
ferent jnd (see footnote above). Data reported in Part II support the
contention that there normally is no decrement in performance on
transfer stimuli (within the scope of a rule) when the stimulus cues are
discrete and readily determinable.

CLASSIFICATION OF LEARNING TYPES

Current interest in taxonomy development has been intense (e.g.,
Gagne, 1965; Melton, 1964; Stolurow, 1964). More important,
there has been a concerted effort to uncover basic similarities between
what have heretofore been considered separate categories (e.g., Fitts,
1964; Gagne, 1964, 1965). The emphasis has been towards genotypic,
rather than phenotypic, bases.

Perhaps the most encompassing classification scheme is one proposed
recently by Gagne (1964, 1965). He identifies eight types of learning:
(1) signal learning — the establishment of a conditioned response which
is general, diffuse, and emotional, and not under voluntary control, to
some signal, (2) S–R learning — making very precise movements, under
voluntary control, to very specific stimuli, (3) chaining — connecting
together in a sequence two (or more) previously learned S–R pairs,
(4) verbal association — a subvariety of chaining in which verbal stimuli
and responses are involved, (5) multiple discrimination — learning a set
of distinct chains which are free of interference, (6) concept learning —
learning to respond to stimuli in terms of abstracted properties like
'color', 'shape', and 'number', (7) principle (rule) learning \(^{15}\) – acquiring the 'idea' involved in such propositions as 'If A, then B' where A and B are concepts; a chain or relationship between concepts, internal representations (of concepts) rather than observables being linked, (8) problem solving – combining old principles so as to form new ones.

According to Gagne (1965, 30–31), these varieties of learning were determined in accordance with the conditions required to bring them about. Thus, for example, the preconditions for signal learning are the nearly simultaneous presentation of two forms of stimulation, UCS and CS. Those for principle learning are the prior learning of the related concepts and the chaining of these concepts.

Although Gagne was only secondarily concerned with the problem of how to represent the knowledge underlying his eight types of learning, his taxonomy also provides a natural basis for comparing the S–R language and the SFL. S–R theorists, of course, would contend that the underlying knowledge can, in each case, be represented as a network of associations. Thus, the knowledge underlying signal learning (1) and S–R learning (2) (i.e., once the learning has taken place) might be represented as a simple S–R association. Chaining (3) and verbal association (4) might both involve a chain of S–R associations and be represented,

\[ S - r_{m_1} \sim s_{m_1} - r_{m_2} \sim s_{m_2} - \ldots - s_{m_n} - R. \]

The only difference between the two kinds of learning would be in the nature of the stimuli and responses which constitute the chain. The knowledge underlying a learned multiple discrimination (5) might be similarly represented,

\[ S' - r'_{m_1} \sim s'_{m_1} - r'_{m_2} \sim s'_{m_2} - \ldots - s'_{m_n} - R' \]
\[ S'' - r''_{m_1} \sim s''_{m_1} - r''_{m_2} \sim s''_{m_2} - \ldots - s''_{m_n} - R'' \]

\[ S''' - r'''_{m_1} \sim s'''_{m_1} - r'''_{m_2} \sim s'''_{m_2} - \ldots - s'''_{m_n} - R''' \]

(The primes are used to distinguish between the various chains of stimuli and responses.) The knowledge presumed to underlie a concept (6) can also be represented as a network of associations, this time with a number of different stimuli (exemplars of the concept) having a

\(^{15}\) Gagne has not made a distinction between rules and principles.
common mediating response whose stimulus properties, in turn, elicit a common overt response.

\[
\begin{align*}
S_1 & \\
S_2 & \rightarrow r_m \sim s_m - R \\
S_3 & 
\end{align*}
\]

When it comes to representing the knowledge underlying rules and principles (7) and problem solving (8), however, serious problems are encountered. Gagne’s (1964) original representations of principles and problem solving, for example, did not use the S–R language. Principles were represented as two concepts leading to a rule,

- concept A
- concept B

Similarly, knowledge underlying problem solving ability was represented as two or more rules leading to a higher order rule. In a later formulation, Gagne (1965) has represented knowledge of a principle as a chain of concepts,

- concept A — concept B —.

Concepts, however, are not directly observable whereas the S–R links in a chain can supposedly be made so. Using chaining mechanisms in both situations may indicate similarities which are more apparent than real.

