NEW DIRECTIONS FOR THEORY AND RESEARCH ON RULE LEARNING

III. ANALYSES AND THEORETICAL DIRECTION

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A new scientific language (i.e., the set-function language, SFL) was introduced in the first article in this series. The second article reports a number of empirical studies, which were motivated in part by the SFL. In this, the third and final article, the SFL is again used as a guide to analyze a number of new problem areas. Theoretical direction is also given. An operational definition of 'what (rule) is learned' is proposed and consideration is given to the form a predictive theory, based on the rule construct, is likely to take.

REVERSAL AND NONREVERSAL SHIFTS

Fig. 1 characterizes reversal and nonreversal shifts. Note that the objects vary on two dimensions, size (large, small) and color (black, white). In the experimental setting, these objects are shown in pairs and S is required to choose the correct alternative, indicated by + in fig. 1.

The experimental paradigm involves learning to make two discriminations, the second after S can reliably make the first. The first discrimination is the same for all Ss; the second depends on the treatment, reversal or nonreversal. On the first discrimination, in the example shown in fig. 1, size is the relevant dimension, the larger object being positive. Color is irrelevant. A reversal shift involves the same dimension, size, but the correct response is the smaller object. A nonreversal shift involves the color dimension, black being correct.

On the average, preverbal children (KENDLER, KENDLER and WELLS, 1960) and animals (KELLEHER, 1956) find nonreversal shifts easier to make than reversal shifts whereas older, more verbal, Ss find reversal shifts easier (BUSS, 1956; HARROW and FRIEDMAN, 1958; ISAACS and DUNCAN, 1962; KENDLER and D'AMATO, 1955). In a study with
kindergarten Ss, Kendler and Kendler (1959) found that fast learners, like verbal Ss, were better able to make a reversal shift whereas slow learners, like preverbal Ss, were better able to make a nonreversal shift. It was suggested that the fast Ss approached the experimental task with verbal labels for the correct stimuli already strongly attached, the verbal labels serving as mediating links in a two-stage S-R paradigm. The learning of the slow and presumably preverbal Ss was assumed to involve a single-stage paradigm. Kendler and Kendler (1962) explained the relative ease of reversal and nonreversal shifts in terms of the number of S-R associations that need to be changed.

This interpretation, however, makes no provision for answering the question of whether the increasing ease of making reversal shifts with age is due to a gradual increase in verbal ability by all Ss or to some specific characteristic had by a larger proportion of faster learning children. Reformulating the reversal-nonreversal problem in the SFL provides a basis for answering this question.

Since this problem is different, in an important sense, from any previously encountered in this paper, some preliminary observations are in order. For one thing, the stimuli consist of pairs of objects. In addition, the critical stimulus (set) properties are relations between objects. Learning to make the first discrimination shown in fig. 1, for example, may involve learning a rule in which \( D = \{ \text{one object larger than other} \} \), \( R = \{ \text{larger object} \} \), and \( O \) is a transformation which maps

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1 Stimuli, bearing a particular relationship to one another need not have any 'physical' property in common.
the relational property in D onto R.² Such a discrimination can also be accomplished by learning two less general rules. Notice that during the first discrimination task, S is never presented with two large or two small objects. Thus, S could learn to always choose the large black object when it appears and the large white object when it appears. The fact that both are larger than the object with which they are shown might go unnoticed, for example, with young children. One such principle (it is necessary to distinguish between exemplars and non-exemplars) might be stated, 'If one of two objects is large and black, then choose the large black object'.³

Learning a single relational rule on the first discrimination should result in reversal shifts being easier than nonreversal shifts. In a reversal shift the critical response determining properties would remain the same, only the operation, O, would need to be changed – pick the smaller, rather than the larger, object. A nonreversal shift would involve learning either a completely new general rule or two new more specific principles, in which both the critical cues (involving color) and operations would need to be identified. On the other hand, if two less general principles are learned on the first discrimination, a reversal shift would involve learning two new specific principles (or one more general rule) whereas a nonreversal shift would involve learning only one. The principle, indicated by the statement, 'If one of two objects is large and black, then choose the large black object', is equally applicable to the original discrimination task and the nonreversal shift task in which black is positive (see fig. 1). In short, the relative ease of shift may be dependent on 'what is learned' on the first discrimination.

So far, of course, this interpretation is quite analogous to that presented by S-R theorists (e.g., Goss, 1961; Kendler and Kendler, 1962). It is the assessment methodology which provides the means for determining whether 'what is learned' is related to relative ease of shift in individuals. In order to determine a particular S's basis for making the first discrimination, it is necessary to employ dimensions, such as color and shape, which have more than two easily discriminated values.

