STRUCTURAL LEARNING AND CONCRETE OPERATIONS

An Approach to Piagetian Conservation

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PRAEGER
To our parents
Clare and Douglas Baker,
Lucy and Joseph Scandura
With love and gratitude
PREFACE

For many years, our major concerns have been with developing the theoretical foundations of complex human behavior, especially those of problem solving and instruction. The emphasis in this work has been on structural/process considerations. There are many things still to be done, some of which we are currently pursuing. Nonetheless, as a result of the work accomplished to date, a degree of theoretical closure already has been achieved. This is perhaps best represented by the work reported in J. M. Scandura's books on structural learning and problem solving (Scandura, 1973, 1976, 1977b).

In this, our initial major collaboration, we have begun to explore areas outside the original domain of interest. The concerns represented by Piagetian training studies seem to meet many of the requirements for which we were looking.

First, the existence of important, recognized problems: A large number of North American training studies have shown that children can be taught to act as conservers, usually within short periods of time. On the other hand, Genevan investigators have maintained their insistence that these studies fail to get at the essentials of what Piagetian theory is about. Can these views be reconciled? And, if so, how?

Second, the existence of problems for which the Structural Learning Theory might help to provide solutions: Piagetian theory has long been criticized as being nonoperational; major gaps exist between theory and observables. In contrast, the Structural Learning Theory is designed to be operational in all its essential aspects. On the other hand, most short-term training studies either are not consistent with Piagetian principles (children tend to learn short-cut methods for responding) or substantially help only children who are near-conservers before they begin Piagetian (for example, incongruity) training (Inhelder, Sinclair, and Bovet, 1974).

And third, during the past few years, a number of longitudinal training studies have yielded promising results. Even here, however, the explanations provided have been relatively incomplete; they lack operational and theoretical rigor. What is it exactly that children are actually learning? Until the underlying competence can be specified operationally, we will never know for sure whether the knowledge associated with concrete operations is truly important, for what purposes it might be important, or, more to the point here, how to bring about related capabilities in individual children in an efficient manner.
Overall, our major hope and expectation is not that a structural learning formulation of Piagetian phenomena will necessarily be inconsistent with Piagetian theory but rather that such a formulation may allow us to make operational many of the essentials, and in the process, clarify some of the unresolved problems in the field. Clearly, we do not plan to deal with the totality of Piagetian theory. That would be a hopeless task for any small research group. Rather, we are looking at one major aspect of his theory—the process by which preoperational children attain concrete operations. Specific attention is given to the acquisition of conservation.

Some readers may be unfamiliar with structural learning and the theories and methodologies that characterize it. Hence, we have included two appendixes; the first, a recent article by J. M. Scandura that helps to place the present research in broader perspective; and the second, a selective review of Piagetian research and school programs. Both these appendixes and the study itself are self-contained and may be read independently.

The research on which this work is based was supported, in part, by National Institute of Health Grant 9185 to Joseph M. Scandura. The research reported in Chapter 2 was conducted in collaboration with Roland Schneider. In addition to helping to perform the research, he contributed a number of useful ideas for which we are grateful. Unfortunately, he had to return home to Geneva before the study could be completed. The authors, therefore, must assume full responsibility for the contents of this study.
# CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREFACE</td>
<td>vii</td>
</tr>
<tr>
<td>1 BACKGROUND AND INTRODUCTION TO THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>A Misplaced Criticism of Piagetian Theory</td>
<td>3</td>
</tr>
<tr>
<td>A Brief Overview of the Structural Learning Theory</td>
<td>6</td>
</tr>
<tr>
<td>Introduction to the Problem</td>
<td>11</td>
</tr>
<tr>
<td>2 EXPLORATORY RESEARCH</td>
<td>13</td>
</tr>
<tr>
<td>Pretesting and Assignment to Training Groups</td>
<td>13</td>
</tr>
<tr>
<td>Exploratory Training and Testing</td>
<td>20</td>
</tr>
<tr>
<td>3 THEORETICAL ANALYSIS OF PIAGETIAN CONSERVATION</td>
<td>29</td>
</tr>
<tr>
<td>Structural Analysis of Conservation</td>
<td>33</td>
</tr>
<tr>
<td>Acquisition of Conservation Competence</td>
<td>55</td>
</tr>
<tr>
<td>4 EMPIRICAL TEST OF THE ANALYSIS</td>
<td>65</td>
</tr>
<tr>
<td>Phase Two Training</td>
<td>67</td>
</tr>
<tr>
<td>Phase Three Training</td>
<td>76</td>
</tr>
<tr>
<td>5 SUMMARY AND IMPLICATIONS</td>
<td>119</td>
</tr>
<tr>
<td>Summary</td>
<td>119</td>
</tr>
<tr>
<td>Limitations</td>
<td>123</td>
</tr>
<tr>
<td>Implications and Future Directions</td>
<td>125</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>131</td>
</tr>
</tbody>
</table>
# APPENDIX A. THEORETICAL FOUNDATIONS OF INSTRUCTION: A SYSTEMS ALTERNATIVE TO COGNITIVE PSYCHOLOGY

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Essentials of Instructional Theory</td>
<td>138</td>
</tr>
<tr>
<td>The Structural Learning Theory</td>
<td>141</td>
</tr>
<tr>
<td>Instructional Systems</td>
<td>169</td>
</tr>
<tr>
<td>Relationships to Traditional Cognitive Theories</td>
<td>172</td>
</tr>
<tr>
<td>Conclusions</td>
<td>180</td>
</tr>
<tr>
<td>References</td>
<td>182</td>
</tr>
</tbody>
</table>

# APPENDIX B. SELECTIVE REVIEW OF PIAGETIAN RESEARCH AND SCHOOL PROGRAMS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Operation Training Studies</td>
<td>190</td>
</tr>
<tr>
<td>Piagetian School Programs</td>
<td>194</td>
</tr>
<tr>
<td>Conclusions</td>
<td>198</td>
</tr>
<tr>
<td>References</td>
<td>201</td>
</tr>
</tbody>
</table>

# ABOUT THE AUTHORS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>205</td>
</tr>
</tbody>
</table>
STRUCTURAL LEARNING AND CONCRETE OPERATIONS
BACKGROUND AND INTRODUCTION TO THE PROBLEM

It is well known that important developmental changes take place between ages five and eight. Just before this period, most children are operating at what Piaget calls the preoperational stage of development. Toward the end of this period, many are beginning to function at the stage of concrete operations. Although the exact time at which the various stages and substages appear may vary over different cultures, the sequence in which these stages are achieved is thought to be invariant in the developmental epistemology of Jean Piaget.

Beginning with the appearance of Flavell's (1963) comprehensive and classic treatment in English of Piagetian research, there has been a rapid increase in interest in related phenomena. Although much of the resulting research has adopted Piaget's largely clinical methodology, a good deal of this research has been based on the more experimental methods characteristic of North American psychology. Among other things, research in the latter tradition has tended to focus on the nonoperational character of many of the basic constructs in Piagetian theory. Contrary to what Piagetian theory would predict, for example, a number of training studies have demonstrated that children can be taught how to conserve various concepts and even to effect positive transfer to other conservation tasks (Gelman, 1969; Bearison, 1969). For the most part, these studies have been based on non-Piagetian behavioral theories (of both the cognitive and peripheral [S-R] varieties). Nonetheless, none of these studies has been uniformly successful in bringing about many fundamental changes associated with the stage of concrete operations, at least not as interpreted by Genevans. (According to Piaget, the only type of training that is apt to be successful is based on naturally occurring incongruities and then only when children have already begun to recognize such incongruities on their own.) Similarly, attempts to tie conservation acquisition with future achievements, for example in basic arithmetic, have
been something less than a complete success (Dimitrovsky and Almy, 1975; Bearison, 1969, 1975).

Conversely, Piagetian theory per se has not been sufficiently developed to provide an adequate basis for explaining such phenomena as horizontal decalage. Although Genevans have been sensitive to such phenomena and have conducted a large amount of related research (Piaget and Inhelder, 1958; Gillieron, 1976), Piagetian theory does not adequately explain why, for example, children show evidence of concrete operations on some conservation tasks (for example, number) before others (for example, volume) or, for that matter, why natural conservers do better in subsequent training in arithmetic than do trained conservers (Brainerd, 1979).

These difficulties, we feel, have resulted in large part because of the lack of scientific contact between Piagetian theory and the more restricted but operationally more precise theories on which many contemporary training studies have been based. Thus, for example, Piagetian theory deals first and foremost with general epistemological considerations. The constructs in operational psychological theories are necessarily tied more directly to observable behavior. Unfortunately, central relationships between corresponding constructs in the respective theories have rarely been made explicit.

The theory explored and research conducted in our laboratory over the past decade (Scandura, 1970, 1971, 1972, 1973, 1974b, 1976), and especially over the past three years (Scandura, 1977a, 1977b), share some essentials in common with both approaches. Like Piagetian theory, the Structural Learning Theory works from the top down. In particular, it has been heavily concerned with overall interrelationships, for example, among content, cognition, and individual differences (Scandura, 1977b). On the other hand, a serious attempt has been made to make contact with more specialized cognitive theories, especially in the area of human problem solving (Scandura, 1973, 1977b; Voorhies and Scandura, 1977).

Perhaps most relevant in the present context, the Structural Learning Theory not only includes a relatively explicit cognitive theory but a "meta-theory" with regard to content. Specifically, it tells how to construct specialized theories of arbitrary content (that is, how to identify competencies that satisfy relevant cognitive constraints and are needed to perform successfully with respect to given bodies of content). Moreover, recent theoretical considerations (Scandura, 1977b, chap. 2) suggest that an approach based on the Structural Learning Theory may help to bridge the gap between nonoperational theories, such as the developmental epistemology of Piaget, and operational, but nonetheless more specialized, cognitive theories. All major cognitive constructs in the Structural Learning
Theory are made operational. The theory tells how, for example, individual behavior potential (rules of knowledge) may be defined in terms of test behavior on predetermined kinds of test items.

A MISPLACED CRITICISM OF PIAGETIAN THEORY

To give some idea of our general concerns, and our approach in dealing with them, consider a recent criticism of Piagetian theory that has been made by experimentally oriented developmental psychologists.* The general thrust of this criticism is that Piaget's stages have no explanatory power, that they are operationally circular.

One of Piaget's main contentions, for example, is that the particular invariant sequences that he postulates will necessarily be invariant in all environmental settings. This would seem to be a theoretical statement that could be supported or refuted by empirical evidence.

Some developmental psychologists, however, have argued (essentially) that the tasks associated with Piaget's preliminary stages are logical prerequisites of those tasks associated with subsequent stages. Brainerd (1978), for example, has called these "measurement sequences," which occur "whenever each item in the sequence consists of the immediately preceding item plus some additional things (1978, p. 14)." In effect, the assumption is that Piagetian stages correspond essentially to hierarchies of tasks.

In a similar view, it is widely believed, for example, that children must know how to add in order to multiply, presumably "because multiplication is defined in terms of addition (Brainerd, 1978, p. 15)." True, it is almost always the case that children acquire these skills in the indicated order. We would propose, however, that this is a result of how our educational system (broadly defined) is organized. In general, the contention is false. A person can successfully be taught how to multiply before knowing anything about addition. The product 3 x 2, for example, is simply the number of pairs in the two-dimensional array

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & (1,1) & (1,2) \ (1,3) \\
2 & (2,1) & (2,2) \ (2,3)
\end{array}
\]

*The first part of this section is based on a commentary by J. M. Scandura to an article by Charles J. Brainerd (1978) in Brain and the Behavioral Sciences.
or 6. Notice that all one has to do is to count and that the procedure is perfectly general. There are any number of other examples of this sort, ranging from decoding in reading to college mathematics.

More to the point, there is a basic flaw in the whole logical sequencing (hierarchical) argument since it assumes implicitly that there is an unique basis for solving any given class of tasks. Critics of Piaget have argued, for example, that successful performance on tasks associated with the stage of concrete operations can only be achieved when children have first acquired the capability of performing successfully on tasks associated with the pre-operational stage. This type of argument simply does not follow; it is impossible to define so-called logical sequences of tasks independently of the structures/processes that underlie them. There are any number of different ways (structures/processes) by which a given class of tasks might be solved (Scandura, 1964, 1970, 1972). Although subordinate/superordinate relationships may exist among various structures/processes, this is not true, in general, of tasks that may be solved by using them.

Here, then, is the source of one important difference between North American empiricism and Genevan structuralism. Clearly, Piaget intends for the structures associated with his various stages to be hierarchically related. The behaviors that these structures make possible, however, may be generated in any number of ways (that is, by any number of processes/structures). (It is a mathematical fact that if there is one rule/procedure for solving a given class of tasks, then there must be an infinitude of others that will do the same thing.) Just because particular solution rules associated with different classes of tasks are hierarchically related, there is no guarantee that this same relationship will exist between arbitrary solution rules associated with these classes.

More important in terms of our planned program of research, it is impossible to determine whether or not such relationships exist in the absence of rigorous, rule-based analyses of the respective tasks (Scandura, 1977b). In short, logically determined sequences of tasks are a myth. Such sequences cannot be defined independently of the structures/processes that underlie their solutions.

The same general misunderstanding seems to underlie various arguments regarding cognitive structures. For example, one might think that problem classes associated with later stages should never be solved during earlier stages. Under such conditions, it might appear that the structural distinction between stages would break down completely (Brainerd, 1978, p. 27).

This is not necessarily the case for the reasons indicated above. In particular, problem classes associated with later stages
do not have unique bases for solution. The relatively simple prescriptions preferred in North American training studies, for example, are surely not identical with the structures postulated by Piaget. Learning a structure of the latter type and successful performance on problem classes used by Piaget to determine the presence of that structure are not necessarily the same thing. Thus, successful performance on problems in such classes does not necessarily imply that a Piagetian-type structure has been learned.

The following seriation task illustrates this fact. In the task, a child is shown a set of sticks seriated by length, but with the relevant ends of the sticks hidden by a screen. The child is given a new stick, x, and is asked to insert it in the right position. To accomplish this, the child is allowed to ask the experimenter how the length of x compares with any of the seriated sticks (one at a time). According to Piaget, if one is to avoid redundant comparisons, success on this task requires the transitivity concept (structure); that is, the child must know that $a > b$ and $b > c$ necessarily entails $a > c$. Such knowledge would avoid redundant comparisons because, given the results of any one comparison, the child would be able to eliminate other possible comparisons as logically dependent. (Note that, as defined, transitivity is not fully operational but clearly involves more than knowing any particular solution rule. Rather, in structural learning terms, it corresponds to a class of higher-order rules that might be used to derive a variety of solution rules for arbitrary ordering tasks—for example, involving number or weight as well as length.)

Nonetheless, the child also could succeed on the task by simply applying the following rule. Compare x with the first seriated stick. If x is shorter, put x before the stick and stop. If x is longer, compare x to the next stick, and test x as above.* This example also illustrates why most successful North American training studies are not directly relevant to the structure of Piagetian theory. In the example, transitivity corresponds more to the construction of solution rules (such as the above) than to the a priori knowledge of such rules or their application.

More generally, Piagetian structures appear to be related more to the construction of solution rules than to solution rules themselves (or their application). To illustrate this difference, consider an analogy between the teacher as a programmer (a

*After reading a draft of J. M. Scandura's commentary, one of his doctoral students, Roland Schneider, suggested this example. His commentary is gratefully acknowledged.
constructor of solution rules) and the child as a computer (a user of solution rules). It is evident that the programmer and the computer do not need the same "cognitive" structures to succeed on a given task. Moreover, it would appear that the only kind of training experiments that could be relevant to Piagetian structures would be experiments where the child is taught how to construct (and/or select) solution rules (that is, not only how to use given ones).

Presumably, of course, one would want a precise operational (behavioral) definition of just what a structure is. Piaget, to our knowledge, has not done this, and this is an important limitation that many people have pointed out. The arguments advanced, however, in themselves are not especially damaging to Piaget's theory. Piaget's formulation is an idealization; it is a theory of what behavior would be like under certain "idealized conditions" (see Scandura, 1978a). Unfortunately, Genevan psychologists have not adequately specified just what those idealized conditions might be. Until they do, the theory will necessarily remain nonoperational.

We also are inclined to believe that the formalism introduced by Piaget to represent knowledge may not be an especially useful one. While it may have been the best available at the time Piaget initially developed his theory, this may no longer be the case. Indeed, if Piaget himself had had access at that time to some of the modern information-processing tools that are presently available for representing cognitive structures and processes, we suspect that his theory might have taken a quite different turn.

Although a variety of contemporary formalisms might be used for this purpose, we believe that the structural learning formalism (Scandura, 1971, 1973, 1977b) may be especially useful in this regard. The notion of higher-order rules, or rules that operate on other rules and select and/or construct new ones, and the explicit provision for diagnosing individual differences in cognitive potential seem especially relevant.

A BRIEF OVERVIEW OF THE STRUCTURAL LEARNING THEORY

It is impossible within the space of a few pages to summarize adequately even the main features of the Structural Learning Theory. The earliest presentation (Scandura, 1971), although somewhat outdated, is still perhaps the best introduction. J. M. Scandura's book (1973) on structural learning provides a relatively formal treatment, but his most recent book on problem solving (Scandura, 1977b) furnishes perhaps the clearest version, along with important refinements, extensions, and applications to education. Nonetheless, it may be helpful to include a brief overview here.
The Structural Learning Theory provides a unifying theoretical framework within which to view the concerns of the competence researcher (for example, the artificial intelligence, linguistics, or subject matter specialist), the cognitive psychologist, and the individual differences specialist (see Figure 1).* Although it shares many concerns with theories in these respective fields, however, the Structural Learning Theory differs in a number of important (but sometimes subtle) respects. Generally speaking, these differences play an important role in integrating theory in different areas, in explicating relationships among them, and in providing constraints on more specialized theory and research within given fields.

*This section is based on Scandura (1977a).
For one thing, the theory is basically relativistic. What individual subjects know and what they are able to do is always judged in relation to the cognitive structures and processes underlying some predetermined content (problem domain) and associated with idealized, prototypic members of some subject population. The prototypic processes that collectively make it possible to solve problems in a problem domain are referred to as rules of competence.* Collectively, the set of competence rules is called a competence account of the problem domain.

In the theory, the term problem domain is used in a broad sense and, in principle, may encompass anything from simple arithmetic to language or moral behavior. Similarly, the subject population might be either "multicultural" or highly homogeneous. Depending on the problem domain and target population, then, the underlying competence might provide a detailed account of highly prescribed behavior (for example, borrowing in subtraction) or a molar account of a broad range of phenomena (for example, concrete operations). In most applications to date, both the problem domains and the subject populations have been relatively well delineated, but,

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*The term rule has a technical meaning in the Structural Learning Theory. A rule is a triple consisting of a domain, a range, and a restricted type of procedure.

Domains, in turn, are sets of data structures to which the procedures apply, and ranges are sets of anticipatory data structures. (Structures in the theory also consist of finite n-tuples of elements, relations, and rules, including higher-order relations and rules.) Ranges in this sense are not necessarily identical with the image (output) sets of corresponding procedures (formed when procedures are applied to domains). Rather, they reflect (degrees of) prior awareness of actual image sets. (Note that the latter can only be determined after the fact, that is, after application of procedures.)

The procedures in rules are restricted in the sense that the generation and application of new procedures are not both allowed; that is, while a (higher-order) procedure may generate new procedures, it may not turn around and then "call" that new procedure (so as to apply it). This latter function is reserved in the theory for a hypothesized universal control mechanism (for which relatively strong empirical support has been obtained, for example, in Scandura, 1977b).

Examples of rules that are directly relevant to Piagetian theory are given in the section dealing with structural analysis of conservation tasks.
in principle, this is not an essential limitation (see the section dealing with structural analysis of conservation tasks below).

In theory, any given problem can be solved in any number of ways. In practice, however, only a small number of alternatives normally will be compatible with how a knowledgeable member of the target population might solve it. The subject population places definite constraints on the processes (rules) that may be introduced. For example, German children are taught the equal additions method of subtraction, whereas U.S. children are taught borrowing. Analogous constraints may be imposed by the kind of developmental stages through which Piaget postulates that children go. Such constraints severely limit the theoretically infinite number of competence accounts associated with any given problem domain.

Idealized competence, of course, is not the same as rules of knowledge that characterize individual behavior potential. It is assumed in the theory that what an individual does and can learn depends directly and inextricably on what he already knows. More particularly, it is assumed that the human information processor may be adequately characterized in terms of universal characteristics and individual knowledge that is judged in relation to the competence associated with given problem domains (and subject populations to which the individual belongs).

Control mechanisms are among the most important universal characteristics. Control mechanisms serve to tell the organism which processes (rules) to use and when to use them. They are essential in all information-processing systems, whether man or machine. Whereas all complete information-processing theories make a distinction between process (rule) and control, control in most cases either plays a subordinate role (Newell and Simon, 1972) or is distributed among a variety of different control mechanisms whose coordination, in turn, is often left unspecified (Pascual-Leone, 1970).

In contrast, the Structural Learning Theory postulates a single, goal-switching control mechanism that makes minimal assumptions about the processor but that, nonetheless, has been shown to adequately account for many different kinds of behavior (Scandura, 1973, 1977b). This mechanism is hypothesized to be common to all humans and to govern all cognition, irrespective of the specific knowledge involved. (A considerable amount of empirical support for this contention has been obtained, for example, in Scandura, 1977b.)

A second general characteristic of the theory, which has been empirically tested, is processing capacity (Scandura, 1973; Voorhies and Scandura, 1977). Again, almost all contemporary information-processing theories assume in one form or another
that "working memory" has a limited capacity. In the Structural Learning Theory, working memory is assumed to hold not only data (the stuff on which rules operate) but rules themselves. While capacity per se is assumed to be fixed (although it may vary over individuals), the memory load associated with any given task depends directly on the process (rule) used in attacking it. Thus, for example, whereas it may be impossible to multiply large numbers in one's head using the standard algorithm, many people know shortcut processes that enable them to perform successfully. The theory also allows for the inclusion of other general constraints, such as processing speed, but this part of the theory has been only partially developed (compare Scandura, 1977b).

Each universal characteristic of the human information processor says something about behavior but not all. Accordingly, one can conceive of a succession of deterministic partial theories, each of which in turn says progressively more about human behavior. Each partial theory is deterministic, in the sense that it deals with the behavior of given subjects in particular situations (Scandura, 1971; 1977b, chap. 1).

Deterministic predictions may be expected to hold, however, only in situations that satisfy appropriate boundary conditions. For example, the "memory-free" (partial) theory fully accounts for behavior only in situations where all relevant knowledge may be assumed to be readily available. (This partial theory involves only the control mechanism and does not take processing capacity into account.) To the extent that processing capacity is involved, for example, theoretical predictions can be expected to deviate from obtained results. The idea is directly analogous to the situation involving the law of the inclined plane in elementary classical physics. This law allows one, for example, to calculate the force needed to move a given cart up an inclined plane, but only where the inclined plane is perfectly smooth and the wheels on the cart are frictionless. Deviations from prediction may be expected just to the extent that the inclined plane is bumpy and/or that friction otherwise plays a role.

In effect, the structural theory of cognition is a "top-down" theory. Progressively more structure may be added to the theory by adding more and more (possibly universal) constraints. Thus, adding processing capacity to the "memory-free" theory, which involves only the control structure, makes it possible to account for behavior under a wider variety of conditions.

The possibility of adding more structure implies a particular (structural) approach to theory construction that is an essential aspect of the Structural Learning Theory. In turn, this aspect of the theory has important implications for empirical testing. By way of
summary, suffice it to say that each partial theory must be tested under appropriate idealized conditions in the same sense that the law of the inclined plane must be tested using smooth inclined planes and frictionless wheels.

In contrast to general cognitive constraints, specific knowledge is assumed to vary over individuals. The theory shows how competence, corresponding to the knowledge possessed by idealized, prototypic members of given populations, may be used to define operationally the knowledge possessed by actual individual members of such populations. The rules of competence serve effectively as "rulers" or standards against which individual rules of knowledge may be measured.

Originally, the Structural Learning Theory was primarily schematic insofar as competence was concerned (Scandura, 1971, 1973). Detailed illustrations and general requirements dealing with how such competence should be represented were provided, but, along with other contemporary theories of knowledge, relatively little was said about how to identify such competence (compare Ehrenpreis and Scandura, 1974; Scandura, 1977b). In this sense, the theory is not fully operational because predictions in the theory depend directly on the competence associated with given problem domains and subject populations and because the number of different domains and populations is indeterminately large. In effect, it is essential that a fully operational Structural Learning Theory include a "meta-theory" (systematic method) for identifying arbitrary competence. (Contrast this requirement with that in linguistics, where competence is more sharply prescribed.) Although a complete solution to this important problem is beyond current reach, the constraints imposed on competence by universal characteristics of the human information processor make it possible, apparently, to proceed in a quasi-systematic manner (which we have called structural analysis).

INTRODUCTION TO THE PROBLEM

As originally conceived, the present study was designed with two goals in mind. First, we wanted to determine the feasibility of conducting a type of structural learning analysis of a significant subdomain of Piagetian tasks. The domain of conservation tasks served admirably in this regard. It is highly familiar to most psychologists and, moreover, is perhaps better understood than any other aspect of Piagetian theory. Hence, any further increments in knowledge made possible as a result of structural analysis would more clearly demonstrate its potential in applications to less developed aspects of Piagetian theory.
Second, we wanted to determine whether we could teach initially naive children how to deal with various conservation tasks. Our primary concern here was not whether we could bring about changes in behavior (we already knew that was possible with transitional children). Rather, we wanted to determine whether we could effect changes in pretransitional children in ways that are consistent with Genevan principles (although not necessarily only through incongruity training).
EXPLORATORY RESEARCH

PRETESTING AND ASSIGNMENT TO TRAINING GROUPS

Our original plan was to conduct, in a parallel fashion, a structural analysis of the conservation domain and a flexibly organized exploratory training study. We hoped that the experimentation would provide both a source of ideas for our analysis and a laboratory for trying out hypotheses derived from our analysis. In the process, we also hoped to determine the feasibility of achieving real growth relating to concrete operations in the context of a flexibly organized clinical training environment, of a type that might well go on in an ordinary classroom.

Toward these ends, our first step was to obtain and pretest suitable training groups.

Subjects

The subjects (Ss) were 32 kindergarten children (in morning and afternoon classes) at the Walnut Center, between 4.73 and 5.89 years of age on the day of testing. The Ss were pretested individually on conservation of length and conservation of number.