More particularly, characterizing a rule or principle as a chain of concepts does not make clear why each exemplar (i.e., functional stimulus) of the first concept is assigned to a particular exemplar (i.e., response) of the second concept. With certain principles (I, D, O, R) (Scandura, 1967), it is possible to ‘patch up’ the formulation by asserting the existence of a second chain of associations between corresponding stimuli and responses (i.e., exemplars). Non-corresponding stimuli and responses are connected by only one chain, the chain of concepts. But, as yet this has not been done for arbitrary rules. With most rules, it
would seem most natural to postulate the existence of a general transform or combining operation which maps stimulus properties into response properties (Scandura, 1967). Presumably, it was with some such notion in mind that Gagne (1966) proposed the notion of an ‘Action Concept’ in his most recent statement on rule learning.

No such difficulties are encountered when the rule is taken as the basic unit of analysis. As we have seen, a rule (7) can be represented in the SFL as an ordered triple (D, O, R) where D, O, and R are as previously defined.

The other seven types of learning, identified by Gagne (1964, 1965), turn out to be either special cases or a composite of rules. Concepts (6) are simply rules in which the properties in D refer to a number of distinct classes of stimuli, and those in R refer to a single class of equivalent (overt) responses. In most studies of concept learning, O is a logical rule for combining relevant attributes (e.g., Haygood and Bourne, 1965; Bruner, Goodnow and Austin, 1956; Hunt, 1962).

In order to represent simple S–R associations (2) and signal learning (1), the SFL characterization of a rule is restricted still further. The set D consists of one element (which may consist of a multiple property such as ‘black triangle’) corresponding to a single class of equivalent (nominal) stimuli. The set R similarly corresponds to a single class of equivalent (overt) responses, and O to a simple mapping (i.e., association) between D and R. In this case, the nominal stimuli and responses, respectively, are all functionally equivalent.

Chaining (3) and verbal association (4) also have a natural counterpart in the SFL. Just as associations can be joined end-to-end so can rules (i.e., functions), the output of one rule serving as the input for the next. As suggested above, however, there is a major difference between a chain of associations and a composite rule (i.e., a ‘chain’ of rules). Associations in a chain refer to connections between either: (1) overt stimuli and functional stimuli (mediating responses), (2) mediating response-produced-stimuli and mediating responses, or (3) response-produced-stimuli and overt responses. Component rules, on the other hand, serve as a bridge between determining properties (i.e., functional stimuli; mediating responses) and internal responses (i.e., functional

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16 All many-one pairings, between stimulus properties and internal responses (in R), are not concepts in this sense. Such pairings may also consist of a number of discrete associations (having no common transformation) with only one distinctive response.
responses; response-produced-stimuli). In the SFL characterization of the rule, connections between overt stimuli and determining properties and between internal responses and overt responses are indicated by the common properties of the equivalence classes of overt stimuli in the class of determining properties and the common properties of the equivalence classes of overt responses in the class of (internal) response properties.

In much verbal association learning, the response learning phase (i.e., establishing a connection between what are called mediating stimuli or internal response properties and overt responses) is of considerable importance (e.g., UNDERWOOD and SCHULTZ, 1960). Of perhaps even greater concern in most higher forms of learning is the identification of the critical stimulus cues (in D). The internal response-overt response pairings, such as between numbers (internal) and numerals (overt), are often well-learned prior to rule learning. For example, in applying the summing rule, \( [(A + L)/2]N \), where \( A \) refers to the first term of an arithmetic number series, \( L \), to the last, and \( N \), to the number of terms, the user must be able to determine the properties in the set \( D \). Determining the number of terms \( (N) \) in long arithmetic series, however, has proved to be a non-trivial task for junior high school \( Ss \) (SCANDURA, WOODWARD and LEE, 1967).

The size of the 'gaps' between equivalent overt stimuli and responses and the classes to which they belong can be reduced by introducing what might be called 'perceptual' and response learning skills, skills which turn out also to be rules. Thus, it is often possible to introduce an additional 'gap-filling' rule which is somehow simpler and more general than the 'one in the middle'. Such rules may operate on more readily discernible (stimulus) properties.\(^\text{17}\) The number of terms, \( N \),

\(^{17}\) According to this view, determining characteristics and perceiving characteristics are essentially the same process - the difference being one of degree. Thus, just as the sum (16) of the number series, \( 1 + 3 + 5 + 7 \), can be derived from the properties 1, 7, 4, by computing \( (1 + 7)/2 \) 4; the number of terms 4 can be derived by counting the numerals in '1 + 3 + 5 + 7' - the individual numerals being more easily discernible than the properties 1, 7, and 4 used to derive the sum 16.