² A rule of this sort is analogous to what Bruner, Goodnow, and Austin (1956) have called a relational concept. Although not treated here, there is reason to believe that transposition phenomena (cf. Hebert and Krantz, 1965) may also be reformulated in similar terms.

³ It should be emphasized that S need not be aware of verbal labels in order to learn such a principle.
In this way, it would be possible to use two values of each dimension in presenting the first discrimination problem, leaving the other values for assessment purposes. Suppose, for example, the four objects used in training are either black or white and a circle or a square. Suppose, further, that color is the critical dimension on the first discrimination. Then, the assessment procedure might involve a new discrimination in which the objects used are describable in terms of the two colors (black, white) used during training, and a shape (e.g., triangle) not so used.

In order to help minimize 'strategy shifts', between learning and assessment, positive reinforcement might be given at each choice point, no matter what the response. Under these and appropriate instructional conditions, choosing as positive the transfer object, having the same color as the positive training object, would be indicative of learning a general rule on the training task were it not for the high probability (1/2) of choosing this object by chance alone. Assessment 'certainty', of course, might be increased by using more than one test discrimination task. This could be made possible by increasing the number of values per dimension. Following the same line of reasoning, one could devise reversal-nonreversal type tasks involving more general principles and/or principles which do not involve relational stimulus properties. Under such conditions, assessment problems can be minimized.

Unfortunately, there is little data which is directly relevant. Tighe (1965), however, has published a paper which provides general support. She demonstrated that the relative ease of making reversal and non-reversal shifts with 5- and 6-year-old children can be manipulated by prior training designed to emphasize the independence and *dimensional* nature of the properties of stimuli used in subsequent discrimination shifts. These results provide support for the current interpretation in that learning a general rule on the first discrimination requires that S recognize the independence of the two object dimensions. With the more restrictive principles only object, and not dimensional, differences are important. Because of the obvious implications for future research in this area, early verification of this analysis is urged.

**Conservation versus non-conservation**

Questions relating to the conservation (i.e., invariance) of such properties as amount and number (e.g., see Flavell, 1963), comprise
another problem area in cognitive development that may be reformulated in similar terms.

Consider a procedure that is frequently used to determine whether a child has learned to conserve amount. $E$ shows the child two balls of clay, both of the same shape and size as in display one of fig. 2. (Indeed, $E$ may let $S$ make them the same size.) $S$ is then asked whether each ball, in turn, contains more clay than the other or whether they contain the same amount. Invariably, the normal child says they are the same. Next, $E$ rolls one of the balls into the shape of a sausage as shown in display two of fig. 2 and, then, asks the same question. If the child says that they are the same, and he does so consistently on this and related tasks, he is said to conserve amount. If not, he is a nonconserver.

The important question, from the present point of view, is not merely whether $S$ is or is not a conserver, but on what basis $S$ is responding. The objects shown in the fig. 2 displays are related in many ways—relative volume, weight, height, length, width, shape—besides relative amount. Any one of these (not necessarily independent) relational properties could provide a basis for responding. In display one, for example, the two objects contain the same amount and have the same length (as well as having just about everything else in common except position). Thus, a correct response to display one could signify the operation of any number of rules, including those described by 'the response depends on the relative amount', or '... relative length'. A child's reaction to display two, however, might make it possible to determine which of these two rules is operating. Thus, if the child says that the sausage (in display two) contains more clay, the operating principle probably
involves length. If he says 'the same', amount is most likely the determining factor.  

Clearly, this sort of analysis is quite similar to that applied to the problem of reversal and nonreversal shifts. Both involve relational properties and assessing what the child has previously learned. The essential difference is that, with reversal and nonreversal shifts, the relevant prior experiences (on the first discrimination) can be manipulated. In the latter situation, conservation of amount is assumed to have been acquired or not acquired prior to the experiment.

**Syntactic and Semantic Learning**

All of the stimuli (as well as rule statements) considered so far are symbolic representations of an abstraction. A stimulus such as '1 + 3 + 5 + 7', for example, symbolizes an abstraction reflecting the structure of a variety of other stimulus situations - e.g., four stacks of pennies, the first containing one penny, the second three, the third five, and the fourth seven; a figure representing the produce of four countries, . . . , etc.

Suppose a young child has been taught a rule which makes it possible to say '16' when shown '1 + 3 + 5 + 7'. What happens when he is presented with the four stacks of pennies and is asked how many there are? The answer to this question undoubtedly depends largely on the significance to $S$ of the number symbols (i.e., numerals) in the symbolic stimulus. If the numerals refer to properties of collections of sets, each including a common number of elements or objects, and '+' signifies combining, positive transfer would not be unexpected. If, on the other hand, the numerals and the arithmetic operation of addition has been learned entirely without concrete referents, say with flash cards, one could feel fairly certain that $S$ would see no such relationship.