Materials, Problems, and Pretest Procedures

The materials were typical of those used traditionally in testing number conservation and length conservation. In the number pretest, plastic chips (3 cm. in diameter) of two different colors (A and B) were used. Initially, each child (S) was asked to choose his preferred color. Thereafter, according to the indicated prefer-
ence, the chips were consistently referred to by the experimenter as "yours" (A) and "mine" (B). To help determine whether S understood the meaning of the word "more," where the word was not used spontaneously, some of the test items (for example, 41 in Figure 2) involved dolls (height 25 cm.).

In the length pretest, wooden sticks (lengths between 5 cm. and 15 cm.) were used with most tasks, in particular where rigid transformations were involved. In the task that involved a deformation, three pieces of yarn were used (two 30 cm. in length, one 29.5 cm.). The difference (0.5 cm.) was easy to discern upon careful inspection. Analogous to the use of dolls with number, familiar tokens were used to test understanding of the meaning of the word "longer." In this case, the comparisons involved two necklaces of different length. The shorter necklace was half as long as the longer.

The major goals of the pretest were to assess S's level of functioning in terms of conservation of length and number and understanding the words "more" and "less." To help reflect Genevan principles, an interactive method of testing was used with the content and order of testing dependent on responses Ss gave to earlier items. The primary goal in this case was to determine basic cognitive capabilities of the child, rather than simply to find out whether S was able to perform successfully on a fixed set of predetermined tasks (which, as noted in Chapter 1, may be accomplished in any number of ways).

Testing with respect to number proceeded as indicated in Figure 2. The testing procedure used with length directly paralleled that with number. All but the final six Ss were videotaped during testing as a means of increasing test reliability. One or two observers monitored the testing and independently scored it. The videotape was viewed later by the experimenter and observer(s) to confirm pretest scores. In those cases where the videotape was not available (during the last six pretestings), the experimenter and observer(s) discussed their scores immediately after pretesting. In each case, agreement was readily achieved.

In addition to reflecting Genevan principles, the content and order of test items (that is, problems) involved several relatively noncontroversial assumptions. For example, the transformation \[ \equiv \longrightarrow \equiv \] was assumed to be more difficult than the transformation \[ \equiv \longrightarrow \equiv \], and the transformation \[ \equiv \longrightarrow \equiv \rightarrow \equiv \] more difficult than the transformation \[ \equiv \longrightarrow \equiv \rightarrow \equiv \] .
FIGURE 2. Algorithm of Pretest
Similarly, spontaneous and correct use of a word (for example, "more!") was assumed to denote a more comprehensive knowledge than simple comprehension of the word.

The nature of the test problems, in the technical sense described earlier, can be inferred directly from Figure 2. Consider, for example, an illustrative sequence of test problems, in particular one corresponding to test path N1 of Figure 2. As indicated in Figure 2, this path requires the experimenter to pose in order problems 1, 2, 4, 6, and 8, interspersed with decisions A through D, which require evaluating children's responses. Operations 3, 5, 7, and 9 on this path all involve the experimenter's (E's) recording of S's behavior. (Step 9 also designates "STOP," that is, termination of the path.) More complete descriptions of these activities follow.

In problem 1, the S is given a neatly arranged row of objects (B) and a larger disorganized set of objects (A) and is asked to make a row of A objects (just above row B) in which there is exactly one object in row A for each one in row B.

Step A refers to a branching or decision point. The numbers over the labeled arrows exiting from the decision correspond to the statements in the decision box. In each case, the arrow indicates the next step in the testing procedure. If S gives a correct answer to problem 1, E presents problem 2. If S gives a definitely incorrect answer, E records the answer (step 29) and then presents problem (step) 30 to S. Problem 30 is similar to problem 1 except that the solution can be obtained in the latter case by a largely perceptual (for example, subitizing) strategy. (Research suggests that problem 1 requires a more systematic strategy, such as one-to-one matching.) If S does not understand problem 1 or does not pay enough attention, E presents the problem once more.

As indicated in decision A, problem 2 is presented only if S gives a correct answer to problem 1. This problem (like almost all conservation problems) is a two-step problem. First, S is shown two sets (A, B) and is asked whether they have the same number. Then, only if S agrees on their sameness, E makes the transformation. Second, S is asked if A has more than B or if B has more than A. In some cases, a third option also is given explicitly (that is, A and B have the same number of chips).

Step B refers to another decision point. If S gives a nonconserving response to problem 2, then E goes through steps 13 to F in order to see if S persists in his response (problem 16). Then, E gives S another (supposedly easier) conservation task. If S gives a conserving response, then E goes to step 3.

In step 3, the E records S's answer. (This step is redundant because if step 4 [or 5] is reached at all, the only possible record at step 3 is "same." Generally, steps like this one are omitted.)
In step 4, the E asks why S thinks A and B are the same, and, in step 5, the E records the reasons given.

Question 6 was designed to test the stability of S's answer by focusing S's attention on extraneous cues. For example, the E may say: "But my row (A) comes all the way out here [pointing to the end that extends further to the left]. Does your row (B) have more chips? Or, does my row (A) have more chips?"

In decision C, if S changes his mind after question 6 and gives a nonconserving answer, E records the answer (that is, goes to step 12) and then calls S's attention to the opposite endpoint (for example, "But this [row] comes all the way out to here [to the right]")—question 16—and records S's response (step 17). Then, E presents another (supposedly easier) conservation task (step 18) and records the answer (step 19). If S does not change his mind after question 6, E goes to step 7, in which the E records the answer/reason S gives to question 6.

In step 8, the E tests the stability of S's answer against social pressure (for example, "Another child said that this one has more. What do you think?")

In decision D, if S does not change his answer, E presents a more difficult conservation task (10) and continues as indicated in Figure 2. If S changes his answer, E goes to step 9.

In step 9, the E records the path followed in testing the child, in this case path N1.

The test procedure for length was directly analogous to that for number. In this case, for example, the phrase "has more" in problem 8 above was replaced by "is longer." Due to the nature of the content, of course, certain other modifications were also necessary. For example, corresponding to the transformation OOOOOO → OOOOOG on number, the transformation

\[ \begin{array}{c}
\text{XXXXXXX} \\
\text{X} & \text{X}
\end{array} \]

was used with a string instead of a stick (for the B object). All other transformations involved sticks and had the same effects as the corresponding transformations on sets (that is, number).

In a problem related to one-to-one correspondence (problem 1 in Figure 2), E presented S with a small pile of sticks (A) of different lengths and asked S to pick the stick in A that was exactly the same length as B.*

*The purpose of this task, as with that of one-to-one correspondence, is to see if S has a systematic strategy for comparing
Figure 2 and the corresponding test procedure for length both have 13 distinguishable paths. Because testing always proceeded along exactly one of the paths, it was possible to characterize each S's capability by one path for each conservation concept (that is, for length and number).

A summary description of the performance capabilities associated with each path is given in Table 1. This table indicates the relationship between the various paths and the test problems included in Figure 2 (for testing number conservation). The rows in Table 1 correspond to paths and the columns to the 14 problems included in Figure 2. The + (success) and − (failure) signs and blanks (not tested) in the rows indicate the patterns of responses corresponding to the various paths. For example, to be assigned score N1, a child would either have to succeed on problems 2, 6, and 8 and fail on problem 10 or have to succeed on problems 2 and 6 and fail on 8. Analogous relationships exist among the paths and test problems associated with length conservation.

In both cases, the paths are ordered in the sense that the higher-numbered paths indicate a greater deviation from concrete operational behavior (that is, indicate that subjects are operating at a lower level). Consider, for example, Table 1. In every case, if one path (Ni) has a lower numerical value (i) than another (Nj with numerical value j), then there is at least one problem on which an Ni child would succeed and an Nj child would fail. For example, a child operating at path level N1 would succeed on problem 2, whereas a child operating at path level N2 would fail.

Experimental Design

Based on pretest performance, each S was assigned to one of three categories. Those Ss who were successful on all of the items (paths LO and NO) were designated as conservers. Those who were not conservers but were able to perform successfully on tasks involving one-to-one correspondence (test path N5 or better) appeared to be operating successfully at the preoperational stage with respect to both number and length and were designated as high preoperational lengths. In this task, it is difficult to pick the correct stick by direct perception. By way of contrast, problem 30 (in Figure 2), which involved one-to-one correspondence, had only three A chips so that S might succeed using only a perceptual strategy.
<table>
<thead>
<tr>
<th>Path</th>
<th>Conservation</th>
<th>Resistance</th>
<th>Social Pressure</th>
<th>Conservation</th>
<th>Social Pressure</th>
<th>Reverse Transformation</th>
<th>Can Determine Whether Two Rows Have Same Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>N0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>n.t.*</td>
<td>n.t.</td>
<td>n.t.</td>
</tr>
<tr>
<td>N1</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>n.t.</td>
<td>n.t.</td>
<td>+</td>
</tr>
<tr>
<td>N2</td>
<td>n.t.</td>
<td>-</td>
<td>n.t.</td>
<td>n.t.</td>
<td>+</td>
<td>n.t.</td>
<td>+</td>
</tr>
<tr>
<td>N3</td>
<td>n.t.</td>
<td>-</td>
<td>n.t.</td>
<td>n.t.</td>
<td>+</td>
<td>n.t.</td>
<td>+</td>
</tr>
<tr>
<td>N4</td>
<td>n.t.</td>
<td>-</td>
<td>n.t.</td>
<td>-</td>
<td>n.t.</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>N5</td>
<td>n.t.</td>
<td>-</td>
<td>n.t.</td>
<td>-</td>
<td>n.t.</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(1) Can Determine Whether Two Rows Have Same Number</th>
<th>(32a) Uses &quot;Less&quot; Spontaneously</th>
<th>(32b) Uses &quot;More&quot; Spontaneously</th>
<th>(36) Understands &quot;Less&quot;</th>
<th>(40) Understands &quot;More&quot;</th>
<th>(43a) Repeats Words Verbatim (i.e., More)</th>
<th>(43b) Uses Synonyms but not Exact Words (e.g., &quot;Some&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N6</td>
<td>-</td>
<td>+</td>
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<td>n.t.</td>
<td>n.t.</td>
<td>n.t.</td>
</tr>
<tr>
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<td>-</td>
<td>+</td>
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<td>+</td>
<td>n.t.</td>
<td>n.t.</td>
</tr>
<tr>
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<td>-</td>
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<tr>
<td>N9</td>
<td>-</td>
<td>-</td>
<td>n.t.</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>N10</td>
<td>-</td>
<td>-</td>
<td>n.t.</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>N11</td>
<td>-</td>
<td>-</td>
<td>n.t.</td>
<td>+</td>
<td>n.t.</td>
<td>n.t.</td>
</tr>
<tr>
<td>N12</td>
<td>-</td>
<td>-</td>
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<td>n.t.</td>
<td>n.t.</td>
<td>n.t.</td>
</tr>
</tbody>
</table>

*Not tested.

Source: Compiled by the authors.
subjects. With two exceptions,* the remaining subjects were assigned to the low preoperational level.

By these criteria, four children were identified as conservers and 26 as nonconservers. Half (13) of the nonconservers were designated as high preoperational and half (13) as low preoperational. Children in the latter two categories (that is, high and low preoperational level) were randomly assigned to training and control conditions.

The names of the subjects, ages, pretest data, and experimental categories (including whether subjects came from the A.M. or P.M. class) are summarized in Table 2.

EXPLORATORY TRAINING AND TESTING

As indicated in the introduction, the original purpose of our exploratory training was twofold. First, we hoped that the experimentation would provide both a source of ideas for our theoretical analyses and a laboratory for trying out hypotheses resulting from these analyses. And second, we hoped to determine the feasibility of achieving real growth relating to concrete operations in the context of a flexibly organized clinical training environment.

As the training progressed, two things gradually became apparent. Most of the children were operating at a level far removed from concrete operational performance. Hence, it was necessary to begin work at a far more basic level than had originally been planned. Although we experienced some success in devising and administering Piagetian-like tasks, the relationships between these tasks and those associated with conservation were not always as clear as might be desired. Consequently, it became increasingly

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*The two exceptions were assigned to a special category because their pretest scores differed substantially from the others: Ke (N9, L1) and Ad R. (N12, L11). Ad R. (5.42 years old) performed at the lowest level on the number conservation tasks and the next-to-the-lowest level on the length conservation tasks. Ke (5.06 years old) was almost a conserver on length but performed at a very low level on number. (When asked why he answered as he had on length, Ke indicated that he had "noticed this while playing around with sticks."). Because of the special challenge presented, and because the two special cases were themselves so different, we arbitrarily decided to institute training procedures with Ad R. Ke was essentially eliminated from the experiment, although subsequent tests were administered.
### TABLE 2

**Summary of Ages, Pretest Data, and Experimental Categories**

<table>
<thead>
<tr>
<th>Name</th>
<th>Age</th>
<th>L</th>
<th>N</th>
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</tr>
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<td>N4</td>
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<td>N4</td>
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<td>N4</td>
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<td>N4</td>
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<td>N4</td>
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<td>L11</td>
<td>N12</td>
</tr>
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<td>N9</td>
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<td>—</td>
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<td>N10.5</td>
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<tr>
<td>Ya</td>
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<td>L0</td>
<td>N0</td>
</tr>
<tr>
<td>Ki</td>
<td>5.10</td>
<td>L0</td>
<td>N0</td>
</tr>
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<td>La</td>
<td>5.36</td>
<td>L0</td>
<td>N0</td>
</tr>
<tr>
<td>Va</td>
<td>5.63</td>
<td>L0</td>
<td>N0</td>
</tr>
<tr>
<td>Means</td>
<td>5.27</td>
<td>L0</td>
<td>N0</td>
</tr>
</tbody>
</table>

*Source: Compiled by the authors.*
difficult to provide the children with cumulative knowledge leading toward a concrete operational approach to conservation. In general, it was hard to say, not only what students should be taught next, but what they were actually learning. The experiment was not only flexible and open-ended, it was apparently too flexible and open-ended. Although one of the experimenters had been trained in Geneva and the others were widely read in that literature, even the experimenters could not always tell for sure what to do next. (In retrospect, this is probably why Inhelder, Sinclair, and Bovet [1974] were only successful in training "near-conservers."

More important perhaps, our structural analysis of the conservation domain proceeded more rapidly than we had originally expected. Not only did we begin to identify the sources of some of the problems we faced in the training, but we also began to see more clearly exactly what the children needed to learn and the order in which they needed to learn it.

In effect, the desirability of continuing the planned informal training seemed less pressing, and we began to think in terms of the possibility of a more structured training environment. Instituting changes at this point, of course, posed major risks, but, after considering the alternatives, we decided to go ahead in any case.

In spite of their limitations, we did learn some important lessons from our explorations. Moreover, in view of their importance in redesigning and interpreting our subsequent research, the exploratory training and results are summarized below.

**Initial Instruction**

Initially, most of the instruction took place in small groups, for the most part ranging from one to six Ss.

Lessons were taught to four, more or less intact, training groups: Low/Experimental (A.M.), Low/Experimental (P.M.), High/Experimental (A.M.), and High/Experimental (P.M.). The High/Low (preoperational) classification was based on pretest scores. (See Table 2 for individual scores.) The A.M./P.M. distinction has no theoretical significance and simply refers to whether the kindergarten children attended school in the morning or in the afternoon.

Some feeling for the instruction may be obtained from the following brief description of the first two lessons.

The objective of the first lesson was to teach the relational concepts of "more/less" (amount), "longer/shorter," "on/off," and "bigger/smaller" using play dough, water, sticks, room lights,
chips, and chairs. Pairs of "objects" (for example, two playdough balls, two cups of water, or room lights that were on or off) differing in size or amount were presented, and Ss were asked a "which" question (for example, "Which cup has more water to drink?"). The Ss were corrected if they responded incorrectly. Very few Ss demonstrated mastery; the concept of "more/less" was particularly troublesome.

The objective of the second lesson was to introduce the relational concepts/properties of "bigger/smaller," "longer/shorter," "taller/shorter," "thicker/thinner," and "more/less." Each S was given play dough and asked to make a shape "bigger/smaller," "longer/shorter," "taller/shorter," "thicker/thinner," and "more/less" (amount) than the shape made by the experimenter. The shapes made by the E consisted of play-dough balls, horizontal and vertical cylindrical solids, thin and thick cylindrical solids, and very small balls. The E also removed a plastic shape from a box and asked each S to pick one that was "smaller/larger." Some Ss had difficulty making the play-dough shapes, and most could not make balls "less" in amount.

The instructional procedure initially was group-based. However, as personality interactions among Ss increasingly became a problem, we switched to individualized training. Although a variety of instructional techniques were used in introducing new concepts, we frequently began by asking S to tell us what the corresponding term (for example, "longer") meant. If S appeared to understand the term, a set of paired examples of the concept (generally two to three) were presented in which the critical attribute was pointed out. These were followed by two to four test items, one or two of which required transfer.

Summary of Results and Discussion

Clearly, the preliminary intervention with the children was strictly exploratory. Various children were provided with different amounts of training on classification, various relations (shorter, longer, and so forth), one-to-one correspondence, and compensation tasks. The total training times for each child (in hours) were as follows:

<table>
<thead>
<tr>
<th>High/Experimental</th>
<th>Low/Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.M.</td>
<td>P.M.</td>
</tr>
<tr>
<td>Ti            2:35</td>
<td>Ra        2:50</td>
</tr>
<tr>
<td>Da            2:35</td>
<td>Ty        2:50</td>
</tr>
<tr>
<td>Ma            2:35</td>
<td>Do        2:50</td>
</tr>
<tr>
<td>Sp            2:35</td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
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<td>P.M.</td>
</tr>
<tr>
<td>An            2:05</td>
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</tr>
<tr>
<td>Ro            3:00</td>
<td>Mi        3:40</td>
</tr>
<tr>
<td>Ni            3:25</td>
<td>Br        3:25</td>
</tr>
<tr>
<td>Ad.R.         3:50</td>
<td></td>
</tr>
</tbody>
</table>
Not only did different children often receive different training but no serious attempt was made to insure performance at some prespecified level on the tasks in which they were trained. Consequently, it is impossible to say exactly what individual children did and did not learn as a result of this intervention.

In effect, the intervention served more as a source of insights for the experimenters than as an organized training environment. However, informal interaction with the children led to two important observations. We do not claim any originality in this regard, but it is a fact, nonetheless, that their critical importance became clear to us only after working with the children over a period of time.

First, it quickly became apparent that the children frequently had difficulty understanding the problems that we were presenting for them to solve. This made it difficult (or even impossible) not only to obtain reliable estimates of their capabilities but also to maintain their attention. More specifically, our observations suggested that attention varied directly with the extent to which the children knew what they were supposed to do. When the problems seemed ambiguous, the children tended to lose interest rather rapidly. Conversely, attention span seemed to "magically" increase when a special effort was made to insure that the children properly understood the problems and, in particular, when they were able to evaluate their own responses reliably. In the latter case, the children apparently were able to determine for themselves whether or not potential solutions did indeed satisfy the relevant goal conditions.

In order to overcome this problem in subsequent instruction, children in the experimental groups were not asked to solve problems until we were sure that they understood the problems (that is, had demonstrated their ability to evaluate potential solutions).

A second major problem involved the terminology used. Whereas terms like "more/less," "longer/shorter," "bigger/smaller," "thicker/thinner," "shorter/taller," and so forth, have fairly well-defined meanings for adults, children clearly found it hard to separate near synonyms from different concepts. To make matters worse, individual children tended to prefer various terms for similar ideas. This made it difficult to tailor vocabularies in the instruction to individual needs.

An especially important difficulty involved apparent inconsistencies in children's responses to what appeared from an adult perspective to be equivalent problems. Asked if A is longer than B or if B is longer than A, for example, a child might say that A is longer. Then, when asked if that meant A and B are the same, the same child might respond "yes." One way to explain such behavior is to assume that children are irrational or, put more palatably, have not yet acquired the necessary logical reasoning abilities. In fact, the latter view is characteristic of Piagetian stage theory.
An alternative way of conceptualizing such inconsistencies is in terms of the rules of knowledge available to the children in question. In particular, it seems apparent that the semantic meanings assigned to the terms "longer" and "same" in these contexts overlap for the young child. That is to say, in comparing pairs of objects, the terms "longer," "shorter," and "same" do not necessarily signify mutually exclusive events for the child.

To see this, one has only to consider how an intelligent adult might respond to the two illustrative tasks—if warned in advance that by "same" the experimenter is not necessarily referring to length. The child must learn that the terms "more," "less," and "same" in a given context are normally understood to refer to a common property—to form a trichotomy. And, moreover, the child must agree when one says that two objects are about the same with respect to a property that any difference on that property is understood to be ignored in further discussions. (One can, of course, always find a difference in the real world if one tries hard enough.) The situation is further complicated for the child because of the richness of the English (or other natural) language. Thus, "taller," "higher," "more height," even "bigger," and so forth, are near but not perfect synonyms. It takes time for children to learn the adult connotations of these terms.

Notice in this regard that the role of logic per se is suppressed in rule-based explanations. On the other hand, such an analysis would appear to have greater flexibility and precision. Perhaps more important, we have shown that the characterization of underlying competence (and individual knowledge) in terms of rules lends itself to operational definition in terms of observable behavior (Scandura, 1971, 1973, 1977b).

According to this view, more precise use of language clearly was needed if our training was to be successful—at least with children operating at the level ours were. Only in this way would we be in a position to present problems that were meaningful to the children and/or provide training in solution rules for solving them.

In order to facilitate training with regard to "inconsistent" responding, we decided to work with a simplified syntax and use it consistently. Specifically, our strategy was to introduce single terms to convey the various concepts that we had in mind. To words or phrases, such as "height," "area," and "amount to drink," we would add the word "more," "less," or "same." Hence, we would talk about one object having "more height" or "less height" than another but never about one being "taller" or "shorter." Similarly, one stick might have "more length" than another stick, but it would never be called "longer."
More/Less Test

In spite of the obvious limitations of our exploratory training, a short test was constructed and administered to determine the ability of experimental and control subjects to use the critical terms "more" and "less." The test was designed to determine if there were significant differences between the control and experimental groups in the use of "more" and "less" and to determine the source of difficulties in children's understanding and use of the words "more" and "less." Ideally, we wanted to know (a) given a visual situation involving two balls of clay of different colors and sizes, if S could recognize or reconstruct this situation (some items required memory, some did not); (b) given a visual situation, if S could tell which of the four statements—"red more than white," "white more than red," "red less than white," and "white less than red"—were true and which were not (with and without memory); (c) given a statement, if S could construct or recognize the corresponding visual situation (all items required memory); and (d) given a statement, if S could tell which of the four statements—"red more than blue," "red less than blue," "blue more than red," and "blue less than red"—were true and which were not.

These four types of items may be represented graphically as follows (numbers in parentheses identify which test items correspond to the various types):

Our original goal, then, was to determine whether failure was due to inadequacies in the conceptual field, in the lexical field, or in the passage from one to the other. To achieve this goal, however, we were compelled to ask many similar questions, increasing the possibility of unwanted sequence effects.

As can be seen from Table 3, there were no clear differences favoring the experimental group. In fact, the control subjects as a whole did slightly better on the test.
### TABLE 3

Number of Mistakes on Test Items

<table>
<thead>
<tr>
<th>Test Item</th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3a</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>c</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>d</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>6a</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>d</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>9a</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>b</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>d</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>10a</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>b</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>c</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>d</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Note: The number of errors per child was as follows:

E = 8.38 (n = 13), C = 7.17 (n = 12).

Source: Compiled by the authors.

The gradual increase in the number of mistakes from items 1 through 10 is a direct consequence of the increasing complexity of the items (that is, moving gradually from memory independent, visual, and direct to memory dependent, verbal, and inferred).

The patterns of responses elicited by a couple of children were apparent (for example, Sh answered "yes" to each question, while Ra gave alternate answers), but otherwise there was not enough data to reliably identify possibly important sequence effects.
Getting at such effects would have necessitated development of a conditional testing procedure, such as that used with the pretests, but our expectations of the reliability of the training (borne out by the results) did not warrant the investment of the time that would have been required.
Structural analysis of conservation tasks proceeded concurrently with the exploratory empirical work reported in the previous chapter. Our original intent was to determine the feasibility of applying this method to a large, global, and relatively unstructured problem domain, such as is inherent in the Piagetian stage of concrete operations. Rather than attempt to analyze all tasks associated with concrete operations, however, we selected a subdomain that was sufficiently large to be challenging but not so large as to be intractable. Piagetian-like conservation tasks (for example, conservation of number, length, amount, and so forth) seemed to serve both purposes and constituted the basis for our analysis.

As is well-known, conservation tasks as commonly administered (for example, by Genevans) involve the following steps. First, the experimenter must ascertain whether or not the child can determine or construct two objects, sets of objects, and so forth, denoted A and B, that are the same with respect to some critical concept (property). (Typically, the A and B "objects" share a variety of properties.) The critical concept might involve number, length, area, liquid quantity, amount, or, in principle, almost anything else. For example, the child might be asked to make two balls of clay that "contain the same amount to play with." In this case, the two balls would be expected to have the same height, shape, and so on, as well as the same "amount of clay" per se. Given that a child can accomplish the above, the experimenter then transforms the B object and asks the child whether the original object (A) has more or the transformed object (t(B) or B') has more. (The child also is allowed to say "same" if he believes that neither has more or less.) Finally, the child is required to justify his response. "Why does this one have more?" The experimenter may probe with additional questions until sure that the child not only is able to give the correct response but can rationally and reliably justify that response, usually by giving several reasons.
It is important to emphasize in this regard that successful performance on particular conservation tasks is not necessarily the same as performing at the concrete operational level. For example, consider number conservation. Given any two sets of objects, it is possible to determine which has more (or that they have the same) by simply pairing the A and B objects in a one-to-one fashion or by counting, subitizing, and so forth (Scandura, 1972). In the discussion below, we shall refer to rules for comparing A and B with respect to c (Ac and Bc entities) as concept comparison rules (CC rules). (The procedures that Gelman [1977] calls "estimators" in discussing number conservation are just one kind of CC rule.) Clearly, one could teach a child to solve number conservation problems in this way, and, as we shall see below, similar training might be provided for any other conservation task. One could also teach the child to give appropriate answers to typical "why" questions.

It is a basic mathematical fact, however, that if there is one systematic procedure for solving any given class of tasks, then there is an indefinitely large number of other systematic procedures for achieving the same end. (Behavioral implications are discussed, for example, in Scandura, 1970, 1972, 1973, 1977b.) In particular, solution methods of the above sort (on which many North American Piagetian training studies have been based) deal only indirectly with the way Genevans would have the child perform. The latter purportedly corresponds to the way children naturally come to know how to perform on conservation tasks. (More particularly, as argued below, CC rules are prerequisites to achieving concrete operational capability.)