In most experimental studies, which directly involve perception, the concern has been primarily with less abstract characteristics - typically physical properties of the stimulus. In such cases derivation rules are probably well-learned. Still, all \( Ss \) at one time or another had to learn how to determine even physical characteristics. The new-born infant presumably is very limited in its
in an arithmetic series, for example, can be derived by use of the rule, 
\((L + C - A)/C\), where values of \(C\), the common difference between 
adjacent terms, are presumably easier to determine than values of \(N\).
In this case, the composite rule, \([(A + L)/2 \quad (L + C - A)/C]\), could 
serve equally as well as the original rule, \([(A + L)/2]N\). One can 
conceive of 'gap-filling' rules operating on the response side as well.
Thus, the ability to write any given numeral (e.g., '5') when shown a 
corresponding numeral can be thought of as a rule in which \(D\) might 
refer to those characteristic properties of pencil drawings (e.g., straight 
lines, half-circles, etc.) which the \(S\) had already educed from his 
experiences, \(R\) to the total arrangement (a property) of the numeral 
(a mark on the paper), and \(O\) to the 'actions' which must be taken to 
produce the numeral. A composite rule, of course, has the same prop-
erties as any other rule. Unless the stimuli and responses corresponding 
to each constituent rule are actually observed, however, guessing the 
stages a learner went through would appear to serve no useful purpose.

In order to represent multiple discriminations (5), it is necessary to 
refer to those properties of each stimulus situation which identify the 
appropriate rule. Presumably, a number of discrete associations (i.e.,
degenerate rules) become well-learned when the identifying properties, 
corresponding to the distinct associations, become disjoint and inter-
fERENCE free. Multiple discrimination learning can also be of a more 
general sort. Thus, rather than simple associations, discrete (i.e., non 
overlapping) rules might be involved.\(^\text{18}\) Presumably, Gagne (1964,
1965) had such things in mind as composite rules and generalized 
multiple discriminations when he referred to problem solving (8).

\(^{18}\) It may be worth noting that a single principle may refer to the same 
stimulus class as do a number of more restrictive principles taken collect-
ively. The principles, 'If large and black, then (the response varies with) shape' and
'If large and white, then shape,' may be considered special cases of the principle,
'If large, then color and shape'. The latter, more general, principle at once has 
fewer identifying cues and more response determining dimensions than the other 
two. In effect, it appears that critical response dimensions are traded off with 
critical rule identifying cues. The more general the rule, the more stimulus 
attributes vary with the responses; the more specific the rule, the more stimulus 
properties are required to identify the rule. The total number of critical 
properties remains constant.
This discussion provides another argument for adopting the rule, rather than the association, as the basic unit of behavior in meaningful learning. Not only does the rule have the important advantage of mathematical parsimony but it is possible to view more different kinds of knowledge in terms of rules than in terms of associations. Each kind of knowledge in Gagne's (1964, 1965) taxonomy can be characterized as a rule or as a composite of rules. On the other hand; no one has yet devised a way to satisfactorily represent rules in terms of associations. Furthermore, the rule representations are not subject to criticism for failure to make an explicit distinction between constructs and observables (e.g., see Berlyne, 1965).

Other Comments

Directions serve an important role in almost all experimentation with humans. In meaningful learning, this role may be critical (e.g., Maier, 1930; Gagne, 1962, 1964). According to Gagne (1964, 305), directions may serve to: (1) identify the terminal performance required, (2) identify parts of the stimulus situation, (3) aid the recall of relevant subordinate performance capabilities, and (4) channel thinking. The characterization of a rule as an ordered triple (D, O, R) would seem to reflect each of these functions, at least superficially. R corresponds to the desired class of responses; D corresponds to critical stimulus cues; and information about O corresponds to aiding recall and channeling thinking.

In other situations, directions may serve a higher order role. They may simply define a context or provide a highly nonspecific objective and thereby limit the number of rules which might be evoked in a given situation. This function is served, for example, when S is told to pay attention only to the material 'printed in red'. Similarly, such directions as, 'Try hard to learn', would appear to serve a motivation-like function and are often used in the laboratory in attempts to manipulate S's orientation. Detailed consideration of the underlying mechanisms is beyond the scope of this paper but they do not appear to be incompatible with the SFL.

References


Smedslund, J., 1953. The problem of 'what is learned?' Psychol. Rev. 60, 157-158.