Some hint as to the factors involved can be obtained by characterizing the rules involved in the SFL. Assume that the rule, corresponding to the symbolic stimulus, has been determined, by assessment

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4 Note that both of these two responses eliminate *height* from consideration as a basis for responding. Saying that the sausage contains more clay, however, does not eliminate *shape*.

5 A child, of course, may acquire both the concepts of amount and length in a generic (nonverbal) sense (e.g., Braine, 1959) before he knows what the words 'amount' and 'length' actually mean. All that can be said with confidence is that the phrase, 'which contains *more* clay', may be interpreted by a child in any one of several alternative ways.
procedures to be \((D = \{1, 3, 5, 7\}, \ R = \{16\}, \ O = \text{sequential addition})\). The requisite for applying this (symbolic) rule in a \textit{concrete} situation, once it has been learned, is precisely that rule which makes it possible to go from the concrete situation to the corresponding numbers. A \textit{composite} rule, including this rule, might be characterized \((D = \{1 \text{ penny}, \ldots, 7 \text{ pennies}\}, \ R = \{16 \text{ pennies}\}, \ O = \text{translate the property } X \text{ pennies into the higher order (more abstract) property } x (\textit{the number}), \text{ perform repeated addition, and translate } x \text{ back into } X \text{ pennies}\). Learning the symbolic rule statement, without being able to recognize its concrete referents, would be like having an egg shell but no egg.

In any case, I would propose a need for similar analysis in other situations. Even partial identification of the competencies required for various kinds of behavior could have important practical as well as theoretical implications and is long overdue.

\textbf{AN ANALYSIS OF LEARNING MATHEMATICS BY EXPOSITION AND DISCOVERY}

With the machinery and body of data built up, we are now in a position to provide a fairly detailed analysis of learning by exposition and discovery. Because of space limitations, I shall limit my analysis to the question, 'Is it better to learn by exposition or by discovery and, if there is no unique answer to this question, what are some of the conditions under which a particular method will be better?' In this regard, only the contention that learning by discovery enhances the learner's ability to solve new problems will be considered. Before attempting to answer this question, let me reemphasize the differences between reception learning and learning by discovery (Scandura, 1964).

Reception learning involves the decoding of information presented directly to the learner. Usually, but not necessarily, such information is presented in statement form. Understanding is said to occur, if having learned a principle statement so that it can be ‘parroted back’ on cue, \(S\) is also able to use it to successfully determine the responses to the specified set of stimuli. As formulated in the SFL, \(S\) is presented with a statement that \textit{may} be put in the form, 'If I', then \(O'(D') = R'.\) Then, having memorized the statement, \(S\) is tested on stimuli arbitrarily selected from the domain (i.e., the set of stimuli) of the corresponding \textit{denotation}, \(\{(S_i, R_i) \mid i \in I\}\).
Learning by discovery, on the other hand, requires that the learner abstract the common rule or principle from a subset of the S-R pairs in the set \( \{(S_i, R_i) \mid i \in I\} \). S must, therefore, identify the determining characteristics, D, the operation, O, and if required to discriminate between exemplar and nonexemplar stimuli, he must also determine the identifying characteristics, I. In effect, reception learning presupposes the ability to interpret and apply that information represented by the symbols 'I', 'D', 'O', 'R'; learning by discovery requires the ability to identify, in a nonverbal fashion, and to use I, D, O, and R.

Unfortunately, most existing studies comparing exposition and discovery have little to say about whether discovery enhances the ability to solve new problems as is so often proposed by pedagogical enthusiasts (Davis, 1960; Hendrix, 1961). Suppose, as is often done, the exposition group is presented directly with a rule or principle whereas the discovery group is simply presented with stimuli, to which the principle relates, and required to determine the responses. Presumably, this is accomplished by discovering the associated rule or principle. The two groups are then frequently tested with the original stimuli (e.g., problems), new stimuli within the domain of the principles involved, and stimuli requiring new rules for their solution. What is gained by comparing the expository and discovery groups on several measures, however, is lost in another way. Whenever differences in performance on withinscope problems are found, comparisons on any more general transfer tests will necessarily be inconclusive. Thus, obtained differences in the ability to obtain solutions to new problems, based on new rules, might equally well be due to differences in original learning as due to any advantages inherent in a method itself (Scandura, 1967a; Scandura and Roughead, 1968). In short, such studies can never hope to determine whether discovery methods actually improve the learner’s capacity for dealing with new problems.