Children who use CC rules and children who truly perform at a concrete operational level—for example, in terms of operators (Gelman, 1977)—may be expected to respond similarly when presented with given conservation tasks. Children who solve conservation problems in the former way, however, are more likely to become confused when faced with new conservation tasks (that is, tasks whose solutions require different CC rules from those in which the children were trained). In effect, whereas alternative modes of solution may be equivalent with respect to given problem domains, they will not necessarily be equivalent on other, typically more encompassing domains (Scandura, 1977b, chap. 2). Similarly, the reasons children give when responding to novel problems may be meaningless in the sense that children may simply memorize them with respect to given contexts. With no real understanding of why given answers are correct, such children might reasonably be expected to have difficulty in answering "why" questions about new problems.
Genevan interactional methods of testing are designed largely to distinguish between alternative methods of solution. Moreover, Piaget has developed an elaborate theory not only to explain the child's functioning at the concrete operational (and other) stages but also to explain how the child progresses from stage to stage. Specifically, when responding to conservation tasks, the child at the concrete operational stage, in contrast to one at the preoperational stage, would be expected to concentrate more on the initial states and dynamic transformations (t) involved, rather than make static, more perceptually based comparisons between the A-t(B) pairs resulting from such transformations; be aware of the possibility of reversing such transformations to arrive at the initial comparative (A-B) states; and evidence decentration, the ability to take into account the relationships between a critical concept (for example, number or length) and two or more covarying irrelevant properties (the relative placement of the two endpoints in a length conservation task, for example), as opposed to being restricted to single irrelevant properties.

Despite recent attempts in this direction, at least partially at the prodding of more experimentally oriented psychologists, Genevan investigators have not yet been able to adequately make operational critical concepts in behavioral/observational terms. Thus, for example, the writings of Piaget and other Genevans are highly suggestive about what it means for a child to evidence operativity, reversibility, and decentration. But, as a growing amount of recent research amply demonstrates (for example, Cooper et al., 1978), this is not the same as defining these constructs directly in terms of observables. Piagetian theory must be made operational if it is to lend itself to definitive empirical testing.

A related limitation of Piagetian theory is its relative lack of behavioral precision, expressions of the theory in formal terms (for example, logic or group structures) notwithstanding. In particular, universal constructs, such as operativity, reversibility, and decentration, refer to ideals that appear to capture certain important aspects of cognitive development. They leave much unsaid, however, about the behavior potential of individual children. It is well-known, for example, that children learn to conserve various concepts/properties (for example, number, length, and volume) at different times—a phenomenon that goes under the rubric of "horizontal decalage." In effect, Piagetian theory deals far better with the commonalities among various conservation tasks and various subjects than with decalage and other frequently observed behavioral differences among individuals.

Although we only mention the problem here (detailed discussion is prevalent in the literature, for example, in Piaget, 1955), similar
limitations appear to exist with regard to the acquisition of concrete operations. It is one thing, for example, to appeal to assimilation, accommodation, and equilibration as generalized mechanisms of knowledge acquisition and quite another to provide a detailed account of how individual knowledge is acquired (that is, acquired as a result of individuals interacting dynamically with problem situations in the real world over potentially long periods of time).

As noted in the introduction, the Structural Learning Theory (Scandura, 1971, 1973, 1977b) has features that might make it possible to avoid the above limitations. The theory is designed to be operational in its essential aspects, it deals specifically with the behavior of individual subjects in particular situations, and it provides an explicit basis for learning. Moreover, recent extensions of the theory (Scandura, 1977b, chap. 14) make it possible to explain, predict, and manipulate the growth of knowledge over potentially long periods of time.

In order to apply the theory to the domain of conservation tasks, however, indeed to any new problem domain, one must first subject the domain to a more or less detailed structural analysis—that is, "more or less detailed" depending on the desired/required level of behavioral precision (Scandura, 1977b, chap. 2). Generally speaking, this is not a trivial undertaking. On the average, the amount of effort that goes into devising detailed structural analyses is comparable to doing analogous research in artificial intelligence. (Structural analysis is more demanding because of explicit constraints on the way competence is represented. These constraints are both universal and [in terms of content/population] specific in nature. Structural analysis is less demanding and more reliable [that is, replicable], on the other hand, in that the analyst has a definite method of analysis to use. In addition, for most purposes, he can ignore technical, but often major, problems associated with computer implementation.)

On the other hand, structural analysis is relatively efficient and can be used effectively to analyze complex domains in greater detail than appears feasible using traditional normative experimentation (such as has characterized contemporary information-processing research in cognitive psychology). In contrast to the latter, however, most existing structural analyses have not dealt with latency, although there is no reason in principle why this should not be possible (Scandura, 1973, chap. 10; 1977b, chap. 2; Voorhies and Scandura, 1977).

In general, the precision of a structural analysis must be commensurate with the degree of behavioral detail one wants to explain. Judgments regarding the latter normally will depend on such things as the complexity of the domain, the time and money available for the analysis, and the relative state of understanding in
the field. When behavior associated with a problem domain is poorly understood, for example, even a crude structural analysis might have considerable practical significance.

STRUCTURAL ANALYSIS OF CONSERVATION

Specification of Conservation Problems

In carrying out a structural analysis of the conservation domain, the first step is to more exactly specify and delineate the domain. What precisely is a "more/less/same" problem? Put differently, how are "more/less/same" problems to be represented cognitively? Similarly, what is a "why" question? Until such problems can be formulated unambiguously, it will be impossible to determine what a child needs to know in order to perform at a concrete operational level on these tasks, much less to know how to go about training the child to perform in this manner.

Without loss of generality, the standard conservation situation can be represented as follows:

(1) \[ A_c \xrightarrow{t} A_c \]
\[ B_c \xrightarrow{t(Bc)} B'_c \]

where \( A \) and \( B \) represent objects, sets of objects, displays, and so forth, and the subscript \( c \) refers to the critical concept/property under consideration. The arrow denotes some transformation \( t \) of \( B \) (yielding \( t(Bc) = B'_c \)). Conservation of number, for example, might be represented

(2) \[
\begin{array}{c}
\text{OOOOOO}\\
\text{XXXXXX}
\end{array}
\xrightarrow{t_1}
\begin{array}{c}
\text{OOOOOO}\\
\text{XXXXXX}
\end{array}
\]
where number is the critical property and \( t_1 \) denotes the particular transformation in question. Similarly, in conservation of liquid quantity, for example, the water in a beaker having one shape typically is poured (transformed) into a beaker with another shape (\( B' \)) and might be represented as follows:

(3) 
\[
\begin{array}{c}
//\\
//\\
\end{array}
\xrightarrow{t_1}
\begin{array}{c}
////\\
////\\
\end{array}
\]
In the Structural Learning Theory, the term "problem" has a technical meaning. In specifying a problem, it is essential to explicitly list both the goal variables and the given elements, properties, relations, and operations (including higher-order relations and operations). (Note that goal variables may have relations or operations for values as well as simple elements.) Specifically, in the typical conservation problem, after constructing (or selecting) B so that A and B have the same measure of concept/property c, B is transformed through t, giving t(B) = B'; usually t is carried out in view of the subject. In either case, the child is asked: "Does A have more c? Or, does B' have more?"

The given of this problem consists of the basic elements of which A and B are constructed (A, B, and B' themselves, and so forth), relevant properties of A, B, and B' (not only property c but other properties to which the child might attend), various equivalence relations between A and B (that is, A and B are not only equivalent in terms of concept c but in terms of various irrelevant properties as well; all feasible possibilities must be included in an adequate representation), the operation t, and various (not necessarily equivalence) relations between A and B'. All must be represented at a level of detail compatible with the development level of the children (that is, the target population). In particular, if the representation is to provide an adequate basis for explaining individual behavior, the components (of the representation) must be atomic, in the sense that they affect each child's behavior uniformly or not at all. In principle, such a level of detail can always be determined (Scandura, 1973, 1977b).

The goal, in this case, may be satisfied by any one of the answers "A has more c," "B has more c," or "They have [about] the same." Put somewhat more abstractly, without loss of generality, these alternatives define a three-valued goal variable that can be represented as follows: Ac > Bc', Bc' > Ac, Ac = Bc'. (For our purposes, the child may be assumed to know the correspondence between these relations and the corresponding verbal descriptions.)

It also is important to emphasize that true conservers (children at the concrete operational level) are expected to respond with certainty, speed, and little effort. Hence, the effective goal not only is to state the proper relationship between A and B' (that is, A > B', B' > A, or A = B') but to do so in a manner consistent with the above. These requirements are an integral part of the goal.

Of course, even a correct response is not sufficient to indicate that a child is operating at the concrete operational level. After responding correctly to a "more/less/same" problem, the child also is required to give an acceptable reason for his response. "Why" problems effectively include as part of the given information
not only the givens of the corresponding "more/less/same" problems but the correct responses themselves. "Why" problems are intended to distinguish real from pseudo conservers and, hence, are meaningful only in the context of "more/less/same" problems that have been solved correctly.

The goal of "why" problems, then, is to provide an acceptable reason. More precisely, we think, Genevan investigators require an indication that the child can derive the consequent relation between A and t(B) = B', given only the nature of the transformation t and the initial equivalence of A and B with respect to c before transformation. Moreover, it is assumed that the child can make this inference without answering the question of whether or not it is feasible to make the comparison between A and t(B) directly (for example, in number conservation, by one-to-one matching). A schematic representation of a "why" problem follows.

\[
\begin{align*}
\text{(4) } & \quad \begin{cases}
\{ & O \quad O \quad O \quad O \quad O \quad O \quad t \quad O \quad O \quad O \quad O \quad O \\
X \quad X \quad X \quad X \quad X \quad X \\
X \quad X \\
X \quad X \\
\end{cases} \quad \text{--- } \quad \text{"same" } \\
\text{? Why?}
\end{align*}
\]

In this regard, it should be stressed that the child's verbal fluency, or the particular words that he uses to describe his reasons, is not at issue. What is at issue is the ability of the child to state or to otherwise give the observer an indication that he knows what the correct relation should be, based on the initial state (that is, the A-B pair) and the nature of the transformation t. As with "more/less/same" problems, a child is not normally considered to be a "true/stable" conserver unless he can give such an indication quickly, reliably, and with conviction (that is, in a manner not susceptible to countersuggestion or deception). Put as precisely as possible in the present context, the child's responses to a range of "why" questions, taken collectively, should indicate that the child can distinguish between those transformations t that affect a given concept c and those that do not. In this regard, the child also must be able to rationalize, where appropriate, apparent perceptual differences between A and B' (after transformation t). This might be accomplished, for example, by giving a compensation argument—"This one looks longer but the other is more bunched." Moreover, the child is supposed to be able to make his judgments more or less automatically (that is, quickly).

The final point to stress in this regard is that the intended problem domain is not limited to any given (finite) number of concepts.
(that is, to a finite number of "more/less/same" and "why" conservation problems associated with such concepts). In principle, a child operating at this level should be able to perform successfully on conservation problems about any concept (for example, roughness or height, as well as the more familiar number, length, and so forth). Clearly, it would be impossible to test any given individual on more than a relatively small number of such conservation problems, but the child at the concrete operational level ideally should be able to generalize to all conservation tasks.

Selection of a Representative Sample of Problems

Following the prescription for carrying out a structural analysis (Scandura, 1977b), the next step is to select a representative sample of both "more/less/same" and "why" conservation problems. In general, the objective is to select problems that are sufficient in number and variety to include all important features of the problem domain. Hence, for purposes of efficiency, a structural analysis ideally should include only problems that are essentially "different" in some way. For purposes of completeness, all such problems should be included. In the present structural analysis, we consider a conservation domain that is large enough to be suggestive as to generalization of the analysis, but not so large as to be unwieldy or impractical for experimental purposes.

For present purposes, we selected "more/less/same" and "why" problems dealing with number, length, area, amount (of clay), (a restricted form of) liquid, and the nonstandard conservation concepts of height and "turn." (For detailed descriptions, see below and the sections on experimental method.) Clearly, each such problem may be represented in one of the two forms indicated above. However, because height is a nonstandard concept in the conservation context, and because it is influenced differently by certain transformations than are other conservation problems, an additional observation is in order. Specifically, we want to make clear that height, as it is normally defined, is not the same as length. Height (h) refers to the vertical distance between the base on which the object rests (for example, the ground) and the top of the object (that is, the highest horizontal plane touching the object), as is illustrated by the following examples. Notice, in particular, that the height of the object is affected by a rotation, whereas the length of the object is not.
Specification of Prerequisite Competence: Concept Comparison Rules

In structural analysis, the next step is to formulate solution rules for each of the sampled problems. It is not sufficient, in this regard, to identify just any solution rule. There are any number of ways in which a child might respond to "more/less/same" problems. In evaluating responses to conservation problems (to determine concrete operational behavior), for example, Genevans are singularly uninterested in solution methods that compare A and B' directly (with respect to some concept/property c). Children who can make such comparisons correctly would be characterized in our formulation as knowing a rule or rules that can be applied to arbitrary A-B pairs to make "more/less/same" comparisons. Confronted with a "more/less/same" number conservation problem, for example, the child might determine which has more by simply matching the elements in A and t(B) in one-to-one fashion (compare Gelman, 1977; Scandura, 1972).*

The domains of such concept comparison (CC) rules consist of A-B pairs (for example, pairs of sets/arrays, in the case of number); the ranges, of "more/less/same"; and the operations, of explicit (restricted) procedures for making the intended comparisons. (Note that constructing A and B so that they have the same amount of property c, as is required of subjects before testing for conservation, may be accomplished through more specialized, that is, less general, competence. Perceptual cues may be used in constructing A and B to be equivalent in property c [but not in comparing

*Before we became seriously involved in Piagetian theory, Scandura (1972) proposed an analysis of precisely this sort. Whereas the identified rule was quite sufficient for explaining the particular behavior in question, however, it clearly is not consistent with Genevan criteria.
arbitrary A and B in property c], for example, because A and B after construction typically are identical in any number of respects, not just with respect to property c.)

Genevans appear to view the acquisition of CC rules as a necessary, but not sufficient, condition for concrete operational performance. Only children who know some such rules are viewed as possible "conservers." In general, children who are able to make such comparisons are likely to be more advanced than classically defined preoperational children. Thus, the preoperational child, as defined by Piaget, may be thought of as basing his (comparative) responses on perceptual characteristics of the A-B' pairs. (For example, upon viewing two sticks, A and B', of the same length, with B below but more to the right, the child might say that B' is longer because the right endpoint is further to the right—ignoring the compensating deficit on the left.)

Detailed examples of CC rules are given in the next section. For present purposes, it must be emphasized that knowing appropriate CC rules is a crucial prerequisite to (but not sufficient for) concrete operational performance in the Piagetian sense. In particular, once automated, such rules may serve to define "more/less/same" problems in the more general sense described above (that is, when account is taken of initial A-B equivalences and transformations t). Specifically, automated CC rules provide the child with an independent basis for evaluating responses based on initial states and transformations.

During the normal course of development, children may be expected to learn a variety of CC rules, some of which may be more efficient (automatic) than others. Consequently, it is reasonable to expect that seemingly similar conservation problems may differ in meaningfulness at any given time for any given child. Note that when rules are well learned, that is, automated, or when differences among alternative, behaviorally equivalent rules are not of primary concern, the underlying competence may be characterized in terms of relations. In effect, relations are equivalent to classes of rules in the structural learning formulation and may be used in place of particular rules when details of the latter are not of direct interest.

Hence, relations might be used to characterize the concept comparison capabilities of children who have already mastered the requisite CC rules. Such ability plays an essential role for the child in defining "more/less/same" problems (as above). Because emphasis here is on the additional competence (that Genevans believe children acquire through normal development of concrete operations), specifying cognitive details involved in comparing concepts would be superfluous. Such detail would be necessary, of
course, where children have not yet acquired needed CC rules. Since this was the case with many of our five-year-olds, specific CC rules were identified and taught in the study described below. These rules are detailed there in the method section.

Limitations of Piagetian Structures

Whatever other knowledge they might have (for example, concept comparison rules), children at the concrete operational level, according to Genevan accounts, are able to generate correct responses by appealing to the transformations involved—that is, the A-t(B) relationship may be determined by reference solely to the initial A-B state and the transformation t.*

*Even here, it is possible to give correct answers without necessarily satisfying Piagetian conditions. Thus, in number conservation, for example, the child might learn or be taught to key his responses solely on whether the experimenter adds or takes away elements in the process of transforming B and ignore other aspects of transformations. Whereas responding on this basis would be sufficient in dealing with many "more/less/same" conservation problems (and whereas the child might also be trained to give correct verbal responses to corresponding "why" questions), such solution rules would have at least three major limitations. First, such rules may be taught with no reference to underlying concepts. As far as the child is concerned, the same rule might be expected to work with height, say, as well as number—and without modification. In effect, the child would have no reliable basis, for example, for determining how to respond to height conservation tasks when the critical transformation in question is a rotation. Similarly, in area conservation, the experimenter might "add houses to a toy farm," which would have the effect of reducing "the amount of grass for the cows to eat." Second, unless the child knows how the concepts and transformations are related, it may be possible to trick the child. In particular, unless the child knows an appropriate CC rule, he will have no independent means of comparing A and t(B) (after transformation t) even when critical changes might have been made in transforming B without the child's awareness (for example, as in surreptitiously removing objects). Finally, whereas adding or taking away with number has fairly direct analogues with respect to many conservation concepts, there are both subtle differences, as with area (above), and some not so subtle,
According to Genevans, the reasons children at the concrete operational level give in responding to corresponding "why" problems further indicate that such children take into account the entire display (and not just the final A-B' states), recognize the possibility of returning to original A-B states by reversing observed transformations, and can rationalize incompatibilities between perceptual comparisons of terminal A-B' states and comparisons based on transformation-based reasoning. (We are referring here to the ability of the child at the concrete operational stage to give compensation-type arguments.)

The main difficulty with the above account, as was the case in specifying conservation problems, is not one of omission but rather one of precision. It is not, however, precision just for the sake of precision. Rather, we believe that greater precision would have some important benefits that have so far eluded Piagetian theory. As noted previously, for example, it has been difficult to "educate" children in concrete operations in a way that is at once stable, generalizable, and consistent with normal development. On the average to date, only about half of the children trained in conservation training studies have learned to conserve (Murray, 1978). According to Inhelder, Sinclair, and Bovet (1974), for example, children do not become true conservers as a result of such training unless they are already at a transitional stage.

More recently, some success has been achieved in longitudinal studies designed to teach preoperational children to conserve (Lawton and Hooper, 1978). Even here, however, generalization of the results to new conservation tasks has not been obtained (Lawton, personal communication). Until we succeed in specifying exactly what it is that children must learn in order to function at the concrete operational stage, it will be difficult to systematically plan efficient learning environments that reliably promote such transitions.

Perhaps even more important from a theoretical perspective, we believe that more precise specification of underlying competence may be essential if we are to achieve an adequate understanding of as with roughness. In either case, appropriate generalization of the ability to distinguish transformations that involve adding/taking away from those that do not requires reference to the concept in question. What, for example, corresponds to adding/taking away elements when dealing with roughness? The answer obviously involves physically increasing or decreasing the surface texture, but just how this relates to adding/taking away with number can only be made precise by reference to the underlying rules.
such phenomena as horizontal decalage. Why is it, for example, that children generally conserve with respect to some concepts before others? According to Piagetian theory, all conservation tasks involve the same underlying structures. Although all conservation tasks share certain characteristics, however, there also are important differences. An adequate understanding of horizontal decalage may never be achieved until these differences can be spelled out in terms that have direct behavioral significance.

Just as there is a need for greater precision regarding underlying processes and structures, learning mechanisms also must be specified in an operational manner. We do not believe that assimilation, accommodation, and equilibration serve this purpose adequately and propose instead to base our training on structural learning mechanisms. It will be assumed, for example, that when trying to solve a problem for which they do not already know a solution rule, all people automatically attempt to derive one.

Specification of Solution Rules for Conservation Problems

The above discussion, it is hoped, provides sufficient justification for another major aspect of structural analysis, the precise specification of solution rules underlying sampled ("more/less/same" and "why") problems. As is the case with problems, the term "rule" has a precise, technical meaning in the Structural Learning Theory (Scandura, 1970; 1977b, chap. 2; 1978a). To review, a rule consists of a domain (the set of entities to which the rule applies), a range (the set of outputs "expected" as a result of applying the rule), and a restricted type of procedure (in which it is not possible to derive the equivalent of new "subroutines" and then use them in the same rule application). The last component, of course, constitutes the operative aspect of the rule; the others are static (or, to use Piagetian terminology, "figurative").

In the present context, the basic problem is that of how to represent the prototypic competence that Genevans believe underlies conservation behavior—that is, to devise some rule or rules that make it possible not only to generate conservation responses quickly and reliably but to do so by keying on the given transformation. Ideally, such a rule should apply to all manner of conservation tasks, both standard (for example, those dealing with number or length) and nonstandard (for example, those dealing with height or roughness), and might be represented as shown in Figure 3.

In addition, the idealized child at the concrete operational level also must have available a CC rule corresponding to every conceivable conservation concept (for example, for comparing pairs of sets to determine which contains more elements). In the case of
number, for example, the domain of the CC rule might consist of sets of pairs of A-B objects; the range, of "more," "less," and "same"; and the restricted procedure, of one-to-one matching together with the additional simple steps needed to determine which set has the larger number (for example, determining if there are unmatched elements left after the B objects have been physically paired with the A objects). Without such rules, as observed previously, "more/less/same" problems would lose much of their intended meaning for the child.

![Diagram](attachment:image)

**FIGURE 3.** Idealized, automated higher-order t rule for arbitrary conservation tasks. This rule operates on arbitrary conservation situations, including t, generates appropriate responses to more/less problems, and involves a single unitary decision.

The well-known phenomenon called "horizontal decalage," of course, suggests that the knowledge available to most conserving children, certainly those from ages five to seven, at best only approaches this ideal. In general, research has shown that otherwise identical rules may differ considerably as to scope (Scandura, Woodward, and Lee, 1967)—that is, in the inclusiveness of their domains. Thus, the conservation rules of knowledge available to particular children in this age range are apt to be less general and/or less automated. Thus, for example, at one extreme higher-order t rules might apply only to a subset of conservation tasks
associated with a given concept (for example, conservation of number when each of the A-B sets involves no more than three elements). In the case of "conservers," the available higher-order t rules would almost certainly encompass all tasks associated with given concepts and would be more likely to include a number of more or less related concepts. A specific higher-order t rule of the latter type is detailed below in the discussion of the acquisition of conservation.

In the present context, notice that the higher-order t rule depicted in Figure 3 is a complex general rule that has been automated. It applies to all "more/less/same" conservation problems (appropriately distinguishing among them) and yet involves only a single atomic decision—one that attends solely to observable characteristics (components) or arbitrary transformations t in relation to concepts c (that is, responses depend solely on the nature of t and c). The basic question here is, Where do generalized, automated rules of this sort come from?

More Basic Solution Rules

In the present context (that is, using established principles in structural analysis), the answer to this question resides in a set of more basic rules—rules, for example, that are less complex and less automated and, hence, are more likely to be prototypic of less skilled, younger conservers.

What are such rules apt to be like? For one thing, they are probably less flexible. Thus, the rules may be keyed to particular types of transformations, such as adding elements to a set or pouring liquid into a container, rather than to more abstract classifications, such as having the property of increasing (or decreasing) some arbitrary concept.

Number conservation provides an instructive prototype (see display 2 above). In particular, consider A-B situations where B is transformed slowly in front of the child so that he can observe the kinds of changes brought about. We have seen that solutions to conservation problems based solely on CC rules would not be consistent with the cognitive structures that Genevans associate with the stage of concrete operations. Rather, the true prototypic conserver appears able to determine the correct A-B' relationship based on the initial A-B state and the transformation t.

In particular, it would appear to be sufficient for this purpose to have available a higher-order transformation (t) rule that operates on the class of transformations t (that is, on the class of those t's that apply to concrete, finite sets of objects, such as B) and
distinguishes (partitions) them as to whether or not they involve, among possibly other things, adding or taking away objects (in B). Figure 4 represents one such rule.

**FIGURE 4.** Higher-order t rule for number conservation tasks. This rule operates on the class of transformations t that apply to concrete, finite sets of objects (for example, set B). The symbol +n (-n) means that some unspecified number of elements has been added to (taken away from) set B.

There are, of course, any number of possible transformations that might be applied to such concrete sets as B. Most of the more common compound transformations appear to be combinations of the more basic ones shown in Figure 5A (see also Figure 5B). In effect, the domain of the higher-order t rule of Figure 4 might be defined more precisely as the class of transformations that can be generated by combining one or more of the basic types of transformations illustrated in Figure 5A. For this purpose, of course, cognitive representations of the transformations t must include specification of all relevant basic transformations. Correspondingly, the range consists of the responses "B > A", "A > B", and "A = B".

The operational aspect of the rule (that is, the decision in Figure 4) consists of the following categories: involves adding to set B (+n), involves taking away (-n), involves neither. As
FIGURE 5A. Illustrations of basic types of transformations on number (that is, concrete sets).
represented in Figure 4, notice that this higher-order \( t \) rule involves only one decision and, hence, is relatively automated. Essentially, what the rule does is to automatically check transformation \( t \) to see if it involves adding or taking away. (Note that the output of this particular rule is unambiguously specified only when exactly one adding/taking away component is involved.)

**States Resulting from COMPOUND TRANSFORMATIONS**

Linear (horizontal)
Rotation (45°)
Density (decreased)
Substitution (\( Y \) for \( X \))

\[
\begin{align*}
A & \quad O O O O O O O \\
B' & \quad Y Y Y Y Y Y Y
\end{align*}
\]

Linear Displacement (horizontal)
Density (decreased)
Shape (changed toward circular)
Homogeneity (nonuniform)

\[
\begin{align*}
A & \quad O O O O O O O \\
B' & \quad X X \\
& \quad X \\
& \quad X
\end{align*}
\]

**FIGURE 5B.** Illustrative compound transformations on number.

With less advanced children (that is, those who have not reached automation), it is reasonable to assume that the basic decision (in the rule of Figure 4) would not be atomic (automated). Consequently, an adequate rule characterization of competence would require more detailed specification of the individual steps (including subdecisions) that such children might go through in deciding whether or not any given \( t \) involves adding or taking away. Although it was beyond the scope of this analysis to actually identify
a suitable competence rule of this type, it is easy to envisage more detailed rules that specify and operate on components of transformations, such as those illustrated in Figures 5A and 5B.

Analogy and Generalization

As argued above, it is not sufficient that the conserving child give correct answers and adequate reasons when dealing with number conservation, or for that matter, with conservation of any specific type. The child must be able to solve a wide variety of (ideally all) conservation problems.

Obviously, it would be impossible to learn de novo solution rules for every conceivable conservation concept. Fortunately, this might not be necessary in the present case because of close similarities among the needed solution rules. It is easy, for example, to envision higher-order t rules for other conservation concepts analogous to (that is, of the same form as) that show in Figure 4 for number. In each case, the major differences would involve the critical (conservation) concepts/properties involved (for example, length, volume, and so forth) and the corresponding CC rules and transformations (components) that affect these properties (for example, extending/cutting, pouring in/out, and so forth, as opposed to adding/taking away).

In situations like these, J. M. Scandura and his associates (Scandura et al., 1971; Ehrenpreis and Scandura, 1974; Scandura, Durnin, and Wulfeck, 1974; Scandura, 1977b; Scandura and Durnin, 1977) have found that parallels among solution rules can often be represented in the form of higher-order rules.* Here, for example, analogous higher-order t rules for other conservation concepts can easily be constructed from one for number by simply substituting

*More generally, they have found that it is possible to identify more basic sets of rules, consisting of both higher- and lower-order rules. In these cases, not only was it possible to regenerate the original solution rules by application of the former to the latter, but it was also possible to derive solution rules for new (unsampled) problems as well. In effect, basic rule sets of this type tend to be more powerful than the original set of solution rules from which they are derived (Scandura, 1977b), in the sense that the basic rules collectively can be used to generate solutions for a wider variety of problems. Moreover, the individual basic rules tend to be simpler than the original solution rules.
the relevant concept for number and the critical transformation (component) for adding/taking away. Given the higher-order t rule depicted in Figure 4, for example, a higher-order t rule for length conservation can be derived from it by substituting "length" for "number" in the domain and "extending/cutting" for "adding (+n)/taking away (-n)" in the critical decision.

It is easy to formulate higher-order analogy rules of this type (compare Ehrenpreis and Scandura, 1974; Scandura et al., 1971). Thus, one that would serve present purposes might have a domain consisting of higher-order t rules paired with transformations that affect the (new) conservation concepts. The range, clearly, would consist of the corresponding class of (new) higher-order t rules. The procedure in the higher-order analogy rule would involve the indicated substitutions.

Higher-order generalization rules could take precisely the same form. In this case, however, instead of substituting particular critical transformations (for example, extending/cutting), higher-order generalization rules would involve substituting variables—or classes of critical transformations (for example, adding to/taking away from the substance of an object). The result of such substitution would be a more general higher-order t rule. *

Substitution of classes of transformations, of course, presupposes that they exist as cognitive elements. The acquisition of such classes amounts essentially to identifying their common features through a process that is essentially one of concept formation.

Since concept formation has been extensively discussed in the literature, there is no need to consider the process here. We note only that the classes of transformations involved (for example, adding to/taking away from the substance of an object) may be defined in terms of that which is common to adding/taking elements from sets (number), extending/cutting (length), adding/taking away substance (amount), pouring in more/taking out liquid (liquid), and adding/taking away surface (area). In effect, the common denominator in these cases would appear to involve adding to or taking away from an intrinsic property of object B. Notice that the concept

*The indicated higher-order analogy and generalization rules are quite simple and appear to be rather basic. Presumably, their ontogeny involves generalization of more (domain) restricted analogy and generalization rules of the sort known to be available to young children. The latter type of rules, for example, is manifest when playing "cowboys and Indians" and when letting match boxes represent trucks and lines on a carpet represent roads.
height would not qualify in this sense because transformations other than adding/taking away, such as rotation, may affect height. Thus, whereas rotation, for example, may affect height, it does not affect traditional conservation concepts.

To summarize, whereas idealized conservation behavior may be attributed to a single complex (highly differentiated), general, and automated rule (see Figure 3), acquisition comes about gradually through interaction among more basic rules. Among others, this more basic set of rules includes higher-order t rules that are less differentiated, less general, and possibly less automated (for example, see Figure 4) and higher-order analogy and/or generalization rules. In addition, some provision would have to be included to allow these complexes of higher-order t and analogy rules to become automated.

Process of Automatization

Presumably, automatization comes about gradually through practice on (detailed) higher-order t rules (and analogy/generalization rules) that attend to a broad variety of components associated with conservation tasks. These more specialized, and/or less efficient, higher-order t rules could play an important role in responding to certain variations on conservation problems (for example, surreptitious removal). Hence, their availability, along with their more automated variants, could be crucial.

Although we shall not attempt here a detailed analysis of the process by which automatization takes place, in principle, this process also might be explained in terms of higher-order rules. Specifically, automatization, in this case, can be thought to result from application of higher-order automatization rules to nonautomated rules (for example, higher-order t rules) in situations where the latter are inadequate (for example, where they do not yield desired responses quickly enough). Higher-order automatization rules, for instance, may have the effect of eliminating redundant steps in a rule or of otherwise making more efficient some portion(s) of a given rule. In the present case, the results of such application, presumably, would be more efficient versions of the higher-order t rules. (Higher-order automatization rules were referred to as higher-order "elimination" rules in Scandura, 1973. Further discussion of such rules is provided in Scandura, 1978a.)

In this initial work, no attempt was made to identify specific, prototypic automatization rules. Sole reliance was placed on the less specific, but more generalized, "rule of thumb," to the effect that automatization typically takes place gradually as a result of
practice (on tasks that require use of the rule in question). The process may be enhanced when practice takes place under implicitly speeded conditions (so that the learner will actively look for short-cuts).

Initial Higher-Order Transformation Rules

Even the restricted preautomatized higher-order t rules are relatively complex. They involve anticipating the effects of given transformations (for example, adding or spreading out) on given concepts (for example, number). Consequently, such rules are referred to as anticipatory higher-order t rules. The question, then, arises as to where the nonautomated anticipatory higher-order t rules come from in the first place; that is, How do children learn to coordinate particular transformations with particular concepts?

In the present form of analysis, the answer to this question involves specification of still more basic rules from which the anticipatory higher-order t rules might be derived. (As in other forms of task analysis [Gagne, 1962], analysis continues until the basic rules identified are sufficiently simple that they can be assumed to be uniformly available to children in the target population on an all-or-none basis—that is, when the rules are so simple that it is impossible to teach part of one without teaching it all. What sets structural analysis apart from other forms of task analysis is the specific attention to higher-order rules, precision, and more general theoretical considerations [Scandura, 1977b].)

Clearly, before a child can anticipate transformational effects, he must know which types of transformations affect which types of concepts. (Strictly speaking, the critical effects involve concept comparisons.) In order to provide an unambiguous basis for instruction, of course, it is necessary to specify a rule (or rules) that deals explicitly with relationships between CC rules and observed transformations t. As a minimum, such a rule should allow a knower to determine whether or not a given transformation did in fact affect a given concept comparison.

In principle, there might be any number of such rules, but perhaps the most obvious one is based on simple consistency. Does the posttransformation comparison (through the CC rule) give the same result as the pretransformation comparison? Thus, whether transformations are assumed to affect or to not affect the concept comparison depends solely on whether or not the two comparisons give the same result.

A consistency higher-order t rule of this type is represented in Figure 6. In this rule, the inputs consist of conservation
FIGURE 6. Form of higher-order t rule for arbitrary conservation problems. Unlike automated higher-order t rules, the rule does not apply to anticipatory conservation problems but only to conservation problems in which the final A-B' state is visible and where the child has already determined the correct relationship between Ac and B'c (that is, through a CC rule).
situations (including transformations $t$), corresponding CC rules, and their application in making the $A_c-B'_c$ comparison. (Strictly speaking, application of CC rules is part of the input, as required by restrictions on rule procedures [Scandura, 1978a], although for ease of interpretation, it is designated in Figure 6 as an operation [that is, rectangle].) The critical step in the rule involves comparing the pre- and posttransformation comparisons for consistency. The only assumption necessary to guarantee a correct result is that one know an appropriate CC rule. If the two comparisons give different results, then the transformation is identified as one that affects the property. Otherwise, it is not so identified.

This rule, clearly, is an extremely basic and simple one. Given an appropriate CC rule (which we have assumed as being prerequisite to the analysis of Piagetian conservation), the essential operation is that of evaluating two comparisons in terms of equivalence. It is hard to imagine a more basic ability (with respect, say, to five-year-olds) other than perhaps the ability to compare two simple properties (concepts). Notice, in particular, that making the latter types of comparisons, involving such simple properties as length, liquid, and so forth, is precisely what CC rules are supposed to do. Hence, if one were to push the analysis any further than we have done, the only gap remaining to be filled would be that of specifying the higher-order rules by which CC rules are generalized to allow comparison of comparisons as well as comparison of simple properties.

The desired anticipatory higher-order $t$ rules do not follow directly from the consistency higher-order rules, of course. In addition, a complete characterization of the requisite competence would include a (higher-order) process by which the former may be derived from the latter. As was the case in passing from the simpler, less general anticipatory higher-order $t$ rules to more idealized forms, the process involved is largely one of increasing automaticity and will not be detailed here. For present purposes, it is sufficient to note that the desired derivation involves the transition from identifying particular transformations as affecting or not affecting a concept comparison to forming a more efficient, explicit rule utilizing that information to anticipate effects of that transformation. The nonautomated, concept-specific anticipatory higher-order $t$ rules of the previous sections serve the latter purpose.

Additional Analysis

It is not always the case, as assumed above, that a child can systematically analyze the transformation $t$ as it happens. The
experimenter may perform a transformation so rapidly, for example, that the child cannot tell for sure whether or not the critical property is affected even if he otherwise has the means to do so. Moreover, the changes might actually be shielded from the child, either directly, by means of a screen, say, or surreptitiously.

In a situation of this type involving number conservation, for example, a child could obviously resort to one-to-one matching of the elements in A and B' (that is, use a number CC rule). For instance, given A and B', the child might determine whether A has more or less than B by transforming B' into a form that parallels A as closely as possible (doing so in a way that does not change the number). For example, given the A-B' problem,

```
OOOOOOO
XX
XX
XX
```

the child might line the X's up one by one under the O's starting on the left. This would give the transformed array

```
OOOOOOO
XX XX XX
```

The final step, then, might be simply to inspect the transformed array visually.

Although this solution rule might appear to involve transformations, notice that the hypothesized solution rule is qualitatively different from the mode of solution discussed previously, when transformation t was directly observable. In the latter case, the solution rule involved an anticipatory higher-order transformation rule that operated on t. Solution of the above problem (where t is not observable) was based primarily on a type of inverse transformation (one not involving adding/taking away). This inverse transformation amounts essentially to a type of CC rule.

Again, however, as assumed in Piagetian formulations, the child at the concrete operational level is not restricted to such rules. Among other things, such a rule might be relatively inefficient to apply where the number of elements is large. More important theoretically, as most developmental psychologists now agree (Kuhn, 1974), knowing an inverse transformation and Piagetian reversibility are two different things.
Specifically, one reasonably can argue that the latter corresponds essentially to a higher-order rule for generating inverse transformations (that is, CC rules). Thus, inverse transformations might be derived (in situations where they are not already available) by applying a higher-order t rule or number analogous to that described in Figure 6. In this case, however, the higher-order t rule would apply to A-B pairs before and after (A-B') some unobserved transformation t and generate the inverse transformation needed to make the A-B' comparison (for example, by direct observation). More generally, one might expect the child truly at the concrete operational level to be capable of generating any number of (equivalent) inverse transformations, as demanded by the particular problem.

Verbal justification, of course, also would be somewhat different. Thus, confronted with a "why" problem, a child might be expected to respond, "Because I can put them back," implicitly conveying the idea that he can do this without changing the critical property (for example, without adding or subtracting elements).

Summary of the Structural Analysis

Let us briefly summarize the essentials of our analysis. First, we began by more sharply delineating the domain of conservation problems and by defining such problems more precisely. Then, we distinguished between two alternative ways of generating correct responses to "more/less/same" problems: CC rules and rules based on initial states and transformations. The essentials of CC rules were described, and a promise was made to specify these precisely in the empirical study that follows.

The CC rules provided a base level for the remainder of the analysis, which began by specifying a perfectly differentiated and automated anticipatory higher-order t rule, corresponding to an idealized conserver in the Piagetian sense. Continued structural analysis yielded a simpler, seemingly more basic set of rules from which the idealized rule might gradually evolve through rule-based learning. This set of rules included simpler, less differentiated, and special purpose anticipatory higher-order t rules, higher-order analogy and generalization rules relating to the former, and an informally specified process (corresponding to a higher-order rule) by which the above complexes of rules might become both integrated and automated. Although the higher-order analogy and generalization rules appeared to be both extremely simple and basic, it was still not entirely clear where the anticipatory higher-order t rules might come from. Subsequent analysis showed that the ontogeny of these rules might be traced to a general, very basic
consistency higher-order t rule together with complementary automatization processes (rules). Finally, it was noted that the consistency higher-order t rule was a natural extension of the basic CC rules upon which the entire analysis was based—thereby completing the structural analysis.

ACQUISITION OF CONSERVATION COMPETENCE

The preceding structural analysis tells us what kinds of rules need to be acquired by the conserving child at various stages of development. The analysis is also suggestive of how the acquisition of anticipatory higher-order t rules might tie in with the normal sequence of development (that is, assuming availability of appropriate CC rules). For example, we have seen that successful application of anticipatory higher-order t rules implicitly assumes that the child already knows which kinds of transformations (for example, adding/taking away, extending/cutting, and so forth) affect given properties (that is, number, length, and so forth).

Let us now look at the problem from the opposite direction. Assuming that we know the source of such development, the basic ontogenetic question is, How does the learning underlying this development come about?

The basic assumption on which the above analysis is based is that such learning is the result of interaction among available higher-order and lower-order rules in concrete, specific problem situations, all under the control of a hypothesized, universal, goal-switching control mechanism (along with certain other assumed and tested basic constraints on rule use). For present purposes, it is sufficient to simply note that the control mechanism operates essentially in the following manner. Given a problem for which the learner does not already know a solution rule, this mechanism has the effect of directing attention toward the derivation of such a rule. Where a needed rule is available, the control mechanism mandates that it be used.*

*As simple as this mechanism appears to be, its formalization, empirical testing, and generalization (so that it applies to arbitrary knowledge) has been anything but a trivial matter and involves a number of fundamental assumptions, including assumptions concerning the methods one uses to study human cognitive processes. For detailed discussion and empirical tests, the interested reader is referred to Scandura (1973, 1974b, 1977b).
The main problem addressed here is that of specifying the kinds of concrete, specific problem situations that might be expected to lead a child who knows the requisite CC rules to acquire generalized conservation competence. In this regard, the basic question would appear to be how the child might come to know which kinds of transformations affect which kinds of properties (actually, comparisons or relations between properties).

This ability (representable as a type of higher-order t rule) might come about in the following manner. The child is confronted with one A-B pair and then another. (In a training situation, the child would be asked in each case which has more of the critical concept.) The latter pair is obtained from the former by some observable transformation. This process may be repeated any number of times—with what in effect amounts to a series of conservation situations.

\[
\begin{align*}
A_c & \rightarrow t \rightarrow A_c \\
B_c & \rightarrow t(B_c) = B'_c
\end{align*}
\]

First, the child might be asked to compare the before-and-after comparisons and to draw natural inferences concerning possible effects of the transformations involved. Eventually, in this way if the initial transformations are observable and sufficiently simple, the child may learn (by use of consistency higher-order t rules) which transformations affect which critical concepts (that is, affect the outputs of the relevant CC rules) and which do not.

Under normal developmental (everyday) circumstances, rules for anticipating which transformations affect which comparisons also may be expected to be learned gradually. Thus, for example, recent research (Cooper et al., 1978) suggests that children first learn to deal with simple conservation situations, such as

\[
\begin{align*}
XXX & \rightarrow t \rightarrow XXX \\
OOO & \rightarrow ooo
\end{align*}
\]

where the A-B and A-B' comparisons are easy to make (for example, with small sets) and, hence, where absence of a critical t-effect is easily recognizable. (Clearly, t does not change the results of one-to-one matching on the A-B pair before t and on the A-B' pair afterwards.) Assuming that a variety of increasingly complex transformations are presented, the domain of situations, where the child can anticipate which transformations change which critical properties, might be expected to gradually increase in scope.
The rate of learning how to anticipate effects in this way, of course, may be affected by the conditions under which the conservation/comparison situations are presented. Thus, for example, learning might be facilitated when the child is asked explicitly to anticipate the effects of given transformations (that is, when the goal of the conservation problem is specified), and/or the final results of the transformations are shielded from view during the transformation and until after the child guesses what the effect might be.

Attending to the transformations also may be encouraged by asking the child to respond as quickly as possible—thereby encouraging him to avoid using a corresponding CC rule. In addition, the essential features of the transformations might be made clear, thereby helping to fixate the essentials.

Once an anticipatory higher-order t rule has been learned, further practice under speeded conditions may be expected to lead gradually to automation (that is, to the derivation of a more efficient solution rule).

To summarize the above, the effective nature of the task is that of presenting the results of one comparison (that is, an A-B pair that has been explicitly compared on some concept by the child). Then, a transformation is performed (either by the experimenter or by the child), and the child is asked a question that requires him to anticipate the effects on the A-t(B) comparison. In this case, since the child is assumed to know the relevant CC rules, he can check his answers independently (of anticipating transformational effects); external feedback is not necessary. (This, we think, is the essential source of the "incongruities" Genevans talk about.) In effect, assuming the availability of needed CC rules, the tasks of correlating various types of transformations with before (A-B) and after (A-B') comparisons amount to classic cases of relational concept attainment, first with the before-and-after comparisons playing the critical role (in the consistency tasks) and then with the before comparisons and the transformations playing this role (in the anticipatory tasks).

For present purposes, the exact processes that children use toward this end—for example, scanning or focusing (Bruner, Goodnow, and Austin, 1956)—are not important. In all situations, the child, by assumption, enters the task situation knowing the corresponding CC rule. When attention is focused (by the child and/or the experimenter) exclusively on the respective A-B states, the CC rule may be applied repeatedly in making A-B comparisons. The desired learning under these conditions may be expected to come about very slowly, if at all.
Alternatively, after making a postcomparison, the child might be asked to indicate whether or not the transformation affected the property in question (by using some form of the consistency higher-order \( t \) rule shown in Figure 6). Once having determined (learned) how some transformations are correlated with some concepts, which is equivalent to deriving (acquiring) a nonautomated anticipatory higher-order \( t \) rule of the type discussed above, the child might be asked to anticipate the results of given transformations and, only then, to check his answers (through the CC rule). This may be done either passively, by asking the child to predict the effects of given transformations, or actively, by asking the child to transform the objects (for example, \( B \)) so that \( A \) and \( t(B) \) bear a desired relationship to one another. In either case, the nature of the task is different. The child must base his responses on initial \( A-B \) comparisons and/or transformations. Finally, where his predictions and postcomparisons give different results, the child might be asked to reconcile the two.

In this way, eventually, the child may gradually learn to correctly anticipate the effects of various types of transformations. Presumably, such learning takes place in stages, as the child gains more familiarity with the tasks.

More specifically, according to our analysis, asking a child to draw inferences about the effects of observed transformations, given the results of pre- and posttransformation comparisons, may lead the child not only to learn the consistency higher-order \( t \) rule but, by using this rule, to also learn (derive) desired relationships between transformations and concept comparisons. These relationships constitute major components of special purpose, nonautomated anticipatory higher-order \( t \) rules.

Presenting anticipatory conservation problems, followed by "forced" feedback in the form of reconciliation of predictions and postcomparisons, may lead the child to transform his relatively inefficient (and thereby unreliable) anticipatory higher-order \( t \) rules into more useful forms. Thus, rather than simply knowing whether or not a given transformation did or did not affect a particular property (based on recalling previous pre- and postcomparisons), the new anticipatory higher-order \( t \) rules would allow increasingly efficient and reliable predictions concerning posttransformation comparisons (based solely on initial states and transformations).

An efficient anticipatory higher-order \( t \) rule of the latter type in the case of number was shown in Figure 4. In that case, recall, the critical step involved deciding whether or not the transformation involved adding/taking away objects from set \( B \). Before automatization, presumably, this step would not be carried out so directly but might involve systematically checking the observed transformation...
(or components thereof) against each kind of transformation with which the child is familiar. In each case, the child would have to first decide whether the observed transformation was of that type and then whether that type affects number.

In view of the indeterminately large number of possible conservation concepts, of course, it would be very inefficient indeed if each child had to learn each concept-specific anticipatory higher-order rule independently. Fortunately, this may not be necessary in practice. Once one such rule has been learned, new ones might be learned more quickly. As with Harlow's (1949) learning sets, as the child learns more and more anticipatory higher-order rules, he should become better and better at correlating critical transformations with CC rules and the properties they evaluate. Ultimately, this may be accomplished with a minimum number of instances, perhaps on the first trial. As indicated above, we feel that rather simple and basic higher-order analogy/generalization rules could play a crucial role in effecting such transfer.

There is, however, no a priori reason to expect all anticipatory higher-order rules to be equally easy to learn. For one thing, the relationships between properties, CC rules, and transformations that may affect properties are closer for some conservation concepts than for others. The analogy between amount and area, for example, is almost certainly closer than that, say, between amount and height. To the extent that this assumption is valid, then, transfer will necessarily be dependent on concept/property. Horizontal decalage, in effect, may be a necessary consequence of the way the world is conceptualized.

In the present analysis, recall, it was not necessary or feasible to identify explicitly all of the analogy/generalization rules that might be involved in going from one anticipatory higher-order rule to another, even though this might be possible in principle. For purposes of the experiment that is to be proposed, it is sufficient here to distinguish between anticipatory higher-order rules associated with conservation concepts (for example, number or length) that are "intrinsic properties" of the A-B entities and those rules associated with conservation concepts (for example, height) that are not. In this regard, the higher-order rule shown in Figure 7 correctly anticipates the effects of given transformations whenever an intrinsic property is involved. This rule is not guaranteed to anticipate A-B' comparisons when the critical concepts are not "intrinsic properties," and the domain appropriately reflects this. Without this restriction, the rule could be applied to any conservation problem (for example, one involving height), albeit perhaps giving incorrect A-B' comparisons. (For examples of this kind, see the posttest items involving height and torque in the main empirical study described below.)
As it stands, this anticipatory higher-order t rule may or may not actually be used with "more/less/same" conservation problems when the terminal (A-B') state as well as the transformation t are in view of the child. This is particularly true when A and B' appear different (that is, when there is a perceptual illusion). In such situations, children at the indicated stage might use either the higher-order t rule or a CC rule to give the correct "more/less/same" response. Giving an appropriate reason, on the other hand, would require reference to the former higher-order t rule.

In future research, it might be of some interest to categorize individual conservation concepts according to their similarity as a basis for making finer distinctions among higher-order t rules. (In addition to its purely theoretical value, presumably, such a taxonomy might be useful in planning more efficient instruction for preschoolers.) In any case, given the present level of analysis, the point to emphasize is that instruction in generalized higher-order t rules, such as that shown in Figure 7, might best take place informally rather than by verbal exposition. In fact, it is unlikely
that young children can be taught such rules reliably by purely expository methods. A more effective way to proceed, after the child has learned one or more specialized higher-order t rules, might be to provide hints about relationships—what to substitute for what, and so forth—all in the context of solving particular conservation problems.

According to our analysis, generalization is likely to be maximized by calling children's attention to analogous features when switching from one type of conservation problem (that is, concept) to the next. After learning an anticipatory higher-order t rule for amount conservation, for example, a child might be confronted, say, not only with anticipatory area conservation problems but also might be asked or shown what corresponds to what (for example, adding globs of clay to adjoining surfaces).

An Operational Explanation of Horizontal Decalage

What is more interesting, perhaps, is that children frequently learn to conserve certain concepts before others—that is, the children are said to exhibit "horizontal decalage." In the present view, such differences are the result of two things. First, children may know CC rules for some concepts but not others. As we have seen, true conservation is not possible in the absence of such rules. Second, the anticipatory higher-order t rules that are available to individual children may be applicable to some conservation problems but not to others. Specifically, the availability of such rules may be crucial to conservation in the Genevan sense.

Eventually, by means of clinical observation and more detailed structural analysis, one might reasonably hope to "tease out" and to specify the criteria used for this purpose by children who exhibit various patterns of "decalage." Toward this end, the present study begins with the somewhat cruder and more easily made distinction between "intrinsic" property and "critical" property. (Even here, the distinction is fairly subtle. Given the subtleties of most concepts and natural languages in the real world, it is hardly surprising that detailed specification of children's competence has not been a simple task. It is to be hoped, however, that better understanding of the basic principles and requirements involved may lead to faster progress in this complex area.)

In particular, "intrinsic property" seems relevant to most traditional Piagetian conservation tasks (for example, those involving number, length, liquid, amount [of clay], and area). In these cases, the only transformations that affect the A-B' concept comparisons are those that change the amount of B (for example, by
adding/taking away elements, adjoining/taking away linear substance, pouring in/out, adding/taking away substance, or adjoining/taking away surface). Transformations, such as rigid movements in the plane or space, rotations, or changes in density, shape, and homogeneity (where applicable), do not affect amount of number, length, and so forth. (Incidentally, it should be emphasized that the former and the latter types of transformations may act independently of one another [see Figure 5]; that is, it is possible to perform one type with or without concurrently performing the other.)

"Critical property," on the other hand, connotes a broader class of conservationlike tasks. In particular, whereas concepts associated with traditional conservation tasks are not affected by rotations, for example, height may be so affected. Thus, turning an object from the vertical reduces its height (whereas it does not, for example, affect its length).

It also is possible to view a variant of the balance beam in this way. Specifically, a child might (through trial and error or otherwise) put a beam in balance by adding various weights at various distances. Then, the balance might be subjected to a visible transformation t, with the final results (only) hidden from view. Finally, the child might be asked to predict which side went down (that is, to predict the relationship between A and B' after t). Under these circumstances, if the transformation involves exchanging a given weight on side B with a larger one, the above anticipatory higher-order t rule might lead a child to predict that side B would go down, irrespective of whether or not the distance from the fulcrum also is decreased by a compensating amount. An anticipatory higher-order t rule of this sort would have an effect not unlike the "weight" rule described by Klahr (1978) and Siegler (1978)—although the former would have a more general character and would operate on transformations rather than on static quantities. The latter, in fact, amounts essentially to a CC rule (with a restricted domain) for the balance beam. The knowledge had by a child who could take both weight and distance transformations into account would be better characterized by an anticipatory higher-order t rule that explicitly attends to the compound property, torque (turn). (Torque may be defined as the product of mass [weight] and distance.)

The above observations could play a useful role in explaining behavior patterns of the type associated with horizontal decalage and were utilized in designing the study described below.