To make a valid comparison of the sort described, exposition and discovery groups must be equated on original learning. Suppose experimental Ss are set with the task of learning several principles. In

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6 It is not uncommon in such studies, in fact, rather typical, for the rule-given group to perform better on within-scope problems. The symbolism study, described in part II, however, demonstrates quite clearly that this need not necessarily be so. If the constituent symbols are not familiar to the learner, his performance on within-, as well as extra-, scope test problems may be expected to be uniformly poor.
this case, the appropriate tasks on which to equate exposition and
discovery groups are those which require the ability to apply the
principles taught (or discovered). Assuming the necessary equivalence,
tasks, requiring new principles for their solution, may be posed to
compare the groups as to the ability to deal with new problems (based
on new principles).

In order to predict how exposition and discovery groups would fare
on such tasks, explicit consideration must be given to what is required
of the learner in the two situations and the nature of the transfer
problems. Discovery Ss must learn how to derive rules (those involved
in the learning situation) in order to achieve criterion; exposition Ss may
not. Making predictions, then, really boils down to what discovery Ss
learn that exposition Ss might not. It is likely that discovery Ss, in
attaining criterion, may discover a derivation rule by which new
principles, similar to those originally learned, can also be derived. In
this case, discovery Ss might be expected to perform better than
expository Ss on tasks which can be solved via principles within the
scope of the derivation rule discovered. On the other hand, discovery
Ss would probably have no special advantage on problems, requiring
for their solution, principles beyond the scope of this derivation rule.

Suppose, for example, that (as in part II) exposition and discovery
groups are set with the task of determining the sums of number series –
long series so that it is infeasible to perform sequential addition.
Further suppose that two formulas (i.e., rules), say $n^2$ and $n^2 + n$, will
suffice for all of the learning series. The discovery Ss, of course, would
have to discover these formulas whereas they would simply be presented
to the exposition Ss. Assuming equal mastery by the two groups, one
would make the following predictions. If the formula for obtaining the
sum of a new transfer series may be derived in the same manner (i.e.,
via the same derivation principle) as the two original formulas, then the
discovery Ss may be expected to demonstrate superior ability. If not,
no such difference may be expected.

Consider a second example in which the Ss are to demonstrate their
ability to, say, construct models corresponding to each of two finite
geometries.7 Suppose the corresponding axiom systems are:

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7 It is assumed, of course, that referents, such as $'a'$, $'b'$, and $'c$–$d'$, have been
assigned to the undefined terms.
A. 1. there exists at least one point;
   2. each point is on exactly two lines;
   3. each line contains exactly two points;
   4. two points determine at most one line;
   5. there are no two lines not having a point in common.

B. 1, 2, 3, and 4 identical to system A;
   5. to each line, there corresponds exactly two lines which do
      not have a point in common with it.

Then, the models

![Fig. 3a.]

and

![Fig. 3b.]

correspond to systems A and B, respectively.

The task posed might be viewed as involving two discrete principles,
each having a denotation consisting of one stimulus (i.e., list of axioms)
and one response (i.e., model). If the exposition Ss are presented with
the models directly while the discovery Ss are required to derive them,
it would not be surprising if the discovery Ss could derive the model

![Fig. 3c.]

for the system,

C. 1, 2, 3, and 4 identical to systems A and B;
5. to each line, there corresponds exactly one line which does not have a point in common with it, while the exposition Ss could not. On the other hand, an axiom system of the form,

D. 1. there exist exactly nine points;
    2. each line contains at least three points;
    3. ...,

might well pose equivalent difficulties for both groups.

It appears, on analysis, that these experiments were rigged in favor of discovery. The discovery Ss were required to learn a principle(s) for deriving solutions; the exposition Ss were not. Of course, nature is not always as simple as we would like so that it is, indeed, highly encouraging that strong support for the essential correctness of the proposed analysis has been obtained (see part II). Learning by discovery does, in fact, seem to improve ability to solve new (within-scope) problems when original learning is controlled.

Nonetheless, consider what happens when we bring guidance into the discovery situation. Hints might be given, for example, to cue the determining attributes, D. They might also be used to direct the learner toward the appropriate combining operation, O. In fact, the Ss might be given all of this information directly. But, then, is not this something very much akin to teaching by exposition? It would appear to be at least theoretically possible to present derivation rules in an expository manner rather than to depend on discovery.

So far, I have avoided the $64 question — just what are derivation rules and can they be stated in expository form? It turns out that an extremely simple derivation rule can be specified in the case of the finite geometries example. Let the determining attribute be the number of lines not having a point in common with a given line (axiom 5). The combining operation, O, would be that mapping which takes this number into a regular polygon with three more sides than the number in question. Whether this specification concurs with what the reader feels that a mathematics student should learn in such a situation is not the point. I, too, would be unhappy if this is all he learned. The point is that he could solve the problems posed, armed only with the simple derivation rule indicated.  