Solving "Why" Problems

In view of the preceding analysis, what role do "why" problems play in helping to insure conservation behavior? As before, let us
limit the discussion to conservation situations where the child is able to adequately observe the transformation and thereby apply an anticipatory higher-order t rule. In this case, it is sufficient to simply assume that the child can explain why the higher-order t rule works. In the case of number, for example, as long as the child is able to convey the idea that some component of t adds, takes away, or does not change the number of elements in B, it is not important what description rule the child uses.* Since most conservers may be assumed to have this capability (that of describing their knowledge of conservation), one might reasonably assume that the rule is atomic and just identify its domain (for example, anticipatory higher-order t rules involving number) and range (that is, explanations of how the transformation t affects the number comparison between A and t(B)—for example, "You did not add anything").

Notice in this regard, with the major exception of torque,† that the main difference between description rules for giving identity reasons (for example, nothing added or nothing taken away) and rules for giving compensation reasons is largely one of completeness. Specifically, the domains of rules that generate compensation reasons are somewhat more encompassing. Whereas the domains of both kinds of rules involve higher-order t rules, compensation rules also give explicit attention to compensating relationships between irrelevant features of A-B' pairs (for example, "This [B'] looks longer but these [A objects] are more spread out").‡ Implicit in such reasons is the prior determination that transformation t does not change the critical property. Compensation reasons in this view are simply a way of further explicating why A and B' are the same (with respect to the critical property), even though they may appear different perceptually. It is not surprising, therefore, that compensation arguments are frequently elicited in response to questions

---*Our off-hand treatment of conservers' description abilities is not meant to imply disrespect for the large body of research involving so-called metacognition, or awareness, of one's cognitive processes. It is our impression, however, that such research might benefit from more explicit attention to identifying the description rules required in specific instances of metacognition.

†In the case of torque, and certain other compound properties, easily quantified changes in one defining property (that is, weight or distance) may be compensated for by similar changes in other defining properties.

‡Compare this type of compensation with that suggested in the note above, which refers primarily to compensating transformations.
that focus the conserving child's attention on the A–B' pair (as opposed or in addition to itself).

In effect, although giving compensation reasons probably involves additional competence (over and above that required in giving identity reasons), such reasons are not especially crucial in the present analysis. (Clearly, however, one could introduce tasks that require explicit verbal justification for apparent visual incongruities, for example, in conservation of number, area, and so forth. Similarly, it would be a relatively simple matter to teach children who otherwise conserve to give compensation reasons. A few illustrations probably would be sufficient to ensure this with those conservation tasks where compensation arguments are most commonly given or expected, for example, those involving area, volume, liquid quantity, and so forth.)
EMPIRICAL TEST OF THE ANALYSIS

As noted previously, Piagetian constructs, such as reversibility, are suggestive but not sufficiently definitive for instructional purposes. On the other hand, what has been taught and presumably learned in many successful Piagetian training studies is not consistent with the structures that Genevans have found to underlie normal development.

Within the limits specified (that is, the prior availability of concept comparison rules and restriction to the conservation domain), the above structural analysis appears to capture the more important essentials of Piagetian conservation. Still, it seems to be considerably more precise and operational. Individual problems, including their goals and givens, were specified unambiguously in a way that makes it possible to determine the generative sufficiency of specific kinds of solution rules. A major distinction was made, for example, between CC problems and conservation problems, on the one hand, and CC rules and higher-order transformation rules for solving such problems, on the other. In the case of conservation, a further distinction was made between anticipatory conservation problems (and anticipatory higher-order t rules) and typical conservation problems (and higher-order t rules) in which the results of transformation t are apparent.

Although a serious attempt was made to identify rules that are compatible with Piagetian structures, there can be no guarantee, short of empirical testing, that any particular set of rules will correspond to the natural course of development. Thus, for example, whereas higher-order t rules may be adequate to solve conservation problems in a generative sense, it is not at all clear that children actually learn any of the particular higher-order t rules specified above. Nor is it clear that children necessarily learn general higher-order t rules by generalization of more specific ones (for example, the order of learning in the natural environment
might be reversed). There are two main points that must be emphasized. First, the stages of learning identified through the structural analysis appear to parallel those associated with the passage from preoperations to concrete operations (as defined in Piagetian theory); that is, if not identical (due to restriction to a subset of conservation problems), the rules identified and their postulated ordering appear isomorphic to the natural course of development (as determined by Genevans). Second, a training study based on this analysis, if successful, would demonstrate its sufficiency (if not its necessity) as a basis for achieving such transition.

Accordingly, the major purpose of the following two-phase study was to determine the viability of the preceding structural analysis. Is the analysis experimentally viable—that is, is it sufficiently compatible with the natural course of children's cognitive development to be implemented experimentally? More specifically, does the analysis provide the intended clarification of differential transfer (that is, horizontal decalage) by demonstrating the possibility of manipulating such transfer? As a minimum, even if inconclusive, it was hoped that an attempt at implementing the preceding analysis could provide valuable insights that could lead to more definitive analyses in future research.

Concurrent with the structural analysis described in Chapter 3, empirical work progressed in three phases. As described in Chapter 2, phase one was strictly exploratory and was designed solely to provide a source of insights and informal verification for the preceding structural analysis of the conservation domain. During this period, the critical importance of CC rules to conservation behavior became increasingly clear. In particular, these rules were found to play a dual role when applied to conservation-related problems. They make it possible to generate correct responses to "more/less/same" conservation problems (by keying on static states rather than on transformations). They also provide an independent basis for evaluating tentative solutions to anticipatory "more/less/same" conservation problems. (Generating such solutions, at least viable ones, requires explicit attention to the transformations involved.) Moreover, the structural analysis had progressed to the point where it was possible to specify particular CC rules with some degree of precision.

Phase two of the empirical work, then, was designed to evaluate and refine these rules as a prelude to more formal research. The results of this work are described and discussed below in the section on training in CC rules.

Phase three of the research was based directly on the structural analysis (outlined in Chapter 3) and was designed to determine the efficacy of the identified rules as a basis for both acquiring Piagetian conservation and manipulating horizontal decalage.
PHASE TWO TRAINING

Subjects and Background

The subjects were 26 kindergarten children at the Walnut Center School in West Philadelphia, who were between the ages of 5.11 and 6.39. All of these children had participated to some degree in the previously described pretesting and/or exploratory training. In particular, all of the Ss were pretested on number and length conservation according to the procedures described in connection with Figure 2. They also were tested later to determine their understanding of "more" and "less" and the interrelationships of these concepts.

In the interim, half (12) of these subjects, the "experimental" Ss of Chapter 2, were subjected to exploratory training. Due to the informal and incomplete nature of this training, it was impossible to draw definitive conclusions from the results. There were, however, no clear differences in ability to use the words "more," "less," or "same" that could be attributed to this training.

It did not take long, however, to recognize the critical importance of CC rules in determining the entering levels of the less developmentally advanced children. These levels provided a baseline of sorts upon which more directed conservation instruction might build. Similarly, in the previously described structural analysis, these rules corresponded to minimal entering capabilities of the assumed target population (Scandura, 1977b, p. 463).

Training on Concept Comparison Rules

Phase two of the present research consisted of training on CC rules identified and was designed to evaluate and refine these rules as a basis for subsequent research. Specifically, training was provided with respect to height, area (to color), liquid (amount of Kool-Aid to drink), and amount (of clay to play with). In each case, S was taught an explicit rule for making the required comparisons. Whenever possible, these rules required S to perform a physical action of some sort. This helped to both involve the young children in the task and make it possible to monitor the children's learning.

In the case of height, for example, S was required to place his hands at the base from which the tops of the objects were to be measured (initially all of the objects rested on a common base) and to move them up slowly (vertically) at the same speed until one hand came to the top of one of the two objects being compared. The S was instructed to pause at this point before continuing. The hand
that reached the top remained where it was, and the other hand continued upward until it reached the top of the other object. The S then was instructed to respond that the object corresponding to the hand that stopped last had "more height," whereas the other had "less height."

In each case, S was taught first how to identify which object had "more (concept)" and then which had "less (concept)." The specialized "more" and "less" rules effectively amounted to partial forms of the more general, ultimately desired CC rules that involved "more," "less," and "same." Next, S was trained to identify both (that is, which had "less [concept]" and which had "more [concept]"). As one might expect, once having learned two specialized forms of a CC rule, children learned the generalization almost automatically.

When S recognized that "about the same" was an allowable answer, automatic transfer also was the norm in generalizing the more/less CC rules so that they applied when two objects had "about the same (concept)." In all but two cases, once the combined more/less CC rules had been mastered, S responded correctly on the first trial to comparison tasks, when the A-B objects had (nearly) identical values of the crucial concept/property.

It is important to emphasize, however, that no attempt was made to teach CC rules of maximum possible generality nor to insure that the CC rules were entirely consistent or compatible with the actual knowledge children had at the onset of the training. For our purposes, it was sufficient to identify precisely the domains of applicability (that is, the classes of A-B objects to be compared with respect to given properties). As a result of such specification, it was possible to predict, solely on the basis of the training provided, which kinds of comparisons S could and could not make. This information was used in selecting appropriate posttest items—posttest items that were used to determine the extent to which S's behavior was consistent with what one might expect. (Restricting the scope of the trained CC rules, of course, says nothing definitive about how S might perform on comparison tasks outside of the domain. According to the Structural Learning Theory, such transfer would be explained in terms of the prior availability of, and/or training on, appropriate higher-order rules, for example, higher-order generalization rules.)

Precise descriptions of the four CC rules explicitly taught are given in Figures 8A, 8B, 8C, and 8D. The domains of each rule are specified in the oval designated "START." Specifically, notice that limits are imposed on the allowed ratios of measures on critical properties.
PROBLEM: Given: 2 objects with height at least 1", measured from common base (more/less ratio no closer than 9:10), (objects may be vertical or slanted*). Goal: Find object with more/less/same height.

From top of each object move hands down perpendicular to base. From base move both hands at the same speed straight up vertical axis until top of one object is reached. STOP.

**Top of other object reached also?**

- **Yes:** Say, "Both objects have about the same height." STOP
- **No:** While keeping hand on top, move other hand to top of second object.

The object whose top reached last (first) has more (less) height. Say, "This object has more (less) height and the other object has less (more) height."

STOP

**FIGURE 8A.** Schematic representation of height CC rule introduced during experimental group training. This rule includes the generalization introduced into the CC rule training during phase three.
The rules were taught to individual Ss in lessons lasting from five to 20 minutes (and rarely longer than ten minutes). Rules were introduced in the following order: more/less height, more/less area (to color), more/less Kool-Aid (to drink), more/less play dough (to make things with), same height, same area (to color), same Kool-Aid (to drink), and same play dough (to make things with). A record was kept of each child's progress. When S made more than one or two errors during the training, the lesson was repeated on another day to insure that S had learned the rule.

Throughout the instruction, E exhibited a warm, friendly attitude toward the children. (For example, S was asked to participate only when not actively involved in a classroom activity, such greetings as "Hi, how are you today?" were used, E held S's hand when...
AREA CC RULE

START

PROBLEM Given: 2 pieces construction paper of regular shape: circles, rectangles, triangles (more/less ratio no closer than 3:5).
Goal: Find object with more/less/same area.

1. Move finger back and forth from top to bottom of each object (paper) one at a time.
2. Put one paper on top of other. Check. Then put other paper on top of first. Check.

1. Take about same amount of time?
2. Evenly matched?

Say, "Both objects have about the same area."

Paper that:
1. Took longer (less time) to complete finger tracing or
2. Cannot (can) be completely covered by other has more (less) area. Say, "This object has more (less) area and the other has less (more) area."

STOP

FIGURE 8C. Schematic representation of area CC rule introduced during experimental training.
walking to the experimental room, and S was allowed to sit on E's lap when working on the floor.) In addition, positive verbal reinforcement was provided routinely (for example, "You really are a good thinker today" and "I am really pleased by the way you listened so carefully").

**FIGURE 8D.** Schematic representation of amount CC rule introduced during experimental group training. This rule includes the generalization introduced into the CC rule training during phase three.

Due to the age and low verbal skills of many Ss, kinesthetic as well as verbal experiences were provided in teaching the rules. For example, E would say: "Hold this play dough in your hand. See how much of it your fingers can cover. Now, hold this play dough in your hand, and see how much of it your fingers can cover.
We are talking about amount of play dough. The amount is how much play dough you could make things with if you took this [play dough] and made it into things like a plate or an animal."

In addition to making it easier to monitor the child's behavior, having S perform physical acts in comparing objects helped to ensure focus on the property (concept) in question. (For example, S could play with the play dough, touch the top of the blocks, feel the amount of area with his fingers, and drink the Kool-Aid.)

Where feasible, E's normal usage of language was modified to accommodate S's vocabulary range (for example, "floor" or "table" instead of "baseline" and "thing" instead of "object"). With the exception of "more/less/same," as described previously, comments, questions, and answers were sometimes paraphrased to help ensure that S was not just learning or repeating memorized statements.

The general procedure used in teaching the rules involved presenting pairs of A-B objects in a static state, side by side on a table.* Initially, the objects were similar in all respects (for example, their shape and color), but the critical property (for example, amount) and the critical difference were relatively large in magnitude. For example, in teaching the initial (restricted) height CC rule, S was presented first with two wooden blocks both having the same color, width (approximately three inches), and thickness (approximately one inch). One block, however, was about three inches high and the other, 15 inches high. Similarly, in teaching the restricted liquid CC rule, S was presented with two identical transparent cups, with a figure stamped on the side of each. Initially, one cup had about two ounces of grape Kool-Aid and the other, about eight ounces of grape Kool-Aid.

When dealing with the concept of restricted height, for example, S was instructed as follows: "The two blocks are both on the floor. Put your hands near the bottom of the blocks [E's hands

*In teaching more/less height (the first CC rule), E initially tried a strategy where S was told, before training, that, if he "paid careful attention and was able to do it," he could be the teacher for the next child. The E hoped that this might provide a good way to double-check S's mastery of the rules. It quickly became clear, however, that being able to answer questions correctly is a very different skill from being able to formulate such questions for instructional purposes or from being able to evaluate others' answers. (In retrospect, this was hardly surprising.) In any case, E replaced this method with that described below.
were put over S's hands, and E placed the hands near the base of each block. Now, slide your hands up evenly until you reach the top of one block. Stop. Now, move the other hand [that did not reach the top] to the top of the other block. [E directed S's hands.] This one [for emphasis, E lightly squeezed the hand on top of the higher block] has more height, and this one [E gave similar tactile cues] has less height."

Subsequent training on all of the rules proceeded in the same general manner. Once S was able to discriminate large differences reliably, the magnitude of the critical difference was reduced gradually. Ultimately, the A-B objects were very similar. In particular, they were more nearly identical than required by the rule domains. Differences in irrelevant properties also were introduced gradually. For example, one block might be 4.5 inches high and four inches wide and the other, five inches high and two inches wide. In addition, to help avoid position effects, objects with more/less of the critical property were randomly placed on the right and on the left. (Incidentally, during the course of training, some Ss also were tested on other common objects. For example, S was shown two chairs and asked, "What can you tell me about the height of these chairs?")

To make sure that S had mastered the CC rules as intended, each child was tested for each rule on at least two A-B pairs that were similar to those used during training and was asked, "What can you tell me about the height [or other concept] of these two blocks?"

In the vast majority of cases, the Ss were successful on such tasks. Indeed, some Ss appeared to master new CC rules on the basis of only one or two examples—for example, Ti (Chinese was his first language), Da, Ty, and An. These Ss often spontaneously drew conclusions, such as "It depends on which one you have with it" or "This [largest] has more than this [middle size] and this [middle size] has more than this [smallest]." Responses of this sort tended to transcend particular concepts, suggesting the presence of rather general rules.

It should be emphasized, however, that the CC rules introduced were applicable only to limited domains of comparison problems. No attempt was made to insure developmental compatibility or even consistency (although we believe that the latter was achieved). Thus, in the initial height comparisons, all objects rested on the same base. (This was extended in the CC rule training during phase three.) In liquid comparisons, the glasses used were of identical shape, size, and so forth. Similarly, the areas and play-dough objects used were of familiar shapes (for example, triangles, rectangles, circles, balls, pancakes, and sausages). Moreover, all
pairs of objects were sufficiently different in terms of the critical properties to allow for easy discrimination (in more/less comparisons).

Even with these simplifications, a number of difficulties were encountered during the training and had to be overcome. Thus, for some concepts, particularly area, and for some children, the CC rules had to be reintroduced several times. Toward this end, a remedial, usually more concrete version of the CC rule in question was devised and introduced to those children who had had difficulty with the rule. (For example, one girl thought that it was necessary to color a piece of paper when talking about the area of the paper, and a boy erroneously tied area solely to rectangles.)

The following misconceptions were common.

Many Ss thought of "more" as being a synonym for "big" (for example, both balls of play dough have more). It was necessary to emphasize that the terms "more," "less," and "same" represent relational properties (A always in relation to B), rather than absolutes. Where there was reason to believe that S gave the correct more/less response (for example, B more) for the wrong reasons, the A object in the comparison was replaced with C. Under these conditions, by properly selecting C (that is, C more than B), such Ss typically became confused. In this case, remedial instruction was provided.

Some Ss found it difficult ever to say that two objects had the same (height, area, liquid, or mass). They felt more comfortable using the phrase "about the same." Part of the problem was the common misconception of equating "same" with the objects being identical. Thus, many Ss felt comfortable using "same" only if the items were identical in size, shape, color, material, and so forth. These Ss, apparently, were unable to focus on one attribute at a time. In this case, E called the Ss' attention, both verbally and kinesthetically, to that part of the A-B objects relevant to the concept in question.

A couple of Ss used object location (right/left) to determine more/less. This relatively minor problem was easily overcome by random placement of objects in the various comparison tasks.

Perhaps more important, even after the training, a few Ss still did not recognize the mutually exclusive and exhaustive nature of "more," "less," and "same." For example, if asked which of two objects had more of some property, such a S would select one (as having more)—in spite of having just said that they were the same. This misconception is roughly related to that involving "same" discussed above. The major difference here is that S must learn that saying "(about the) same" means, for purposes of a given discussion, that any differences that might exist are too small to
matter and are to be ignored in subsequent discussion. It is important to realize that such usage, which adults view as "correct," is more a social convention than a logical necessity. One can always find some difference if one chooses. (In the misconception about "same" discussed above, S must learn that the terms "more," "less," or "same" refer to specific properties, not to pairs of objects as wholes.) Explicit attention was given to more/less/same relationships in the CC rule training in phase three.

PHASE THREE TRAINING

General Design Considerations

By means of a fortuitous series of events and insights, the structural analysis proceeded more rapidly than had been anticipated (a full year had been scheduled). Consequently, shortly before completion of the phase two training on the CC rules, we decided to conduct a more formal experiment, based on the analysis, in the time remaining. Toward this end, we were forced to use the same Ss because they constituted the only suitable population readily available.

Unfortunately, by this time, phase two training on the CC rules was inextricably confounded with the early (phase one) exploratory training. To obtain some measure of the effect of this early intervention, half of the experimental Ss were randomly selected and yoked to control Ss. These control Ss received precisely the same kinds and amounts of training as did the corresponding Ss in the experimental group. Overall, completing the structural analysis (see Chapter 3), redesigning the study, and providing yoked training for half of the control group took about six weeks.

The yoking, of course, was designed to determine whether the phase one training would differentially affect posttest results. While not taking into account possible interactions, the absence of such an effect among the yoked and nonyoked controls would tend to suggest a similar absence had the experimental Ss entered phase three without the exploratory training of phase one. Moreover, ignoring the slight age differential at the time of the exploratory training, the yoked control Ss would provide an appropriate control for the experimental Ss even if the exploratory training did prove to affect the phase three posttesting. In effect, posttest results that might be found to differentiate experimental and control Ss could reasonably be attributed to differences in experimental phase
two and phase three training on the identified (CC and higher-order t) rules rather than to the informal exploratory training.*

It must be emphasized, however, that normative comparisons involving experimental and control groups are of strictly secondary interest insofar as an evaluation of the preceding structural analysis is concerned. Judging success of the present study depends primarily on the extent to which the identified rules and their proposed order of introduction provide a viable basis for instruction and the extent to which the rules actually acquired—in concert with universal assumptions in the Structural Learning Theory pertaining to the use of such rules (Scandura, 1977b)—make it possible to predict the behavior of individual Ss on specific conservation-related problems, including transfer tasks for which they received no training.

Training on Concept Comparison Rules

In view of the elapsed time between initial training on the CC rules and the resumption of experimental training (six weeks after the last lesson), retention of the individual CC rules was checked before proceeding to the third-phase training. Each child in the experimental group was tested individually, during a single session approximately ten to 15 minutes long, on eight pairs of A-B comparison problems. These problems were presented in the following order:

*Other arguments suggesting a lack of generalizable effects of the exploratory training are the lack of any positive effect on the more/less test that could be traced to the exploratory training and the fact that the exploratory training was not that different from what all of the children were being exposed to every day as part of their normal schooling and, in any case, averaged only about 2.5 hours per child.

The only likely exception to the latter observation involved the training that some of the experimental and control Ss received on one-to-one correspondence. To allow for separation of possible effects due to the exploratory training, no further training was provided on one-to-one correspondence. Moreover, the posttest items introduced to deal with number were planned so as to tap individual differences on number CC rules and to allow use of this information as a basis for explaining performance on number conservation problems.
1. Height
   a. two wooden blocks (one 5.5 inches, one 11 inches) *(ans. more/less)*
   b. two wooden blocks (both 11 inches) *(ans. same)*

2. Area to color
   a. one blue right triangle (three by four by five inches),
      one yellow circle (two inches in diameter) *(ans. more/less area to color)*
   b. one blue rectangle, one yellow rectangle (both 2.5 by 3.5 inches) *(ans. same area to color)*

3. Liquid (Kool-Aid to drink)
   a. two identical glasses (one with two ounces of cherry Kool-Aid, the other with four ounces of cherry Kool-Aid) *(ans. more/less Kool-Aid to drink)*
   b. two identical glasses (both with three ounces of cherry Kool-Aid) *(ans. same Kool-Aid to drink)*

4. Amount of play dough
   a. two white play-dough balls (the one, one inch in diameter; the other, two inches in diameter) *(ans. more/less amount)*
   b. two white play-dough balls (both 1.5 inches in diameter) *(ans. same amount)*

On each of the "more/less" problems, S was asked, "What can you tell me about the _____ [for example, height of these blocks]?
" If S did not respond using the terms "more" or "less," E asked, "Which has more _____ [for example, height]?
" If S answered correctly, E asked, "What can you tell me about the _____ [for example, height] of this one [pointing to the one with less _____]?" If the child did not use the word "less," E asked, "Which one has less _____ [for example, height]?"

On each of the "same" problems, E asked, "What can you tell me about the _____ [for example, height of these blocks]?
" If S responded that they were about the same (or the same size), E asked again, "And what about the _____ [for example, height]?
" to be sure that S was referring to the appropriate attribute.

Throughout, a record was kept of the words Ss used spontaneously. In the case of height, for example, only two of the 13 experimental Ss spontaneously gave the answers "more height/less height." The most common terms used were "bigger," "longer," "taller," "higher," "smaller," "little," "shorter," and "small one." If it appeared that S was uncertain about which attribute was being compared, remedial instruction was provided until it was certain that the corresponding CC rule could be applied uniformly on the intended domain. Three Ss needed a remedial lesson on area, one
of those also needed a review of "same" amount of play dough, and one S had trouble using the word "more," although she clearly understood the concepts.

As noted previously, some of the CC rules had overly restrictive domains. To provide Ss with a basis for checking answers to a wider variety of anticipatory conservation problems (that is, problems where the S must anticipate the effects of given transformations), the domains of the CC rules for height and amount (mass) were generalized slightly. (These generalizations are designated in Figures 8A and 8D.)

In the case of height, the original CC rule was generalized to include situations where the A-B objects whose height was being measured were not resting on the common base and where the axis of one of the objects was disinclined from the vertical (that is, where the height was distinct from the length). The Ss had to learn, for example, that the height of a balloon, say, or of a pencil leaning on a book is the perpendicular distance from the top of the respective object to some baseline. (In the comparisons, all A-B pairs were measured from a common baseline.)

In the case of amount, the domain of the CC rule was generalized to include A-B objects having different shapes. The following shapes were included: single balls, several small balls, "hot dogs," and "pancakes."

To generalize the previously taught height CC rule, S was first shown two different wooden blocks that had been placed on a table, one 11 inches high and the other 12 inches high. The S was told: "Today, we are going to talk about height. Tell me about the height of the blocks." Then, S was asked where he looked when he wanted to decide about height. If S did not respond "top," E suggested this, took S's hands, and put them on top of each block. Then, E moved S's hands downward from the top along the vertical perpendicular to the table. In actually carrying out the CC rule (that is, moving both hands up vertically from the base to the respective tops), E called S's attention to the fact that both blocks were on the table and that height refers to the vertical distance from the table to the top. The E emphasized that determining the height of an object involved three things: the top of the object, the baseline from which the top is measured, and measurement along the vertical.

Next, E called S's attention to two balloons on a wall. "If we were going to talk about the height of the balloons, from where would we be measuring the height?" The dialogue was continued until S understood that the floor was the implicit baseline. To help insure generalization of the concept, E also discussed the height of pictures drawn on blackboards. The E presented examples until S could give five correct answers in a row to height comparison problems involving various blocks, pictures, and balloons.
Finally, E called attention to two identical pencils, one vertical and one tilted at a 20° angle to the floor, and asked about the height. The E noted again that height meant the vertical distance from the table (baseline) perpendicular to the tops of the pencils (that is, objects being measured). The next example involved vertical and angular crayons that had the same height. The E corrected errors, always calling attention to the way height is measured. This procedure was repeated until S could give five quick, correct responses in a row.

Teaching and testing of the expanded height domain took place during one session lasting from ten to 30 minutes. In all test situations, special care was taken to present only static situations. The materials were in place before S entered the room and were not moved during a lesson. In effect, a clear distinction was made between CC rule training and the planned higher-order t rule training. In the latter case, transformations were performed in direct view of S.

A similar procedure was used to generalize amount. The S was shown two play-dough shapes (for example, a "pancake" and a "hot dog") containing the same amount of play dough. The E reminded S of the rule for determining relative quantities of play dough (that is, put one object in each hand, cover as much of the object as possible by "packing in" parts that stick out, and see which hand covers more of the clay). Then E asked, "How could we tell about the amount of play dough—whether this one has more, or this one, or whether they have about the same amount of play dough?" If S did not suggest or do it spontaneously, E said: "Why don't you fit one in each of your hands? Then you can tell." After S fit as much of the objects into his hands (assisted, if necessary) as possible, E asked, "Now, what about the amount of play dough?"

This procedure was repeated with a ball (less) and a hot dog (more), a ball (more) and a pancake (less), and a ball and six small balls (same). Any errors were corrected, and S was told that "about the same" was acceptable when it was too difficult to tell which had more and which had less, or when the difference was so small that it did not matter.

The final step in the CC rule training was designed to insure that S not only could use the CC rules quickly but also could readily determine which one was to be used with given comparison problems and that more/less/same are understood by (adult) social convention to form a trichotomy (for example, it is not possible for A to be both more than B and the same as B in terms of the same property). For this purpose, two random sequences of comparison problems were presented. The first sequence involved discriminations that were easy to make and the second, discriminations more difficult to make.
Two comparison problems (one simple, one complex) for each CC rule were constructed. The simple comparison problems were placed on a table in random order and were presented to S on one day. The complex problems were similarly arranged and presented on a subsequent day. For each comparison problem, E asked S two questions.

E first asked: "What can you tell me about the [height, area, amount of Kool-Aid, or amount of play dough] of these two? Is one more than the other or do they have about the same?" If S answered correctly but did not use the words "more," "less," or "same," E stated: "Yes, that is right. This has more _____ and this has less _____." (Or, "They both have about the same _____.")

If the result of the comparison was "about the same," E added: "Could you also say that this one clearly has more _____? Would you be wrong? Could you also say that this other one clearly has more _____? Would you be wrong?" Where necessary, E stressed that one normally agrees not to use "more," "less," and "same" in the same context. If the result of the comparison was more/less _____, E added: "Could you also say that they have about the same _____? Would you be wrong?"

The materials and their order of presentation were as follows:

**Simple Stimuli**

1. More/less play dough - two shapes (one pancake .125 inches thick, with a diameter of 1.5 inches, one hot dog one by three inches)
2. Same area - two pieces of construction paper (one red and one blue circle, each with a three-inch diameter)
3. More/less Kool-Aid - two plastic cups (containing two and six ounces of Kool-Aid, respectively)
4. Same Kool-Aid - two plastic cups (each with four ounces of Kool-Aid)
5. More/less height - two blocks (one, one by three by four inches high, and a cylinder with a one-inch diameter, seven inches high)
6. More/less area - two pieces of construction paper (one blue triangle three by four by five inches, one yellow square two inches on each side)
7. Same height - two balloons (with no strings, both inflated to the same diameter and placed with tops at the same height from floor)
8. Same play dough - two play-dough cubes (both with sides of .5 inches)
**Complex Stimuli**

1. More/less height - two identical pencils (one vertical, with its top five inches above the table; one at angle, with its top three inches above the table)
2. More/less liquid - two wide containers (both with 2.5-inch diameters, one with two ounces and one with four ounces of orange Kool-Aid)
3. Same area - two pieces of construction paper (both with the same, irregular, amoeba shape, one purple, one orange)
4. More/less area - two pieces of construction paper (one pink, five-inch diameter circle; one green, three-by-four-by five-inch triangle)
5. More/less play dough - two play-dough shapes (one cube with .5-inch sides, one ball with a two-inch diameter)
6. Same play dough - two play-dough shapes (one pancake 2.5 inches in diameter, one ball one inch in diameter)
7. Same height - two blocks (one, one by three by three inches high; one, one by seven by three inches high)
8. Same liquid - two tall transparent glasses (each with the same amount of orange Kool-Aid)

Any S who did not answer all questions correctly and quickly was given remedial instruction. According to the above criteria (that is, five quick, correct responses in a row), six Ss required remedial instruction. Two of these needed two lessons.

**Higher-Order Transformation Rule Training**

After achieving the criterion on the CC rule training, S was given higher-order t rule training. The goal of this training was to lead S to learn some behavioral equivalent of the **anticipatory** higher-order t rule based on adding/taking away (see Figure 7) and to accomplish this in a way that built on earlier learning (for example, on the CC rules). As discussed previously, when applied to **anticipatory** conservation problems, the higher-order t rule of Figure 7 correctly anticipates posttransformation A-B' comparisons only when the critical properties are intrinsic to the objects. This rule does not necessarily anticipate comparisons (correctly) when the critical properties are not intrinsic to the A-B' objects (for example, as with height and turn).

The **consistency** higher-order t rule associated with Figure 6, on the other hand, is not limited in this way. The basic procedure is the same and gives a correct result no matter what property is
involved in the A-B comparisons. The major specific prerequisites to learning the consistency higher-order t rule are the appropriate CC rules. In each case, the main operation (decision) involves checking pre- and posttransformation comparisons for consistency. If the pre- and postcomparisons give different results, then the transformation is identified as one that affects the property. Otherwise, it is identified as one that does not.

In comparison with anticipating transformation effects, checking for consistency would appear to be the more basic skill, one more compatible with the general, "real-world" knowledge that five-year-olds might be expected to bring to the task. Consequently, higher-order t rule training progressed in two stages. First, S was taught the generalized consistency higher-order t rule of Figure 6 and, then, the anticipatory higher-order t rule of Figure 7.

There could be no guarantee, of course, that verbal descriptions of these rules would be understood by young children as intended. Hence, verbal instruction was used only to supplement S's direct involvement with preplanned sequences of problems. In fact, the present approach to higher-order t rule training was not all that different from what has been recommended by Piaget for use with transitional children. Aside from explicit training on CC rules, there were two major differences. Most of our Ss were not "transitional" until after they had learned the relevant CC rules, and the training problems and underlying solution rules were operational and more precisely specified. The latter allowed for the possibility of more effective and efficient instruction, including the possibility of predicting the individual behavior in specific transfer situations.

During phase one of the higher-order t rule training, S was presented with a series of problem sequences designed to insure acquisition of the consistency higher-order t rule. Each sequence of problems involved either liquid (Kool-Aid to drink), area (to color), or amount (of play dough).

The first problem in each sequence involved comparing A and B with respect to the given property. In four out of five cases, A and B were equal. By virtue of the CC rule training, S was expected to make the comparisons correctly. When he did not, remedial instruction was provided. Immediately after S responded, E shielded the A-B objects and asked, "Do you remember which one had more, or were they about the same?" (After S had correctly recalled what he had said on two such problems, it was assumed that S could do this reliably, and further questioning along these lines was eliminated.) This type of question was designed to help insure that S would remember the results later in the pre-posttransformation comparison.
Second, S was told how E was going to transform B (for example, "Now, I am going to pour all the Kool-Aid in this glass into this one"). Immediately thereafter, E carried out the transformation in full view of S. (Note that to avoid ambiguous situations, E never applied transformations where A and B differed initially. In these cases, E simply moved on to the next sequence of problems.) The E then asked S to describe what E had done (that is, the transformation) and, irrespective of S's response, introduced a verbal label for the transformation to serve both as a memory aid and to facilitate future identification.

Third, S was asked to compare A and B' as in the initial comparison problem. After S responded correctly, E asked, "Do you remember which had more before, or were they about the same?" Or "Did B' also have more [less, about the same] before [the transformation]?" The E provided assistance whenever S had difficulty remembering what he had said. If S responded incorrectly, E first told S to use (and/or how to use) the CC rule to compare A-B'.

Fourth, if S correctly indicated that the pre- and postcomparisons were the same or different, E asked whether the transformation "changed the [concept]." Help was provided when necessary. If S responded incorrectly, E had S make new pre- and posttransformation comparisons with new materials until S could tell whether or not the transformation affected the results of the comparisons. (A record was kept of incorrect responses to provide evidence concerning the prior availability of simple consistency skills in five-year-olds.)

Finally, S was asked, "What did I do [to B]?" Where necessary, E reminded S of the transformation performed and provided feedback in the form of the appropriate verbal label. The E then asked: "Why did the [transformation, for example, pouring] change [not change] the amount of _____ [concept, for example, Kool-Aid to drink]? Did the _____ [pouring and so forth] change the amount?" The point of the questioning and feedback was to help S identify which transformations did and did not affect the property in question.

The problem sequences were administered in the order indicated in Table 4. Four sequences involved liquid; four, area; and four, amount. The various concept sequences were presented in random order to help insure generality of the to-be-learned consistency higher-order t rule. Two sequences for each concept were presented in the initial series, then one of the two remaining sequences for each concept, and finally the remaining sequence for each.

The types of transformations introduced for each concept during training or during posttesting are shown in Table 5. These transformations either change the relative amount of A and B with
<table>
<thead>
<tr>
<th>Concept</th>
<th>Initial State</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Liquid  (Kool-Aid)</td>
<td>A = B</td>
<td>adding some</td>
</tr>
<tr>
<td>2 Liquid  (Kool-Aid)</td>
<td>A = B</td>
<td>rotating</td>
</tr>
<tr>
<td>3 Area   (paper)</td>
<td>A = B</td>
<td>rotating (out of plane)</td>
</tr>
<tr>
<td>4 Area   (paper)</td>
<td>A = B</td>
<td>rotating (in plane)</td>
</tr>
<tr>
<td>4' Liquid (Kool-Aid)</td>
<td>A &gt; B</td>
<td>STOP</td>
</tr>
<tr>
<td>5 Mass   (play dough)</td>
<td>A = B</td>
<td>adding</td>
</tr>
<tr>
<td>6 Mass   (play dough)</td>
<td>A = B</td>
<td>taking some away</td>
</tr>
<tr>
<td>7 Liquid  (Kool-Aid)</td>
<td>A = B</td>
<td>pouring all into other glass</td>
</tr>
<tr>
<td>8 Area   (paper)</td>
<td>A = B</td>
<td>adding some</td>
</tr>
<tr>
<td>8' Area  (paper)</td>
<td>A &lt; B</td>
<td>STOP</td>
</tr>
<tr>
<td>9 Mass   (play dough)</td>
<td>A = B</td>
<td>deformation (rolling)</td>
</tr>
<tr>
<td>10 Liquid (Kool-Aid)</td>
<td>A = B</td>
<td>taking some away</td>
</tr>
<tr>
<td>11 Area  (paper)</td>
<td>A = B</td>
<td>taking some away</td>
</tr>
<tr>
<td>12 Mass  (play dough)</td>
<td>A = B</td>
<td>deformation (flattening)</td>
</tr>
<tr>
<td>12' Mass</td>
<td>A &gt; B</td>
<td>STOP</td>
</tr>
</tbody>
</table>

Source: Compiled by the authors.
TABLE 5

Types of Transformations Introduced during Training/Posttesting

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Length (Rigid or Flexible)</th>
<th>Area (to Color)</th>
<th>Liquid Kool-Aid to Drink</th>
<th>Amount (Use of Dough)</th>
<th>Torque (Use of Balance Beam)</th>
<th>Height</th>
<th>Weight (Use of Scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taking away/adding</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Pouring</td>
<td>n.a.*</td>
<td>n.a.</td>
<td>-</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>Rotation</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>n.a.</td>
<td>n.a.</td>
<td>+</td>
<td>n.a.</td>
</tr>
<tr>
<td>Deformation (e.g., rolling, spreading)</td>
<td>-</td>
<td>-</td>
<td>n.a.</td>
<td>n.a.</td>
<td>-</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

*Not applicable.

Source: Compiled by the authors.
respect to the concept (+), or they do not (−). Adding and taking away were used with all concepts. In addition, during training, liquid (Kool-Aid to drink) was subjected to pouring and rotation transformations; area (to color), to various rotations (for example, 90° rotation or irregular movements through space); and amount (of play dough), to various deformations (for example, rolling or flattening). Half of the transformations actually used for training purposes did affect the critical concept; the others did not. (In addition to adding/taking away during posttesting [see below], number and length also were subjected to rotations and deformations. Height also was subjected to rotations.)

Each S was presented with each sequence at least once. If S was correct on all problems in these 12 sequences, training was terminated. Remedial instruction then was given on all sequences involving one or more problems that S did not solve quickly and accurately. Remediation continued until S could go through five sequences in a row quickly and accurately, including all sequences missed previously.

The higher-order consistency training took place during two sessions of 15 to 30 minutes each. During session one, problem sequences 1 through 6 were introduced. The remaining sequences were introduced during session two.

In summarizing what S had found, E reminded S of all of the transformations introduced with respect to each concept. During the review, E helped S to distinguish between transformations that did affect each concept and those that did not. The E then helped S to draw analogies among the various transformations—leading to the generalization that adding/taking away changes the amount of Kool-Aid to drink/area to color/play dough to make things with. The other transformations do not affect these properties.

The next phase of the training involved the anticipatory higher-order t rule based on adding/taking away (Figure 7). In this case, S was presented with a series of problem sequences, each consisting of concept comparison, anticipatory conservation, and post-transformation conservation problems.

Specifically, the first problem in each sequence was an initial A-B concept comparison problem. In three out of four of these initial problems, A and B were essentially identical, not only in the critical concept but in all other obvious respects (that is, properties) as well. These concept comparison problems correspond to questions/directives in traditional conservation tasks pertaining to pre-transformation equivalence of A-B objects. In the other 25 percent of the tasks, either A was greater than B or B was greater than A with respect to the critical concept. In each case, S was asked whether A or B had more of the critical concept.
Correct responding on these problems, as noted earlier, could be due to use either of an appropriate CC rule or of some form of perceptual equivalence rule (because the A and B objects "looked" identical in all respects). If S gave an incorrect answer (especially when S said that A or B had more when they had approximately the same), E explained that the goal was to determine whether A "clearly" had more or less of the concept than B or if A and B had "about" the same value of that property. If necessary, E replaced A and B with a new pair and started over.

In those cases where A and B were identical, E next presented the corresponding anticipatory conservation problem; that is, S was told that E was going to change B in a certain way (that is, perform a transformation on it). (Where A and B were unequal, E simply went on to the next problem sequence.) Then, a transformation of the indicated type was actually performed behind a screen so that the transformation itself (or at least a portion thereof) was visible but not the end result of the transformation—that is, A and t(B). The S was asked if A had more of the concept than B, whether B had more than A, or whether they were about the same.

Next, E showed S the A-B' state resulting from the transformation and asked whether A and B were clearly different or if they were about the same, as before. When incorrect, S was urged to use the corresponding CC rule.

Finally, where S gave an answer that was inconsistent with his anticipatory expectation, or was consistently wrong, S was asked, "Why?" If S could not answer, E asked, "Can you change that one [that is, B'] so that it has the same?"

By virtue of the CC rule training, the experimental Ss were expected to answer the postcomparison problems correctly. However, although the consistency higher-order t rule training introduced earlier makes it possible to determine whether a given transformation affects a given property after the fact, S would not necessarily know how to anticipate the effects of given transformations beforehand. (Since the transformations were identical to those introduced during the consistency training, being able to anticipate transformational effects, that is, knowing an appropriate anticipatory higher-order t rule, amounted to remembering which transformations affect which concepts.)

In order to facilitate such learning, three additional things were done. First, when a transformation (for example, adding) could bring about different degrees of change in B, E started with small changes and gradually moved to larger ones as S demonstrated his capability on the easier ones. The method used was directly analogous to "dimensional analysis" as described by Lowerre and Scandura (1973, 1977).
Second, during the training for each concept (that is, amount of play dough to make things with, amount of Kool-Aid to drink, and amount of area to color), E summarized each major facet of S's learning. Specifically, for the initial A-B comparison problem, E said something to the effect that "That is correct, A and B have about the same [concept] because ____" and then demonstrated with a CC rule. Almost incidentally, E added that A and B also looked about the same. When dealing with the anticipatory conservation problems, after S gave a correct answer, E offered something to the effect that "Yes, it does not change B so A and B' still have the same" (or "Yes, t does change B, so ____"). On the A-B' comparison problem resulting from the transformation, E responded as when S was successful on the initial comparison problem. In this case, E commented, where appropriate, that A and B' had the same amount of the concept, as indicated by the CC rule, even though A and B' might "look different." Perhaps most important, when S responded incorrectly to either the anticipatory conservation or the posttransformation problems, E called S's attention to the incongruities involved.

Third, the E emphasized interrelationships in the anticipatory higher-order t rule training. Specifically, in training on the second and subsequent concepts, E called S's attention to corresponding transformations involving previously taught concepts. For example, E might note that adding more clay to B, for amount, "is like" adding more liquid to B, both of which increase the value of the indicated properties. The E similarly pointed out analogies involving transformations that did not affect such properties (for example, rotations and translations of areas correspond to rotations and translations of liquid and of mass). The E also helped S to recognize in each case that B is invariant under transformations that do not affect the substance of B per se (that is, that do not affect an "intrinsic" amount property).

As in the case of consistency training, the anticipatory higher-order t rule training involved three concepts: (1) amount of Kool-Aid to drink, (2) amount of area to color, and (3) amount of clay with which to make things. In all three cases, the range of situations considered was compatible with the domains of the CC rules on which training was provided. To help facilitate learning, the three concepts were presented in the order 1, 2, and 3. As defined, notice that the CC rules corresponding to this order involve one, two, and three dimensions, respectively.

A summary of the materials, the specific problem sequences (each including an initial comparison problem, an anticipatory conservation problem, a postcomparison problem, and sometimes a final incongruity problem), and procedures for presenting the anticipatory conservation problems is given below.
In problems involving Kool-Aid to drink, materials consisted of four transparent cups with a small figure printed on the side, all of the same size. A spoon was used to take away or add to B. A screen also was used. This screen was slightly shorter than the four cups.

Problem sequences involved pouring B into B', taking away liquid from B with a spoon, rotating B (small to large rotations), and adding liquid to B.

In the anticipatory conservation problems, the screen was used to hide the level of the liquid but was not so high as to cover the tops of the cups. Hence, S could see the transformations as they were being performed.

In problems involving area to color, materials consisted of two squares of paper with the same surface and different colors. Horizontal and vertical pencil lines were drawn on one of the squares.

Problem sequences involved rotating B (in plane, with small to large rotations), cutting pieces from B, rotating B (out of plane), and taping a new piece onto B.

In the anticipatory conservation problems, the transformations were performed in front of the child but with the final results hidden from view.

In problems involving play dough, materials consisted of two balls of play dough and one rectangular piece of wood (30 by 20 cm.), which was used to roll or spread out the play dough.

Problem sequences involved rolling B into B' with wood (for example, into a sausage), taking play dough away from B, squashing B into B' (that is, into a pancake), and adding play dough to B.

The piece of wood served simultaneously to affect the transformations and to shield the latter's effects on B.

As in the consistency rule training, E introduced S to all 12 problem sequences, noting and correcting any errors made. Training was considered completed if S quickly and accurately anticipated the effects of the various transformations with all 12 problem sequences. Otherwise, S was provided with remedial instruction until uniformly successful on five sequences in a row, including all previously missed sequences.

Posttest

Because the school year was rapidly coming to a close, post-testing of the control Ss began shortly before some of the experimental Ss had completed the higher-order t rule training. Each experimental S was tested within one week of receiving his last training lesson.
First, all Ss were tested to determine individual mastery of those CC rules on which the experimental subjects had been trained, that is, on CC rules for comparing liquid (Kool-Aid to drink), area (to color), amount (of play dough), and height. This testing was similar to traditional conservation testing for determining initial equivalence but, as in the CC rule training, was more general in that S was presented with unequal and perceptually distinct A-B pairs as well as identical ones. The test problems were presented in random order and were based on the following materials.

The liquid (Kool-Aid to drink) problem involved two identical transparent cups with the same standard (medium) amount of liquid; two identical transparent cups with the same amount of liquid, one on its base, the other held at a 50° angle from the horizontal; two identical transparent cups with more (than the standard amount of) liquid in one; and two identical transparent cups, the one with less liquid held at a 50° angle.

The area (to color) problem involved two circular pieces of paper of identical size; two square pieces of paper of about the same area, one having a grid drawn on it (like graph paper) and rotated at a 45° angle to the other; two triangular pieces of paper, with the first having more area than the second; and a triangular and a square piece of paper, with the latter clearly having more area.

The amount (of play dough) problem involved two identical balls of clay; a ball of clay and a sausage-shaped piece of clay, each having the same amount; two balls of clay, with the first clearly having more than the second; and a ball of clay and a pancake-shaped piece of clay, with the first having more than the second.

The height problem involved two balloons placed at the same height on a wall; two balloons of different sizes at the same height, each with a string attached (the longer string dangling from the smaller balloon); two identical books placed on a shelf, one standing vertically and the other lying flat; and two balloons of different sizes, with the smaller balloon having the shorter string and being higher than the other.

In each case, S was asked, "What can you tell me about the _____ [for example, height]?" If S answered incorrectly, E asked, "Why?"

Second, Ss were tested on four comparison problems involving number. As before, the problems were presented in a randomly determined order and were based on the following materials: two rows, respectively, of six red and six green poker chips arranged with the chips equally spaced, the green ones directly below and paired one-to-one with the red ones; a row of seven green poker chips as above and a "clump" of seven yellow chips to the right; a row of nine red poker chips and a row of seven blue poker
chips spread farther apart and horizontally transformed so that each end of the second row extended further out; and a row of seven green poker chips and a "clump" of nine blue chips.

Third, Ss were tested on conservation problems in both anticipatory and traditional manners. In general, the following test procedure was used. The S was asked to construct and/or to agree that A and B had the same amounts of the specified properties. (They also were as close to identical as possible in all other respects.) Next, a transformation was performed in front of S. Initially, the resulting A-B' pair was shielded from view. The S was asked to predict the A-B' relationship and to indicate why he felt this would be the case. The results were recorded as evidence of anticipatory conservation performance. Then, the shield was removed so S could see the resulting A-B' pair. In each case, the transformation t was chosen so that the appearance of A-B' and the actual relationship between A and B', as determined by the corresponding CC rule, were discordant. (The goal was to see if S would infer the proper relationship between A and B', perceptual illusions notwithstanding.) Finally, the S was asked to justify his answers. No feedback was given throughout the testing.

Three types of conservation problems were administered.

Each S was tested on within-scope problems, that is, on conservation problems corresponding to the experimental group training in liquid, area, and amount. In the case of liquid conservation, the transformation involved rotating (tilting) glass B 45°. In the case of area, A and B were both squares. The transformation involved rotating B 45° and turning it over, thus exposing a grid of lines (as on graph paper) drawn on the back. In the case of amount, the transformation involved rolling the B object (initially in the form of a ball) into the form of a sausage.

Each S was tested on two number conservation problem sequences. Although not included in the experimental group training, these problems were within the scope of the anticipatory higher-order t rule (based on adding/taking away). Hence, an experimental S who had mastered this rule would be expected to correctly anticipate A-B' relationships resulting from given transformations, depending on whether adding or taking away was involved. Similarly, an S who knew an appropriate CC rule for number would be expected to perform successfully on problems involving perceptual illusions. Without such a rule, S might be confused by the perceptual illusions. To check these hypotheses, S was required in problem one to first construct B so that "this row [B] has the same number as this one [A]." Then, E spread the B objects, which were behind a screen, so that they extended slightly beyond both sides of the screen and asked S to state the relationship between A and B', both
before and after actually seeing A and B' as described above, and to justify his responses. The second number conservation problem involved adding two B objects and transforming them (all) into a circle.

Each S was tested on conservation extra-scope problem sequences. If the higher-order t rule training was successful in leading the experimental Ss to learn an anticipatory higher-order t rule based on adding/taking away, then these Ss would be expected to fail on these problems. Specifically, the availability of this higher-order t rule would be hypothesized to cause experimental Ss to make incorrect predictions.

To test this hypothesis in the case of height conservation, S was asked to select a B pen from a given pile of pens and to place it vertically on end next to one (A) that already was standing vertically on end on a table "so that they have about the same height." If S was able to do this, E put a cap on the B pen (making it longer), "turned" the pen (rotated it 60°), replaced it behind the screen (shielding the result from view), and asked S how the heights compared and why he believed this. Then, E removed the shield and asked the same question. Finally, S was asked to justify his answers.

According to the anticipatory higher-order t rule, based on adding/taking away, one would expect that on the anticipatory conservation task the experimental Ss would respond that the B pen "has more height." In fact, the B pen had less height. This criterion-referenced prediction was primary. In addition, on average, the experimental Ss would be expected to not perform as well on this task as the control Ss. In view of the height CC rule training, the experimental versus control comparison would be more uncertain after removing the shield. Use of the CC rule would indicate that the vertical pen had "more height," whereas the anticipatory higher-order t rule would lead S to expect the other to have more height.

Perhaps more important, availability of both an inadequate anticipatory higher-order t rule and the height CC rule could introduce an incongruity, leading S to learn that rotations do in fact affect height. (We had planned originally to test this hypothesis by introducing a second height conservation problem, this one involving two books of equal height. The critical transformation was to involve letting one of the books fall flat. Unfortunately, due to the extreme time pressures we were under near the close of the school term, this item was inadvertently omitted from the posttest.)

The second extra-scope problem sequence involved an extension of the liquid conservation problems that had been previously introduced. Specifically, after first insuring that A and B contained
the same amount of liquid, as before, the contents of B were poured (behind a shield) into a tall, thin vessel. The liquid in its post-
transformation B' state came up much higher in the thin vessel than
in the glass. It is especially important to notice that the liquid CC
rule introduced previously was not adequate for comparing A and
B'. Hence, to the child, the former may have appeared to contain
"more Kool-Aid to drink."

In this case, then, the experimental Ss would be expected to
predict initially that A and B' would have about the same amount of
Kool-Aid. After actually seeing A and B', however, they might
well become confused and/or want to change their minds.

The test procedure was similar in the case of torque (turn). In
this case, however, S was first allowed to physically handle sev-
eral steel objects (with hooks) of three obviously different weights
and sizes. (Although explicit instruction was not provided, this
experience was designed to help equalize familiarity with weight
and corresponds to weight CC rule training.) Specifically, after a
brief period of exploratory handling, S was asked to compare vari-
ous pairs of weights by holding them in his hand. Next, the S was
presented with a balance beam on which weights could be hung at
various distances from the fulcrum. Initially, S was presented
with the balance beam without any weights attached. In turn, E
added a weight to one side and then an equal weight at the same dis-
tance from the fulcrum on the opposite side. Correspondingly, E
commented on how the first weight made its side go down and how
the second weight brought the beam back into balance (that is, "made
it level again"). The S then was told that E was going to exchange
one weight with a heavier one. The exchange was made behind a
shield, and S was asked to predict which way the balance beam had
turned (if at all) and to explain why he thought so. (The E placed
the heavier weight so close to the fulcrum that the side with the
lighter weight actually went down.) The shield was removed, and
S was asked to explain.

On the basis of the anticipatory higher-order t rule involving
adding/taking away, one might expect S to predict that the side with
the heavier weight would go down. (Control Ss, of course, also
might be inclined to make the same prediction "because heavy things
go down." In effect, what young children may need to learn most is
to distinguish other transformations in terms of whether they do or
do not affect given properties.)