8 One way to get closer to a 'more desired' sort of learning may be by imposing additional restrictions on the expected performance.
Of course, the results of the symbolism study (in part II) indicate that being able to specify and state a higher order derivation rule in symbolic form may not always be sufficient to insure learning. Among other things, the language used must be interpretable by the learner. S might not, for example, know to what the symbol 'c' refers. Unless the derivation rule can be specified in a symbolic form that the learner can correctly interpret, there is no alternative to learning by discovery. If, on the other hand, a comprehensible symbolic (or iconic) representation can be given, the question of whether exposition or discovery is to be preferred will depend on factors such as learning efficiency and retention – two questions to which we have not addressed ourselves.

**AN OPERATIONAL DEFINITION OF 'WHAT IS LEARNED'**

The question of 'what is learned' is inextricably tied to the question of transfer. In S-R formulations, 'what is learned' is usually equated with the ability to give the learned response to the training stimulus. Transfer is then explained in terms of the proximity of the transfer stimulus to the training stimulus (e.g., Kendler, 1952; Smedslund, 1953) or, more directly relevant to rule learning, the proximity of the transfer instance (i.e. stimulus-response pair) (Berlyne, 1965). It is generally assumed that the more similar the transfer item is to the training item, the more likely is transfer to take place.

Nonetheless, the rule has been gaining favor recently among educational psychologists and psycho-linguists, as well as cognitive psychologists generally, as the preferred means of explaining transfer. Perhaps the primary reason for this change is that in meaningful learning similarity cannot normally be measured in terms of distances along a continuum and, although I cannot detail my reasons here for believing so, defining a metric on a set of meaningful stimuli appears equivalent to defining a rule. Other widely mentioned reasons are: (1) transfer is typically an all-or-none phenomenon in meaningful learning (e.g., Greeno and Scandura, 1966; Levine, Miller and Steinmeyer, 1967; Scandura, 1966a), (2) networks of associations seem incapable of reflecting human capacities for language and other subject matter behavior – the number of associations that would be required is much greater than could reasonably be acquired by a person during a normal lifetime (e.g., Chomsky, 1957), (3) mathematical parsimony suggests that the association can best be viewed as a special case of the rule (rather than the reverse as is supposed by S-R theorists – see part I).
In rule interpretations, the tendency has been to explain transfer in terms of 'what (rule) is learned'. Such interpretations, however, have been rightly criticized as lacking operational definition. On strictly logical grounds it is effectively impossible to define in terms of performance 'what (rule) is learned' in any unique sense. There are typically many different routes to the same end. For another thing, rules typically have a large, if not infinite, number of instances. It is practically possible to test for the acquisition of only a relative few.

On the positive side of the ledger, it does not appear necessary to know everything that a subject knows in order to predict what he will do in a given situation. Much of a subject's knowledge becomes irrelevant once a specific context is specified. More important, an increasing amount of evidence (e.g., Le Vine, 1966; Le Vine, Leitenberg, and Richter, 1964; Scandura, 1966b, 1967b) suggests that the relevant knowledge which underlies mathematical and other meaningful behavior can often be specified with a fair degree of precision. In part I, for example, research was reported in which giving the appropriate response to one test instance (of a rule) almost invariably implied the ability to give the appropriate response to any other. My first reaction to these results was that one test instance might, at least for certain relatively simple tasks, be sufficient to determine 'what (rule) is learned'. While the results were generally quite favorable for such an interpretation, however, there were enough exceptions (see part II) where the subject performed well on some instances but not on others that for theoretical purposes a more complete operational definition seemed desirable.

Collectively, these observations place important restrictions on the form a truly adequate operational definition of 'what is learned' might take. First, it is essentially impossible to define 'what rule is learned' in any unique fashion. Second, an operational definition of what is learned must be formulated relative to a particular contextual situation which involves a goal and a stimulus context from which the response is to be derived. Third, any such definition must be based on performance on a small finite number of instances and, if possible, it should be applicable no matter how many test instances are employed.

In view of these restrictions, any attempt to operationally define what particular rule is learned seems doomed to failure from the start. What appears to be needed is a definition which takes into account all feasible underlying rules. Such a definition can be given by specifying what rule
is learned up to a *class* of rules. Thus, given that a particular stimulus elicits a response, 'what is learned' can be defined as some (unspecified) rule in that (specified) class of those rules whose denotations all include the given stimulus-response pair. This definition should be interpreted to mean that *one* of the rules in the class has been used in responding to the test item, not that all of them have been used.

Fortunately, this definition can be generalized directly to include any number of test instances. Given a particular context, and a performance capability summarized by a set of stimulus-response pairs, 'what is learned' is defined as some particular rule in that class of rules whose denotations all include the set of S-R pairs.

The utility of this definition depends on the unstated assumption that some particular underlying rule (which is an element in the class 'what is learned') remains in use as long as the context remains stable. If this were not the case, the definition would not be well-defined where more than one test instance is allowed. 'What is learned' could depend on which of the allowable rules (in the class) was in operation at a given point in time.

As described in part II, my students and I have collected a fairly substantial body of data over the past few years which makes this consistency hypothesis a highly feasible assumption. Whenever the response given by a subject to one test stimulus was in accord with the particular given or derived rule, so was the response to a second test stimulus. We have generally been able to predict second test behavior with anywhere between 80 % and 95 % accuracy.

More important, the results of the Extra-Scope Transfer Study (see part II) strongly suggest that, in general, the effective size of the defined class, 'what is learned', can often be reduced considerably by the appropriate selection of two test instances. In particular, Durnin and I found that successful performance with two stimuli which differ along one or more familiar dimensions implies successful performance with other stimuli which differ only along these dimensions. This result suggests that success on two instances which differ simultaneously along all possible dimensions involved in a rule may be adequate to define 'what (rule) is learned' in an essentially unique fashion. The question remains, of course, as to what is meant by a well-learned dimension.

Particularly since the boundary conditions for our findings and hypotheses are still to be detailed, it is encouraging that other investi-
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Investigators have also found this consistency notion useful. Levine, Leitenberg and Richter (1964), for example, have used performance on reinforced trials to predict performance on non-reinforced trials with a high degree of success. In so far as I can see, it appears that the entire notion of response consistency involves capitalizing on Einstellung to ascertain what is learned and to predict performance on future items. Rather than viewing consistency as an undesirable behavioral tendency to be disrupted whenever possible, my own guess is that the consistency hypothesis is a basic law of behavior which will prove fundamental to any predictive theory based on the rule construct.

THEORETICAL COMMENT

In parts II and III of this monograph, the SFL has been used as a guide to at least partially clarify a number of problems involving meaningful behavior. Nonetheless, this monograph has dealt more with a precise scientific language than with theory. Whatever clarification has been achieved was based not so much on new theoretical assumptions as on logical analyses of the situations involved, sometimes by making formal use of the SFL to help clarify particularly difficult points.

Nonetheless, use of the rule as the basic unit of behavior, is bound to have important theoretical implications. A recurring theme of this monograph has been that the association is too restrictive a unit on which to build a theory of meaningful learning. Although close relationships have been shown between the association and the rule (see part I), the representation of increasingly complex learning situations, in terms of associations, soon becomes prohibitively cumbersome. The rule, on the other hand, is not only an efficient mechanism but it has much in common with a number of cognitive constructs, variously called schemas, TOTE units, etc., which have long been used to account for complex behavior. Equally important, the SFL representation of a rule provides cognitive theorists with a much needed ingredient - precision.

Although it is beyond the scope of this paper to present anything approaching a finely-spun theory, I will try to indicate two ways in which theory development, based on the SFL, might proceed. In the process, I hope to suggest some fundamental differences between statistical theories, designed to predict group behavior, and more deterministic theories, which may make possible the prediction of individual behavior.
The first approach is well exemplified by, but certainly not limited to, stochastic learning models. In these theories, given an initial state, each $S$ is typically assumed on each trial to enter the next state with a certain probability. This process continues until the terminal or absorbing state is reached. To make predictions, based on such theories, the values of the transition probabilities, which presumably are based on underlying, physiological capacities, are estimated from data acquired in situations which are closely related to, and usually the same as, those in which the predictions are made. The predictions, themselves, deal exclusively with statistics defined on the resulting distributions of scores. In short, assumptions are made about individual learning processes and predictions are made about group behavior.

In order to see how such a theory might develop in a context of rule or principle learning, consider a situation in which instances of a rule (i.e., related S-R pairs) are presented until $S$ can reliably anticipate new responses to new, within scope, stimuli. To discover the underlying rule, $S$ must (1) determine the relevant properties of the stimuli and responses and (2) discover the transformation relating them. In addition, if noninstances are included in the test list, $S$ would have to identify those properties of the situation which make it possible to discriminate between instances and noninstances.

These requirements imply a model extension of the sort suggested by Haygood and Bourne (1965). These authors postulated the need for a second process, rule selection, which is independent of attribute selection – the latter providing the basis for Restle’s (1962) theory of concept learning (i.e., attribute identification). In the present, more general, situation, stimulus dimensions, rather than properties, would need to be considered. The identification of the relevant response dimensions would also be involved. Only under these conditions could rule learning be expected. Varying stimuli, although sufficient for concept learning, would not necessarily increase the likelihood of discovering a rule from its instances. The responses would also have to vary. These considerations imply a model with at least three, and possibly more, independent processes.

A major limitation of present-day stochastic models is that they are fundamentally incapable of predicting individual performance. All $S$s are assumed to bring with them the same learning parameters (i.e., transition probabilities); yet, it is common knowledge that $S$s enter these
situations with important experiential differences. Such differences, of course, could be taken into account by assigning different transition probabilities to different groups of Ss. In the limiting case, individual parameters could be based on individual data. But, then, no rational theorist would resort to such nonsense – using an individual’s data to predict the individual’s data.