Finally, S was tested on length conservation exactly as in the
original pretest (see Chapter 2). Although the results of this test-
ing did not exactly parallel the concept comparison and conserva-
tion testing above, they provided an overall measure of growth dur-
ing the year. (Note that length provided a better measure in this
EMPIRICAL TEST OF THE ANALYSIS / 95

respect than number because of the number CC rule training, that is, one-to-one matching, with the experimental Ss during the exploratory training.)

Results and Discussion

As emphasized earlier, the primary purpose of the present study was to determine the behavioral viability of the previously described structural analysis. Consequently, the experimental training was based directly on this analysis. The experimental results, in turn, provide a measure of the extent to which the identified rule set provides a sufficient (if not necessary) basis for explaining, predicting, and, in this case, also manipulating conservation behavior. Unlike other prescribed training studies, the present one provides an explicit basis for predicting horizontal decalage, both positive and negative transfer to new conservation tasks.

Training Criteria

Of the 13 experimental children, seven completed training on each of the training rules as planned before school let out for the summer: An, Ro, Ni, Ti, Da, Ty, and De. Two others were exposed to all of the training rules but did not reach criterion on the anticipatory higher-order t rule and/or missed some of the post-test problems. Mo left school early with only one day's notice. She was close to criterion on the anticipatory higher-order t rule but had to be rushed through to completion on the training without achieving the desired degree of automaticity and/or generality. Even more important, the posttest had only partially been formulated when she left, so that she missed two crucial transfer problems used to measure possible decalages. Although Sp did manage to give the required number of correct responses on some training tasks, he never achieved the level of automatic, consistent behavior required, especially with regard to generalizing across training tasks.

The other four children completed lesser amounts of the training for various reasons. Ma and Ra did not reach criterion on the anticipatory higher-order t rule and showed only limited evidence of being able to generalize across training tasks based on the common "adding/taking away" criterion. Br was absent from school for several days because of a broken foot and did not have time to complete the anticipatory higher-order t rule training. In the time available, Mi was not able to complete even the consistency training.
One major reason that the children did not complete the training was simply a lack of time. As frequently happens, especially in kindergarten, absences increased during the latter part of the school year to accommodate parents' vacations and for other sundry reasons. Moreover, the weather was warm on many days, and children often found it hard to concentrate. Other possible explanations might be given, of course. Thus, one could speculate that some kindergartners are simply incapable of ever learning the indicated rules.

All things considered, our judgment is that all normal five-year-old children can be taught these rules given sufficient time but that it would have been very difficult to induce more learning in the time available (for example, "pressing" harder would almost certainly have caused the children to lose interest).

 Nonetheless, it is possible that the higher-order rules may not have been sufficiently detailed (that is, atomic) in relation to the entering capabilities of some of the children (for example, Mi). In this case, of course, the solution in future studies would be to introduce such detail (that is, to represent the rules in terms of simpler components) and to take this into account in the actual instruction.

**Trained CC Problems**

All seven of the children who completed the training were successful on most of the 16 concept comparison (CC) problems for liquid, area, amount, and height (see Table 6A). The only exceptions were Ti, who failed on the second CC problem on amount (which involved a ball and an equivalent "hot dog") and the second one on height (two balloons of different sizes at the same height and with different-length strings attached), and Ty, who failed on the fourth CC problem on amount (which involved a ball and a very thin pancake of the same diameter) and the third CC problem on height (which involved two books, one lying flat and the other on end). Ti, apparently, was still influenced slightly by perceptual illusions in making certain comparisons. (In the Structural Learning Theory, such behavior would be explained in terms of higher-order selection rules operating on the CC and perceptual rules available to a subject [Scandura, 1973].) Ty's two mistakes are perhaps best classified as idiosyncratic.

Although there were a few more errors overall, the six other experimental children performed similarly. (All had reached criterion on the CC rules for liquid, area, amount, and height.) On the four liquid CC problems, there were a total of two (of 24 possible) errors. There were no errors on the four area problems, four on
TABLE 6A
Summary of Individual Results: Experimental Subjects (CC Tasks)

<table>
<thead>
<tr>
<th>S No.</th>
<th>Lgth.</th>
<th>Liquid CC</th>
<th>Area CC</th>
<th>Amount CC</th>
<th>Height CC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
</tr>
</tbody>
</table>

Proportions correct
7/7 7/7 7/7 7/7 7/7 7/7 7/7 7/7 7/7 6/7 7/7 6/7 7/7 6/7 7/7 7/7 7/7 7/7 7/7

aDashed circle indicates initial error with subsequent correction.
bCircle indicates error.
Source: Compiled by the authors.
the amount CC problems, and five (of 24) on the height CC problems. Four of the five errors involving height were on the second height CC problem, which Ti also had failed. This problem had caused some confusion during the training, apparently because the lower balloon was longer than the "higher one." Four of the five errors appeared to reflect keying on "height of the objects," rather than "height from the floor."

Number CC Problems

Aside from the initial unevenly distributed and nonspecific exploratory training, none of the experimental children had received training on the number CC problems. Hence, the role of these problems is properly viewed as diagnostic—that of specifying entering capabilities of the children with regard to number, rather than of determining the extent to which particular trained rules have actually been acquired. (Note that diagnosis only allows specification of what has been learned up to equivalence classes of rules; see Scandura, 1970.)

The first number CC problem, strictly speaking, did not require one-to-one matching or counting (it could be solved perceptually), whereas problems two, three, and four involved perceptual illusions in varying degrees and required a nontrivial CC rule for solution. Correspondingly, all 13 experimental children were successful on the first number CC problem, nine of 13 were successful on the second, only three of 13 on the third, and seven of 13 on the fourth (see Table 6B). Only An and Ro (after correcting himself on the third and fourth problems) were correct on all four number CC rules. Mo, Br, Da, Ty, and De made only one mistake each. (Ty, however, corrected herself on two problems. In each case, self-correction appeared to result from inconsistencies with respect to implicit answers to corresponding "why" problems.)

Trained Conservation

According to prediction, those experimental children who completed the anticipatory higher-order t rule training would be expected to correctly anticipate the conservation of liquid, area, and amount quantities. (The corresponding transformations used on the posttest were, respectively, tipping a glass, rotating and exposing the grid side of B, and rolling a ball into a "hot dog.") In fact, all seven children who reached criterion not only correctly anticipated the results of these transformations but also gave reasons consistent with "true" conservation—for example, "You didn't add anything," "Rolling it doesn't change the amount," "Nobody put more on," and so forth. The latter is especially significant since
no attempt was made to teach "reasons" that the children could parrot back on cue.

Not surprisingly, all of these children also gave correct and consistent responses on the posttransformation comparisons. These comparisons correspond to traditional conservation problems where, in responding, children must learn to ignore perceptual misinformation. In this case, the experimental children could generate appropriate responses either by applying the corresponding CC rule or by reapplying the anticipatory higher-order t rule, both of which had been learned. Having just applied the latter (higher-order) rule, many of the children appeared to apply the former (CC rule) as a check, saying, for example, "I checked and they are still the same," "They look different . . . but if you put them back you can see they are still the same," "[checking] I told you," and so forth.

Although not achieving the desired level of atomaticity and generality during training, Mo and Sp also performed perfectly on the liquid and area conservation-related problems. On the amount conservation problems, Mo succeeded, but Sp failed to anticipate the effects (on the A-B' comparison) of transforming ball B by rolling it. Of the other four noncriterion children, only Br gave consistently correct responses, including appropriate reasons (for example, "didn't add or take any away"). The other three failed on most of the anticipation problems. In the case of liquid, two of these children (Ma and Ra) switched answers (that is, corrected themselves) after actually seeing the posttransformation A-B' pair. Mi correctly anticipated the effects on liquid of tipping one glass but characteristically gave the opposite response after actually seeing the glasses after the transformation. (Recall that Mi was the only experimental child who did not complete the consistency training.)

**Number Conservation: Positive Transfer**

As noted previously, the anticipatory higher-order t rule based on adding/taking away should provide a sufficient basis for correctly anticipating posttransformation comparisons in the case of number. In addition, in order for the problems to be meaningful (that is, for children to be able to evaluate their own trial solutions), it is essential that they also know a CC rule for number (for example, some form of one-to-one matching). To test these hypotheses, the children were tested on two number conservation problem sequences. The first problem sequence involved spreading the B objects out so that both ends extended slightly beyond the shield. The second involved adding two objects and rearranging them into a circular "clump." (Rearrangements of this type appear to be among the more compelling visual illusions for young children and, consequently, among the more difficult to compare.)
<table>
<thead>
<tr>
<th>S</th>
<th>Number CC</th>
<th>Liquid Conservation Anticipatory Posttest</th>
<th>Area Conservation Anticipatory Posttest</th>
<th>Amount Conservation Anticipatory Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>An</td>
<td>A=B A=B A=B A=B</td>
<td>&quot;same amount&quot; &quot;I checked them first; had same&quot;</td>
<td>&quot;had the same before&quot;</td>
<td>&quot;had the same before&quot;</td>
</tr>
<tr>
<td></td>
<td>1-1 count 1-1 count</td>
<td>even if you tip it&quot;</td>
<td>&quot;I can check them and put them together&quot;</td>
<td>&quot;look different but had same before and didn't add anything&quot;</td>
</tr>
<tr>
<td>Ro</td>
<td>A=B A=B</td>
<td>&quot;had same amount in glasses&quot;</td>
<td>&quot;can turn back and looks same; is same&quot;</td>
<td>&quot;will be same&quot;</td>
</tr>
<tr>
<td></td>
<td>visual 1-1 counted</td>
<td></td>
<td></td>
<td>&quot;still same after rolling it out&quot;</td>
</tr>
<tr>
<td>Ni</td>
<td>A=B A&gt;B</td>
<td>&quot;tipping doesn't make same&quot;</td>
<td>&quot;those blocks don't change area to color&quot;</td>
<td>&quot;rolling doesn't affect sameness&quot;</td>
</tr>
<tr>
<td></td>
<td>extra 1-1 lined up closer out</td>
<td>(not attending to what he was saying)</td>
<td></td>
<td>&quot;I told you same as before&quot;</td>
</tr>
<tr>
<td>Ti</td>
<td>A=B A&gt;B A&gt;B A&gt;B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>counted, but didn't touch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Da</td>
<td>A=B A=B</td>
<td>&quot;nothing poured in or out&quot;</td>
<td>&quot;nothing taken out or put in&quot;</td>
<td>&quot;nothing put on or taken off&quot;</td>
</tr>
<tr>
<td></td>
<td>counted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ty</td>
<td>A=B A&gt;B</td>
<td>&quot;were same before&quot;</td>
<td>&quot;made same area; you just turned it&quot;</td>
<td>&quot;rolling doesn't change amount&quot;</td>
</tr>
<tr>
<td></td>
<td>moved 1-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>De</td>
<td>A=B A=B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>count (B looked longer)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Proportions correct

<p>|   | 7/7 | 5/7 | 2/7 | 5/7 | 7/7 | 7/7 | 7/7 | 7/7 | 7/7 | 7/7 |</p>
<table>
<thead>
<tr>
<th>Mo</th>
<th>A=B</th>
<th>A&gt;(\leq)B</th>
<th>A(\leq)B</th>
<th>A(\geq)B</th>
<th>A(\leq)B</th>
<th>A=B</th>
<th>A=B</th>
<th>A=B</th>
<th>A=B</th>
<th>A=B</th>
<th>A=B</th>
<th>A=B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportions correct</td>
<td>2/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>2/2</td>
<td>2/2</td>
<td>2/2</td>
<td>2/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>(each time put them in stacks and compared them)</td>
<td>A=B</td>
<td>A=B</td>
<td>A=B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
</tr>
<tr>
<td>Ma</td>
<td>A&gt;B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A&lt;B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
</tr>
<tr>
<td>Mi</td>
<td>A&gt;B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
<td>A(\geq)B</td>
</tr>
<tr>
<td>Proportions correct</td>
<td>4/4</td>
<td>3/4</td>
<td>0/4</td>
<td>1/4</td>
<td>2/4</td>
<td>3/4</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(a\) Dashed circle indicates initial error with subsequent correction.

\(b\) Circle indicates error.

Source: Compiled by the authors.
<table>
<thead>
<tr>
<th>S</th>
<th>First Number Conservation Anticipatory Posttest</th>
<th>Second Number Conservation Anticipatory Posttest</th>
<th>Height Anticipatory Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>An</td>
<td>A=B &quot;only moved it&quot;</td>
<td>A=B</td>
<td>A=B &quot;you added&quot;</td>
</tr>
<tr>
<td>Ro</td>
<td>A=B &quot;were same before&quot;</td>
<td>A=B</td>
<td>A=B &quot;added some&quot;</td>
</tr>
<tr>
<td>Ni</td>
<td>A=B &quot;moving doesn't change them&quot;</td>
<td>A=B</td>
<td>A=B &quot;added two&quot;</td>
</tr>
<tr>
<td>Ti</td>
<td>A=B &quot;are same not sure&quot;</td>
<td>A=B</td>
<td>A&gt;B &quot;added and looks different&quot;</td>
</tr>
<tr>
<td>Da</td>
<td>A=B &quot;didn't add or take away&quot;</td>
<td>A&gt;B &quot;none put in or taken out&quot;</td>
<td>A&gt;B &quot;added 2 chips&quot;</td>
</tr>
<tr>
<td>Ty</td>
<td>A&gt;B &quot;just moved over&quot;</td>
<td>A&gt;B</td>
<td>A&gt;B &quot;added 2 chips&quot;</td>
</tr>
<tr>
<td>De</td>
<td>A&gt;B &quot;can see B ones&quot;</td>
<td>A&gt;B</td>
<td>A&gt;B &quot;in a circle&quot;</td>
</tr>
<tr>
<td>Proportions correct</td>
<td>6/7</td>
<td>5/7</td>
<td>7/7</td>
</tr>
<tr>
<td>Mo</td>
<td>A=B</td>
<td>A&gt;B, 1/1→A&gt;B</td>
<td>Did not get tasks</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>--------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Sp</td>
<td>A&gt;B</td>
<td>A&lt;B</td>
<td>&quot;added two more and moved into circle&quot;</td>
</tr>
<tr>
<td></td>
<td>&quot;moved chips&quot;</td>
<td>&quot;moved them out&quot;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A=B</td>
<td>A&gt;B</td>
<td>&quot;in a circle&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;A didn't add or take away&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;B in a circle&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;B in a circle&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;B didn't add or take away&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;B are bigger, A are shorter&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;A lined up, B in circle&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;adding top; adding changes height, turning doesn't&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;doesn't add height&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;made A into a circle&quot;; didn't know why</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Proportions correct</th>
<th>1/2</th>
<th>1/2</th>
<th>1/1</th>
<th>0/1</th>
<th>1/1</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Br</th>
<th>A=B</th>
<th>A&gt;B</th>
<th>A&lt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&quot;didn't add or take away&quot;</td>
<td>&quot;didn't add or take any away&quot;</td>
<td>&quot;moving them out&quot;</td>
<td>&quot;added two chips&quot;</td>
<td>&quot;B moved out&quot;</td>
</tr>
<tr>
<td></td>
<td>&quot;not sure&quot;</td>
<td>&quot;didn't add or take away any&quot;</td>
<td>&quot;A lined up, B in circle&quot;</td>
<td>&quot;added top; adding changes height, turning doesn't&quot;</td>
<td>&quot;didn't add height&quot;</td>
</tr>
<tr>
<td></td>
<td>&quot;didn't add or take away any&quot;</td>
<td>&quot;A lined up, B in circle&quot;</td>
<td>&quot;adding top; adding changes height, turning doesn't&quot;</td>
<td>&quot;didn't add height&quot;</td>
<td>&quot;didn't add height&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ma</th>
<th>A=B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&quot;moved them&quot;</td>
<td>&quot;B are bigger, A are shorter&quot;</td>
<td>&quot;A lined up, B in circle&quot;</td>
<td>&quot;made A into a circle&quot;; didn't know why</td>
<td>&quot;adding changes height; turning height&quot;</td>
</tr>
<tr>
<td></td>
<td>&quot;B are bigger, A are shorter&quot;</td>
<td>&quot;A lined up, B in circle&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ra</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&quot;moved them out&quot;</td>
<td>&quot;B in a circle&quot;</td>
<td>&quot;B in a circle&quot;</td>
<td>&quot;made A into a circle&quot;; didn't know why</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
</tr>
<tr>
<td></td>
<td>&quot;B in a circle&quot;</td>
<td>&quot;B in a circle&quot;</td>
<td>&quot;made A into a circle&quot;; didn't know why</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mi</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
<th>A&gt;B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>didn't know why</td>
<td>didn't know why</td>
<td>didn't know why</td>
<td>didn't know why</td>
<td>didn't know why</td>
</tr>
<tr>
<td></td>
<td>&quot;made A into a circle&quot;; didn't know why</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
<td>&quot;adding changes height; turning changes height&quot;</td>
</tr>
</tbody>
</table>

Proportions correct | 1/4 | 1/4 | 1/4 | 1/4 | 2/4 | 4/4

*Circle indicates error.

*Dashed circle indicates initial error with subsequent correction.

Source: Compiled by the authors.
As with the other conservation problem sequences, all of the experimental children correctly determined the pretransformation A-B equivalences associated with both number conservation problem sequences (see Table 6C). Essentially as predicted, the seven children who reached criterion on the anticipatory training, correctly anticipated the A-B' comparisons in almost every case on both anticipatory number conservation problems. The only exception (among the 14) was De, who predicted on the first problem that "A has more than B" for the seemingly inconsistent reason that she could "see the [B] ones [extending beyond the shield on both ends]." (Note that this problem was the only one in which the shield did not completely block the children's view. By her answer, perhaps De meant that "A has more than B" behind the screen?) On the posttransformation comparison, De first maintained that A had more than B and then switched to "B has more than A" because B "is longer."

Her susceptibility to each type of illusion was reflected in her responses to the number CC problems. On the third CC problem, she counted correctly but incorrectly selected the longer row as having more elements, whereas on the fourth, she selected the "clumped" B objects as having more. Although De responded correctly on the second sequence of number conservation problems (involving clumping), her "preference" for "clumps" might help to explain her anticipatory response on the first sequence. In that case, the A row (which was completely shielded) was more "clumped" than the B row (where some elements extended beyond the shield). Ni, Ti, and Ty also had difficulty on the number CC problems. Predictably, they also erred on the posttransformation comparisons.

In spite of the indicated inadequacies in their anticipatory training, Mo and Sp correctly anticipated the effects of one of the transformations. Sp responded incorrectly on the first. (Mo left school early and was not tested on the second number problem sequence, nor did she get those involving height or extended liquid.) Moreover, Sp had had difficulties on the number CC problems and not surprisingly also was incorrect on both posttransformation comparisons, in each case having to switch from his anticipatory preillusion predictions. As expected, the other four children who did not complete the anticipatory training did not perform as well as the others. Br succeeded on the first anticipatory and posttransformation problem; none of the others did. Ma was the only one of the four who correctly predicted the transformation effect on the second problem sequence. In addition, Br was the only one who had an adequate systematic method (rule) for comparing numbers of chips. She stacked them into two piles and compared the heights. Correspondingly, she was the only one who was successful on the posttransformation comparisons.
Height Conservation: Negative Transfer

Whereas the anticipatory problems involving number are properly viewed as providing evidence of positive transfer to new conservation concepts, those involving height and extended liquid test for the presence of two kinds of negative transfer (horizontal decalage). Since the transformation involving height entails both adding and turning, the children who had learned the anticipatory higher-order t rule would be expected to predict that the turned object would have more height when, in fact, the reverse was true. With only minor exceptions, these children all knew an adequate height CC rule. Hence, one might reasonably expect many of the children to change their minds after actually seeing the transformed objects.

With a margin of error, this is exactly what happened. Six of the seven children who had achieved anticipatory criterion incorrectly predicted that the extended B object (pen with added top) would have more height. (De was the exception.) Of these six, four switched answers on seeing the A-B' objects after the transformation. Of the others, An appeared to sense the incongruity ("They don't look the same.") but was reluctant to contradict herself ("Turning doesn't change height, adding does.") Ro appeared to maintain the same response because he had defined both the anticipatory and posttransformation problems in terms of comparing the A and B objects "when they are [both] standing up."

By way of contrast, Sp, Ma, and Ra (who had not reached criterion on the anticipatory higher-order t rule) all predicted the effects of the transformation correctly. Having also learned the height CC rule, they maintained their predictions on the posttransformation comparisons. Mi and Br thought that B' would have more before the transformation, but they changed their minds after actually seeing the A-B' objects. Not having been exposed to the anticipatory higher-order t rule, for example, Mi did not know which types of transformations affect height (for example, "adding changes height, turning changes height").

Extended Liquid Conservation: Inadequate CC Rule

Similarly, pouring liquid into a tall, thin vessel (that is, into a differently shaped container) might be expected to mislead children who had learned to anticipate effects based on adding/taking away. This time, however, the main difficulty would be expected after the appearance of the posttransformation "illusion"; that is, before actually seeing the results of the pouring, the criterion children would be expected to predict that the A-B' objects would contain the same amounts of liquid. After the transformation, on the
other hand, the restricted liquid CC rule (by design) would be expected to mislead children into thinking that the tall vessel contained more liquid.

This time, of the seven who had achieved the training criterion, all but An correctly anticipated that the posttransformation objects would contain the same amount of liquid (see Table 6D). After the transformation, however, only Ti, of the six who anticipated correctly, refused to be influenced by appearances—although it was clear that he, too, was surprised by what he saw—"I could tell by looking before [the transformation]; to change you have to pour some out." Ro and Ty initially were misled into thinking that the shape of the container affected the amount of liquid, but consistency requirements eventually convinced them to correct their mistakes. For example, Ro said, "This one looks like it has more but they were the same before and you poured out every bit." In changing her answer, Ty asked the experimenter to redo the transformation because she "couldn't see how it could have more in another glass." (To avoid demonstrating reversibility, the experimenter repeated the experiment with new glasses and fresh Kool-Aid.) Ni also was surprised. He did not understand "how it could do that," Da and De, on the other hand, thought the taller vessel contained more liquid because "it's higher up" (De) or "I drank out of a glass like that and had more" (Da). For An, of course, actually seeing that the liquid was higher in the tall vessel simply reinforced her original (erroneous) expectation—"If you change a cup to skinny you have more."

Interestingly, Sp responded as did An. He both predicted and maintained that B' had more than A because "when you pour it into a glass it goes all the way up here." Prior experiences clearly were dominant over both An's (relatively complete) and Sp's (inadequate) anticipatory higher-order t rules. (Note that dominance refers to the availability of higher-order selection rules, which, in this context, give preference to prior experience. For discussion of related issues, see Scandura, 1973, chap. 8.) Of the other four children who failed to achieve criterion during the anticipatory training (Mo did not get the item), the only correct response was that given by Mi on the anticipatory problem. Even here, it appeared that this was simply a matter of guessing. Mi proposed two different answers, stating after each of them that she "didn't know why."

Balance Beam

The results on the balance beam problem may be explained similarly, although here generalized prior knowledge also may have
been sufficient for this purpose. On the anticipatory task, only Sp and Ra correctly, but inexplicably, predicted that the heavy side would go up.* Neither child had completed the training. All of the other children incorrectly predicted that the heavy side would go down. After the shield was removed, all of the children obviously were able to indicate which side actually had gone down. Perhaps more relevant, five of those who had completed the training gave reasons that clearly indicated their awareness that distance (from the fulcrum), as well as weight, is important in determining "turning" power.

Length Conservation

In spite of the above successes, the results on the length conservation posttest indicate that conservation-related predictions cannot be made indiscriminately. Specifically, this posttest item (more accurately, series of items) was administered as on the pretest, so that the items used are not directly comparable to what was taught during the training. It would seem unwise, therefore, to attempt to draw significance from the slight upward trend (mean $L = 3.17$) evidenced on the length posttest scores (as compared to the length pretest; mean = 4.9). To say anything even partially definitive, one must look at specific test problems associated with length conservation. When this is done, it becomes clear that the children were able to select from a set of sticks the one having the same length as a given one. In effect, all of the children were able to compare the objects correctly before the transformation. If a child failed on this task, his path score could be no better (that is, lower) than L6. (All children scored at L4 or better.)

The following key questions (problem sequence) distinguished between children who scored at levels L0 or L1 and L2 to L4. First, the experimenter moved the (B) stick horizontally so that one end was further to the right than the other (A) stick and asked: "Which one is longer? Why?" Second, the experimenter focused the child's attention on the "opposite endpoint" of the transformed stick by asking, "What about this?" The expectation was that a true conserver would give a type of compensation reason.