The purpose of this discussion has not been to discredit stochastic models. Predicting group performance is equally as defensible, and important, as predicting individual performance. What I have hoped to do is to provide a perspective for viewing present theoretical concerns.

In contrast to the stochastic approach, consider a theory which says, in effect, that if the relevant things about S’s present state are known, it would be possible to predict what he would do in any given situation. Such a theory would necessarily be concerned with determining what the relevant state characteristics are and with procedures for determining whether they are within S’s immediate repertoire (see previous section). Rather than making assumptions about underlying physiological capacities (e.g., values of transition probabilities) to make predictions about learning, the proposed approach would involve inferring behavioral capacities in one situation to predict behavior in another. It would be a measurement based theory, unlike that exposed by present day neo-associationism, in which internal factors, determined from past behavior, would play the central role. Such a theory would need to specify how inputs interact with the state of an organism to produce responses. Scandura (1966b) has argued that a theory of this sort would be, to a large extent, independent of traditional learning theory.

The writings of many theorists (e.g., Muller, 1913; Hull, 1935; Berlyne, 1965) indicate that there are at least two aspects of an individual’s cognitive state that may affect what response is given to a presented stimulus, motivation and prior knowledge. Although I am not

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9 There are, of course, situations in which organismic differences may be minimal. The backgrounds of rats and new-born babies undoubtedly differ less than that of normal adults. Similarly, humans are more likely to flutter their eyelids in the same way than they are to solve mathematical problems. In such situations, largely ignoring the effects of prior learning has caused relatively few difficulties.

10 One might, however, use an individual’s behavior in one situation to predict his behavior in another situation (Scandura, 1966b). But, this would seem to require a theory with quite different properties from the usual variety of stochastic model.
prepared to defend the idea here, motivation might conceivably be viewed as a higher order or ‘incomplete rule’ of some sort which determines what unit of knowledge (i.e., rule) is to be applied. (The rule, itself, uniquely determines what the response is to be.) Without some such mechanism, organisms would be incapable of responding in any reasonable fashion. Having been presented with an arithmetic series written on a piece of paper, S wouldn’t know whether to count the number of terms, give the sum, or burn it (the paper).

Although the values of the state variables would necessarily have to be inferred from behavior, all empirical evidence would not have to be in the form of simple R-R relationships. State variables might be manipulated by prior training. Rule learning, for example, can presumably be manipulated, at least within certain populations, by presenting suitable statements directly or by promoting discovery from instances. Assessment procedures would play an essential role in such manipulations, of course, by insuring that the intended learning did, in fact, take place. Directions would appear to be the manipulable counterpart of internal motivation. In effect, a stage approach to research of this type could provide a basis for establishing causal relationships.11

Although clearly speculative, I would be remiss if I did not at least mention what to date have been my working assumptions toward a state theory. First, regardless of how rules are acquired in the first place, behavior depends on the selection and use of some rule. Second, motivation may be viewed as the internal equivalent of a contextual situation within which rules are selected. Thus, for example, identifying number series as the critical feature of the environment and sums as the goal to be achieved would serve to limit the class of potentially applicable rules by specifying both the relevant stimuli and the desired objective. Which rule in a particular class is actually selected, of course, would presumably depend on such things as the amount of cognitive strain (as might be determined from measures of latency) involved in

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11 This is not to imply that a stage approach has not been used before. Much of the research in the Hullian tradition, involving drive and habit strength, would fit this general paradigm.

The research of many investigators (e.g., HILGARD, IRVINE and WHITTLE, 1953; KATONA, 1940; MALTZMAN, EISMAN, BROOKS and SMITH, 1956), who have been primarily concerned with meaningful learning, also has been conducted in stages. More recently, GAGNE (1964) has argued pervasively for a stage approach to problem solving research.
applying the respective rules. Third, responses to new stimuli remain
under the control of a particular rule so long as the then present motiva-
tional state obtains and new input and/or feedback does not otherwise
indicate that the rule is no longer applicable. A tentative fourth postulate,
which in actuality is a special case of the second, is that if $S$ already has
some means available for responding to a particular stimulus, he is not
likely to devise a more general — and, hence, more elaborate — rule for
doing the same thing. Some support for the latter two hypotheses has
been obtained and is reported in part II.

These speculative postulates clearly do not, in themselves, constitute
the basis for any theory. As a minimum, one must specify how new rules
are brought into play and how existing knowledge is modified. We can
only speculate as to the nature of the mechanisms underlying such
changes. Perhaps the simplest assumption is that a rule ceases to operate
when it is no longer applicable — as when the stimulus input does not
correspond to an instance of that rule. There are at least two ways in
which such a situation may arise. The environmental situation may
change as the result of the very responses controlled by the operating
rule or the change may be the result of extra-subject influence. Suppose,
for example, that an $S$ is in the process of nailing planking for the floor
of a house. If the nail is up, the operating rule is 'hit with a hammer'.
The very act of hitting, of course, causes the stimulus situation to
change. The new situation, in turn, could serve to call up the next rule —
e.g., '(If down) move to the next nail', or '(If down and plank secure)
get a new plank'. The introduction of extra-scope stimuli or conflicting
directions, which provide a new context, could also transfer response
control to a new rule. It would be a brave (or stupid) individual, indeed,
who did not cease hammering and run as the result of someone's
shouting, 'Watch out for the rattlesnake!'

Cognitive disequilibrium theories (e.g., see Piaget as reported in
Flavell, 1963; Berlyne, 1965) suggest that a learned rule may be
modified if feedback conflicts with the original response. Suppose
$S$ has learned the rule 'say kitty' when asked the name of a furry
animal with four legs and that he is then shown a furry animal
with four legs which happens to be a squirrel. The dictated response
would be 'kitty'. If, however, feedback indicates that the correct
response is 'squirrel', $S$ is confronted with a dilemma. He can either
retain his rule, (almost) in its original form, and remember the exception
(as a new more restrictive rule) or, revise the original rule and assimilate
both the exception and the original as instances of another more general rule.

The theorist might attempt to reconcile the outcome of such conflict in one of at least two ways. He might, if he has a physiological bent, for example, postulate that the amount of cognitive strain (e.g., Brünner, Goodnow and Austin, 1956; Miller, 1956) is crucial. If committed entirely to a state theory, on the other hand, the theorist would probably resort to higher order rules of some sort. Any complete theory, of course, will undoubtedly involve both inherent physiological capacities for processing information and super-ordinate cognitive processes which can presumably be learned.

CONCLUDING REMARKS

The role of any scientific language is to capture the essence of an area – to provide a means for dealing with the relevant variables while making it possible to avoid irrelevancies. Stated in more operational terms, a language is useful to the extent that it leads the research worker to ask fundamental questions. If it is precise, so much the better.

Throughout this series of articles, it has been argued that the SFL meets both of these criteria. The SFL was shown to be sufficiently broad to encompass a wide variety of behavioral phenomena, from rote to meaningful learning. Taking the rule or principle as the basic behavior unit makes it possible to deal effectively with actual subject matters (like mathematics) and may lead to new and presumably important questions. Both the pilot research, that helped shape our thinking, and the analyses described above are illustrative. Questions concerning generality, abstractness, prior learning, statement interpretability, and discovery – all matters which have long plagued thinking pedagogues as well as psychologists – seem to lend themselves to SFL based analysis. Furthermore, the SFL is precise; problems can be formulated in mathematical terminology wherever necessary or desirable.

In addition, a partial solution was proposed to the problem of how to operationally define what (rule) is learned. Data supporting the response consistency hypothesis made it possible to define what is learned in terms of observable test stimuli and responses. It is hoped that this sort of assessment methodology may make it possible to more effectively implement a multi-stage approach to meaningful learning.

Nonetheless, the SFL is far from a finished product. Many things
remain to be done. First, the validity of the proposed analyses needs to be ascertained. The already completed SFL related research was based on a preliminary and largely inadequate version of the SFL. Theoretical implications need to be more carefully and completely drawn out and suitable empirical tests conducted.

Second, the SFL itself can undoubtedly be further improved. As more problems are analyzed and the results subjected to empirical test, further evolution of the language, and hopefully of related theory, is likely to occur.

Third, more rigorous attempts should be made to contrast S-R mediation and SFL formulations of such phenomena as stimulus-response generalization, cognitive equilibrium, and classical conditioning. Although this might not always prove to be possible, important insights into both languages may be so attained.\(^{12}\)

At this point, no one can argue with any authority as to the ultimate value of the SFL in promoting fruitful research. The outcome is even less certain as regards theory construction. Nonetheless, the SFL appears sufficiently promising to warrant further investigation.

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\(^{12}\) In this regard, I am under no delusions that the SFL can deal effectively with all that the older S-R language can. This need not be a crucial limitation, however, since their primary areas of concern are different. It may be no more necessary for the SFL to incorporate all S-R phenomena than it was for Kepler to incorporate epicycles. To be sure, as a result of his theorizing, he got rid of them. Many of the phenomena dealt with within a given theoretical framework disappear when looked at from a different point of view.

Nonetheless, although apparently lacking, the S-R language may again prove sufficiently flexible to deal effectively with meaningful learning. This possibility must certainly be entertained.


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