*This problem was first introduced on an ad hoc basis to Mo (just before she left school) by a relatively unfamiliar experimenter. With a gleeful look in her eye (as was typical), Mo seemed to purposely give what she felt was an incorrect answer. After the shield was removed, Mo first looked surprised, then seemed to gather herself and said calmly, "See, I told you."
<table>
<thead>
<tr>
<th>S</th>
<th>Extended Liquid Anticipatory Posttest</th>
<th>Balance Beam Anticipatory Posttest</th>
<th>Length Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>An</td>
<td>&quot;it's skinny so changes to more&quot;</td>
<td>&quot;measured; cup = fat; glass = skinny if change to skinny have more&quot;</td>
<td>left side didn't know why</td>
</tr>
<tr>
<td>Ro</td>
<td>A=B &quot;had same before&quot;</td>
<td>&quot;looks like has more; A=B had same before and poured every bit&quot;</td>
<td>left side moved heavy one to end saying, &quot;If moved over would be heavier&quot;</td>
</tr>
<tr>
<td>Ni</td>
<td>A=B &quot;turning makes it still have same&quot;</td>
<td>&quot;(surprised) didn't understand how pouring made&quot;</td>
<td>left side because &quot;more weight on right&quot;</td>
</tr>
<tr>
<td>Ti</td>
<td>A=B didn't know why</td>
<td>&quot;tell by looking before; to change, must pour some out&quot;</td>
<td>left side &quot;if moved to #7 peg would go down&quot;</td>
</tr>
<tr>
<td>Da</td>
<td>A=B &quot;didn't pour in or take out any&quot;</td>
<td>&quot;tall and not fat like cup; drank from glass like it and had more&quot;</td>
<td>left side &quot;heavy one closer to center&quot;</td>
</tr>
<tr>
<td>Ty</td>
<td>A=B &quot;just poured&quot;</td>
<td>&quot;poured into different glass&quot;</td>
<td>left side &quot;light side should go down, heavy shouldn't; had to put on same peg&quot;</td>
</tr>
</tbody>
</table>

TABLE 6D
Summary of Individual Results: Experimental Subjects (Additional Transfer Conservation Tasks)
<table>
<thead>
<tr>
<th>De</th>
<th>A=B</th>
<th>A&gt;B</th>
<th>right side</th>
<th>left side</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&quot;pouring does not change height; adding taking away does&quot;</td>
<td>&quot;B higher up&quot;</td>
<td>&quot;heavier weight&quot;</td>
<td>&quot;put heavy one on; peg #7 would go down&quot;</td>
<td></td>
</tr>
<tr>
<td>Proportions correct</td>
<td>6/7</td>
<td>2/7</td>
<td>0/7</td>
<td>7/7</td>
<td></td>
</tr>
<tr>
<td>Mo</td>
<td>did not get task</td>
<td>did not get task</td>
<td>left side</td>
<td>left side</td>
<td>did not get task</td>
</tr>
<tr>
<td></td>
<td>&quot;heavy one too heavy to go down&quot;</td>
<td>&quot;heavy one too heavy&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sp</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>left side</td>
<td>left side</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>&quot;poured cup into glass&quot;</td>
<td>&quot;poured cup into glass; it goes all the way up there&quot;</td>
<td>&quot;right side heavier; middle made it go down; right heavier&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportions correct</td>
<td>0/1</td>
<td>0/1</td>
<td>2/2</td>
<td>2/2</td>
<td></td>
</tr>
<tr>
<td>Br</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>right side</td>
<td>left side</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>&quot;taller glass&quot;; A&gt;B: &quot;didn't add or take away&quot;</td>
<td>&quot;had more&quot;</td>
<td>&quot;added another weight&quot;</td>
<td>didn't know why</td>
<td></td>
</tr>
<tr>
<td>Ma</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>right side</td>
<td>left side</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>&quot;because glass is bigger&quot;</td>
<td>&quot;the way up&quot;</td>
<td>&quot;added on; has heavier weight&quot;</td>
<td>didn't know why</td>
<td></td>
</tr>
<tr>
<td>Ra</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>left side</td>
<td>left side</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>&quot;poured every bit into glass&quot;</td>
<td>&quot;poured little cup into big glass&quot;</td>
<td>&quot;not heavier; go up because it is heavier&quot;</td>
<td>he was right before</td>
<td></td>
</tr>
<tr>
<td>Mi</td>
<td>A&gt;B</td>
<td>A&gt;B</td>
<td>right side</td>
<td>left side</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td>didn't know why</td>
<td>&quot;because you added some&quot;; didn't know why</td>
<td>&quot;it's got more on&quot;</td>
<td>&quot;too heavy to go down&quot;</td>
<td></td>
</tr>
<tr>
<td>Proportions correct</td>
<td>1/4</td>
<td>0/4</td>
<td>1/4</td>
<td>4/4</td>
<td></td>
</tr>
</tbody>
</table>

*aCircle indicates error.
*bDashed circle indicates initial error with subsequent correction.

Source: Compiled by the authors.
Although the children could have been taught how different variables may compensate for one another, instruction of this type was not included in the experimental training. In effect, the critical problems concerning length conservation not only involved a new concept (length) but a different type of question requiring children to make more subtle discriminations.

Criterion versus Noncriterion Experimentals

As might be expected, the differential success in training the criterion and noncriterion experimentals was directly reflected on the posttest. All of the experimental children reached criterion on the CC rule training. Not surprisingly, there were no significant differences between the two subgroups on the CC posttest tasks. Inversely, none of the noncriterion experimentals succeeded in mastering the anticipatory higher-order t rule. This fact is reflected dramatically in the posttest results. The criterion experimentals performed significantly better on the conservation-related problems involving area, amount, number (first and second), height, and extended liquid. (Exact probabilities were $p < .004$, $p < .004$, $p < .03$, $p < .002$, $p < .04$, and $p < .025$, respectively. The difference involving length conservation also was significant [$x^2 = 2.857$, $df = 1$, $p < .05$].) The difference on liquid conservation was in the same direction but only approached significance (Yates $x^2_c = 1.886$, $df = 1$, $p < .10$), probably because this was the easiest conservation task—almost all of the children had received some higher-order t rule training.

On the other hand, there were no significant differences on the balance beam problems. This tends to support the hypothesis made above to the effect that generalized prior knowledge (for example, heavier things go down) might be sufficient to explain the posttest data independently of what was learned during the training.

Discussion

It would appear that these results are highly compatible with our expectations. We were successful in training all of the children on the CC rules for (restricted) liquid, area, amount, and height, and they performed correspondingly well on the associated CC problems.

Those children who completed training on the anticipatory higher-order t rule also were successful on the anticipatory and posttransformation comparison (conservation) problems for liquid, area, and amount. Although these results are not surprising in themselves (explicit training was provided on the tasks), the attained levels of performance were relatively high in comparison with those of most Piagetian training studies.
Performance on the transfer tasks was even more satisfying. While some degree of transfer has been obtained in some training studies, none to our knowledge has succeeded in explicitly manipulating both positive and negative transfer. Moreover, the predictive detail and the strength of the obtained effects appear unique. They are equivalent to the best results obtained with far more prescribed classes of tasks, typically with older children, and appear to exceed those normally associated with Piagetian training studies. Thus, for example, while all of the criterion children were successful on the anticipatory problems involving number, which was expected, some of these children had difficulty with the postcomparisons. In most cases, these exceptions could be traced to specific inadequacies in available number CC rules. (Recall that no explicit training was provided on number CC rules. The number CC tasks essentially served to measure the prior availability of such rules.)

Other instances of positive and negative transfer were equally compelling. The criterion children clearly overgeneralized the anticipatory higher-order t rule based on adding/taking away and applied the rule to height and the balance beam, where it did not apply. The transformation involving height included "adding on," so it is not surprising that the children predicted an increase in height. Given the availability of an appropriate CC rule, however, they noted the inconsistency and wanted to change their responses. This pattern of results was also evident with the balance beam tasks.

On the extended liquid problem, on the other hand, the correct expectation was that pouring would not change the amount. However, having learned an inadequate, restricted CC rule for liquid, all but one of the children changed their answers after actually viewing the cup and the tall, thin vessel. Even the other child was clearly surprised by the result.

The obtained results on length conservation (and some of the balance beam results) make clear the value of precise specification of underlying competence in quite another way. Specifically, such specification makes it possible to distinguish between problems where performance can be predicted and problems where this is not possible. In the present case, the set of training rules was inconclusive when applied to the critical length conservation problems. Although the training rules did not take compensation problems into account, however, it is clear from the analysis that these "extra-domain" problems could have been considered.

Normative (Comparative) Results and Discussion

As noted above, the major purpose of this study was to test the adequacy of the proposed structural analysis for training pur-
poses. Nonetheless, it also is of some interest to compare overall levels of test performance of the experimental and control groups and of those experimental children who completed the training (criterion experimentals) and those who were classified initially as conservers based on their pretest performance (natural conservers). (In the latter comparisons, although the results are felt to be representative, only three of the original four conservers were available for posttesting.)

First of all, it should be noted that the exploratory training, which all of the experimental children received (in varying degrees) and which half of the controls received (in yoked fashion), had no measurable effect in relation to the posttest. Aside from minor nonsystematic differences on individual tasks, the overall difference was small and, in fact, favored those who did not receive the exploratory training (53 percent versus 44 percent). In the following analyses, the control group was considered as a whole.

**Trained CC Problems**

Since the experimental group received explicit training on CC rules for (restricted) liquid, area, amount, and height, they would be expected to do better on the average than the controls, and, in fact, they did (see Table 7). The experimental group solved an average of 93 percent of these tasks (96 percent, 100 percent, 88 percent, and 87 percent, respectively), whereas the controls solved 73.5 percent (78 percent, 78 percent, 75 percent, and 63 percent). The differences for liquid ($x^2 = 7.432, df = 1, p < .005$), area ($x^2 = 12.969, df = 1, p < .001$), and height ($x^2 = 7.183, df = 1, p < .005$) were all highly reliable. That for amount also was significant ($x^2 = 2.852, df = 1, p < .05$). (Note that one should not be misled by the relatively high level of performance by the controls. The first CC problem in each set of four corresponded to initial comparisons in traditional conservation tasks. The experimental children performed at a 100 percent level on all such tasks, with the controls not far behind. Moreover, one other problem in each set was a "give away"; a correct response would be indicated perceptually as well as by an appropriate CC rule. [Guessing alone, of course, would give an average score of 50 percent.])

The natural conservers performed on the liquid CC tasks at a level comparable to the criterion experimentals (92 percent versus 100 percent). Observing that the restricted form of liquid tested was one-dimensional and, hence, directly equivalent to length, and recalling that the natural conservers had performed successfully on length conservation tasks, this result is not surprising. The difference favoring the criterion experimentals was increasingly larger.
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<td>26/28</td>
<td>26/28</td>
<td>19/28</td>
<td>14/14</td>
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<td>14/14</td>
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<td>7/8</td>
<td>6/8</td>
<td>5/8</td>
<td>4/4</td>
<td>4/4</td>
<td>2/4</td>
<td>1/2*</td>
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<td>8/16</td>
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<td>75%</td>
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<td>15%</td>
<td>20%</td>
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<td>4.4</td>
<td>17/20</td>
<td>15/20</td>
<td>13/20</td>
<td>13/20</td>
<td>13/20</td>
<td>0/10</td>
<td>0/10</td>
<td>2/10</td>
<td>2/10</td>
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<td>2/10</td>
<td>5/10</td>
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<td>16/20</td>
<td>17/20</td>
<td>12/20</td>
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<td>5/10</td>
<td>L = 3.4</td>
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<td>L = .67</td>
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*One or more subjects in the group were not available for testing on the problem.

**Note:** Since all Ss succeeded uniformly on all initial (equality) comparisons involving the conservation-related problems, the percentage scores are based solely on the anticipatory and postanticipatory problems.

**Source:** Compiled by the authors.
with respect to the two-dimensional concept of area (100 percent versus 83 percent \(x^2_C = 2.03, df = 1, .05 < p < .10\)), the three-dimensional concept of amount (93 percent versus 75 percent \(x^2_C = 2.449, df = 1, .05 < p < .10\)), and the more subtle concept of height (93 percent versus 50 percent \(x^2_C = 7.1502, df = 1, p < .005\)).

**Number CC Problems**

Except for the early exploratory training, none of the children received training on the number CC tasks. Hence, it is not surprising that the experimental and control groups performed at about the same level (62 percent and 63 percent, respectively). It is not entirely clear, however, in view of their pretest performance on number conservation, why the three conservers only performed at about the same level (58 percent). One possibility is that the pre-test was concerned with number in only a qualitative sense, that is, the children only had to determine whether A (or B') had more/less/same number of elements, given the initial state and the nature of the transformation. The number CC tasks, on the other hand, required a precise quantitative assessment of the elements, a task in which young children are prone to errors, careless or otherwise. In effect, conservation of number may not necessarily guarantee nor require one-to-one matching per se as long as children have some more qualitative means for comparing numbers (for example, mental grouping) that is relatively resistant to visual illusion.

**Trained and Number Conservation**

Clearly, the experimental children, on average, would be expected to do better on the trained, conservation-related tasks. (Since all children succeeded uniformly on initial equality comparisons involving conservation-related problems, the percentage scores reported in Table 7 are based solely on the anticipatory and posttransformation problems.) In fact, the experimental group did perform at significantly higher levels than the controls on (restricted) liquid conservation (88 percent versus 15 percent \(x^2 = 24.824, df = 1, p < .001\)), area conservation (77 percent versus 20 percent \(x^2 = 14.679, df = 1, p < .001\)), and amount conservation (77 percent versus 20 percent \(x^2 = 14.679, df = 1, p < .001\)). Interestingly, the criterion experimentalists (who completed the training) did better than the conservers (100 percent uniformly versus 67 percent, 67 percent, and 50 percent, respectively). Even with the small number of cases involved, the first two differences approached significance (Yates \(x^2_C = 2.1431, df = 1, .05 < p < .10\)), and the third was significant at the .025 level (Yates \(x^2_C = 4.781\)).
These data simply provide more evidence for the well-known phenomenon of horizontal decalage.

These results also make clear empirically what is already clear from our rule-based analysis; namely, conservation knowledge that is acquired "naturally" need not be more general than knowledge that is acquired through direct instruction (compare Roughhead and Scandura, 1968). Moreover, the criterion experimental not only did better than the conservers on the liquid, area, and amount conservation tasks but also performed at about the same level on the two number conservation tasks (79 percent and 93 percent versus 83 percent and 100 percent, respectively).

Interestingly, whereas the experimental group performed at a higher level than the controls on the first number conservation problem (58 percent versus 28 percent \(x^2 = 3.839, df = 1, p < .025\)), both groups performed at the same level (67 percent) on the second number conservation problem. In effect, the performance of the control group was relatively more influenced by differences in the problems than were the experimental.

While it is not possible to give definitive reasons for this difference, we would be inclined to seek resolution in terms of relationships between the task requirements and what the two groups had learned. From this perspective, there is little reason to expect that the anticipatory problems would differ in difficulty for the experimental group (at least for those children who had learned the anticipatory higher-order \(t\) rule). Similarly, comparing the post-transformation \(A-B\)' sets in terms of number would be expected to parallel in difficulty the number CC problems. Judging from the reasons given, the controls found it easier to anticipate the effects of adding two elements (and rearranging), as in the second problem, than of rearranging alone, as in the first problem. This also influenced their posttransformation behavior in the same direction. Why should this be the case?

Again, it is hard to be definitive, but number conservation is different from the other concepts tested in that there is a closer relationship between the kinds of transformations that change number (for example, adding elements) and number comparison rules (for example, counting). For instance, compare the relationship between adding elements (perhaps one by one) and counting, on the one hand, and, on the other hand, pouring liquid and measuring the height of liquid in a cup.

While the above explanation may appear to belabor a minor result, it is illustrative of a more basic and general point. If one has reliable data and goes to the trouble of finding out both what competence must be known and what individuals in a group are likely to know, then predicting (as well as explaining) normative behavior
often follows directly—even in situations where this might not immediately appear to be the case, such as that above. (This is not to say, of course, that other factors, such as processing capacity, might not also be involved [Case, 1978; Scandura, 1977b].)

**Height and Extended Liquid Conservation**

Turning to the extra-scope height and extended liquid conservation problems, we find little overall difference between the experimental and control groups (54 percent versus 50 percent and 38 percent versus 25 percent). Separate analysis of the anticipatory and posttransformation problems, however, indicates some important differences. The experimental group received training on the anticipatory higher-order t rule based on adding/taking away and, as such, would be expected to do relatively badly on the anticipatory problem for height and relatively well on the anticipatory problem for extended liquid. Thus, in the case of height, the experimentals would tend to predict that the object (pen) with the added cap would have more height, whereas the reverse was true (because the "lengthened" pen was turned toward the horizontal). In the case of extended liquid, on the other hand, they would tend to correctly predict that the quantities of liquid would be the same. Conversely, due to their training on the height CC rule, the experimentals might be expected to do relatively well on the height posttransformation problem. Since no CC rule training was provided for extended liquid, the most reasonable expectation would be for the experimental and control groups to perform at about the same level on the posttransformation problem.

This is precisely the pattern of results that was obtained, even more strongly in the case of those experimental children who had actually completed the training. With regard to height, the experimental group performed at a 33 percent level on the anticipatory problem (the criterion experimentals performed at a 14 percent level) and at a 75 percent level on the posttransformation problem, whereas the controls performed at the 50 percent level on each problem. With regard to extended liquid, the experimental group performed at a 58 percent level on the anticipatory problem (the criterion experimentals performed at the 86 percent level), whereas the controls performed at the 30 percent level. Also as predicted, both groups performed at about the same level on the extended liquid posttransformation problem (25 percent versus 20 percent, respectively).

The pattern of results involving the criterion experimentals and the conservers was very similar. Both groups incorrectly anticipated the effects of the height transformation (14 percent and
EMPIRICAL TEST OF THE ANALYSIS / 117

0 percent, respectively), correctly anticipated the effects of the extended liquid transformation (86 percent versus 100 percent), and performed at about the same level on the extended liquid posttransformation problem (29 percent versus 33 percent). The only difference in level of performance was on the height posttransformation problem (57 percent versus 0 percent). As with the corresponding experimental-control group distinction, this difference was most likely due to the special nature of height and the fact that the experimental group received explicit training on how to make height comparisons.

In each case, the critical differences between the experimental and control (or conserver) groups on both types of problem involved the number of times children switched answers in going from anticipatory to posttransformation problems. The experimentals, for example, switched a total of 14 times (on the two problem types), whereas the controls only switched twice. In effect, by most standard criteria, the experimental children would be more likely to be judged transitional on these tasks. The fact that they gave a greater number of inconsistent responses, sometimes reconciling them and sometimes not, would be presumed, according to Genevans, to motivate the children to reestablish "equilibrium" by reconciling the incongruities. The present point is that such reconciliation is not an all-or-nothing affair but takes place gradually as children acquire new and more complete knowledge.

Balance Beam

The balance beam problems apparently provided a baseline of sorts in that all groups performed at about the same level, both overall (experimentals, 61 percent; criterion experimentals, 50 percent; controls, 50 percent; and natural conservers, 50 percent) and on the anticipatory problem (23 percent, 0 percent, 0 percent, 0 percent) and the posttransformation problem (100 percent, 100 percent, 100 percent, 100 percent). It would appear that essentially all of the children expected the heavier weight to "push down" with greater force than the lighter weight. Of course, all of the children could tell which side went down after actually seeing the posttransformation state.

It would appear, therefore, that whatever had been learned during the experimental training simply overlapped with previously available knowledge, so that behavior of the groups was not affected differentially (compare results of experiment two in Scandura, 1977b, chap. 5). Moreover, the fact that five of the experimental children (all had reached criterion), five of the controls, and two of the (three) conservers gave "good" reasons for switching their answers
suggests that many five-year-old children are already sensitive to (at least certain kinds of) inconsistencies and, if given the opportunity, will seek to resolve them.

Length Conservation

Due to the lack of comparability, the small difference on length conservation in favor of the experimental group over the controls (especially those who completed the training) is probably insignificant (both theoretically and practically). As indicated previously, the critical hurdle children had to overcome involved giving compensation arguments, something that was not addressed during the training and that, consequently, would be unlikely to distinguish experimentalists from controls. (The difference between the controls and conservers on the pretest, however, was maintained on the posttest [exact probabilities, p < .04], suggesting that training did have some effect. Similarly, there was no difference between the criterion experimentalists and conservers.)
SUMMARY AND IMPLICATIONS

The purposes of the present study were to determine the feasibility of conducting structural analyses of significant Piagetian constructs and to determine the utility of such analyses as possible bases for effecting significant and predictable developmental changes in individuals. In this context, however, the most fundamental concerns were neither empirical nor even theoretical, although both were obviously important. Instead, they were largely methodological.

As noted in Chapter 1, traditional experimental methods have been notably successful in dealing with directly observable behavior. But, they have been less so in capturing the essentials of cognitive development generally or of Piagetian theory in particular. Nonetheless, most experimentalists would maintain that all cognitive development is largely a result of learning—learning that takes place gradually over long periods of time. Genevans, on the other hand, and other developmentalists who deny the possibility of reducing cognitive development to learning, have erred, we think, in the opposite direction. While addressing basic developmental phenomena, Piagetian constructs, for example, refer to epistemic ideals that have never been satisfactorily made operational in terms of individual behavior.

The basic questions confronted in this study were whether some reconciliation between these two divergent views is possible and, more specifically, whether the Structural Learning Theory and the approach to research that it represents may help to bring about such a resolution. How have we done? At this point, it seems desirable to summarize briefly what we have accomplished and also to discuss some of the things that remain undone.

SUMMARY

Much of our initial work was exploratory, a sort of groping in the dark, trying to establish a comfortable fit between the precision
characteristic of rule-based structural learning theories, on the one hand, and, on the other hand, the more global, less operational constructs of Piagetian theory and the unpredictable character of young children's behavior. Nonetheless, we learned early on something that we had already suspected. Apparent inconsistencies in children's behavior are often not so much a result of inconsistencies per se but of inadequate specification of the competencies needed to perform on the tasks in question.

Our structural analysis of the conservation domain was designed to provide such specification. In this analysis, we were guided not only by sufficiency considerations but also by necessity. Thus, the mere possibility of generating solutions to conservation problems was not deemed adequate. In addition, the identified solution rules were designed to reflect the Piagetian constructs associated with concrete operational behavior.

In the analysis, the first thing that had to be done was to devise an adequate representation of traditional conservation tasks. In this regard, we found that such tasks involve three separable kinds of problems: pretransformation comparison problems, anticipatory conservation problems (in which judgments of post-transformation comparisons must be based on the results of initial comparisons and observed transformations), and posttransformation comparison problems (which may be viewed as comparison problems and/or as anticipatory problems). The last of the three types of problems, of course, forces the child to make a choice as to preferred method of solution.

The second step in the structural analysis was to select a representative set of conservation problems and to identify rules for solving each of them. In this regard, the anticipatory and posttransformation problems were most crucial. Building on our informal observations and those of others, along with Genevan descriptions of the process, we found that solution rules used by conservers to solve such problems appear to be highly general, automated, and quite simple.

The next step in the analysis was to identify a more basic set of prerequisite rules—rules (including higher-order rules) that one might expect children to learn first and from which conserverlike solution rules might be derived. This search led to the discovery of special purpose solution rules (called anticipatory higher-order t rules) of relatively limited scope (that is, of limited applicability). These special purpose higher-order t rules were adequate for solving particular types of conservation problems, number, for example, but they were keyed only to particular types of transformations.

The requirement that conservers generalize across a wide variety of concepts, then, led to the introduction of higher-order
analogy/generalization rules by which new higher-order t rules (that is, for solving new conservation problems) might be generated from existing ones. Special attention was given to a generalized anticipatory higher-order t rule that was keyed to whether the transformation in question involved adding or taking away. This rule is adequate for solving all anticipatory conservation problems in which the critical concept refers to an "intrinsic" property of the A - B' objects (for example, number, length, and amount are intrinsic properties but not height). The process of automatization also was discussed, as were means of making operational this process in the conservation context.

Subsequent analysis of the higher-order t rules resulted in the identification of still more basic rules. Specifically, in order to learn how to anticipate transformational effects, a child must first know which types of transformations affect which types of concepts. For this purpose, checking consistency (between pre- and posttransformation comparisons) was found to be crucial. What were called consistency higher-order t rules, together with still higher-order rules for translating consistency rules into anticipatory (higher-order t) rules, were shown to be more basic (that is, available to younger children) than the anticipatory rules they produced. Moreover, determining the consistency of (that is, comparing) comparisons actually turns out to be quite analogous to comparing pairs of arrays or objects. Comparing comparisons is more general in the sense that what are being compared are comparisons, rather than objects or arrays. Such comparisons also are more basic in the sense that they involve logical equivalence, rather than specific concepts, such as number, liquid, and so forth.

No attempt was made to extend the analysis to a level more basic than concept comparison rules (for comparing pairs of objects/arrays). These rules provide a baseline of sorts that effectively defines (that is, makes meaningful) the conservation problems. In particular, they provide an independent basis for checking answers generated by the anticipatory higher-order t rules. The present analysis, then, by design would be expected to accurately characterize the cognitive development of only those children who begin already knowing the identified CC rules.

Correspondingly, in the empirical evaluation, the first step was to insure that all of the children knew some prespecified set of CC rules. Given this common baseline, training progressed in accordance with the analysis. Although hardly surprising from an analytical perspective, it is of some interest to note that the children's posttest behavior was highly consistent with the rules on which they had been trained. This was equally true of performance on the CC, anticipatory, and posttransformation problems involving
(restricted) liquid, area, and amount. Moreover, the trained sub-
jects spontaneously gave "good reasons" for their answers, even
though no training in this regard was provided.

Available time to complete the training appeared to be the
main limiting factor. In fact, success levels on the training tasks
were quite comparable to those we have obtained in other rule-based
research, most of which has involved older subjects and/or more
structured content.

What was perhaps less clear was what would happen on the
transfer problems involving number, height, extended liquid, and the
balance beam. To be sure, the training was designed to help insure
that the children would generalize the higher-order t rules introduced
(for example, by emphasizing analogies). In most cases, however,
trained CC rules also were necessary for solving the transfer prob-
lems. There could be no guarantee before actual testing that the
desired generality of training had actually materialized or that the
trained rules would interact as predicted.

Hence, we were gratified to find a pattern of results that was
almost perfectly in accord with what one might expect on the basis
of the analysis. Children who had completed the training not only
correctly anticipated the effects of the given transformations on
number comparisons, but their willingness to maintain given an-
swers, as expected, was largely a function of the extent to which
they had mastered number CC rules (for example, one-to-one
matching) that gave results consistent with their predictions. Also
as predicted, all but one of the trained children were moved to
anticipate incorrectly in the case of the height and correctly in the
case of extended liquid. In the former case, the children expected
an increase in relative height when, in fact, the composite trans-
formation in question led to a relative decrease. On extended liquid,
by way of contrast, the original expectation was for no change (which
was correct and as predicted). However, the children changed
their predictions after seeing the results of the transformation, as
would be expected in view of preplanned limitations of the liquid CC
rule introduced during training. The balance beam results were
similar but perhaps more ambiguous, since naturally available
knowledge could have led to the same results. Here, all of the
children incorrectly predicted "which side would go down" before
actually seeing what had happened.

In general, it can fairly be said that the obtained results were
strongly supportive of the analysis—within rather stringent limits—
both absolutely and in comparison with other training studies. More-
over, in view of the complexity of the predicted pattern of results,
it is difficult to envisage parsimonious explanation of the phenomena
that would not be equivalent to the proposed rule-based analysis.
Indeed, in most Piagetian training studies, achieving genuine transfer has been an illusive goal. Attempts to specifically manipulate the nature and extent of such transfer have to our knowledge been essentially nonexistent.

Although normative (group) effects were of secondary interest, performance of the control and conserver groups provided useful bases for comparison. The control-experimental group comparison provided a normative measure of the influence of training. That involving the conserver and criterion-experimental groups provided some estimate of the extent to which the proposed analysis differs from the "natural" course of cognitive development. Interestingly, these comparisons resulted in both positive and negative training effects. In essentially all cases, these effects were in accord with the aforementioned structural analysis. Similarly, the responses given by the trained children, including the reasons they gave to justify apparent inconsistencies, not only were predictable but were, in most cases, indistinguishable from those of the "natural" conservers. All in all, the performance of both the trained and the conserver groups, decalage and all, was quite compatible with what one might reasonably expect of transitional, nearly conserving children.

LIMITATIONS

Perhaps the most obvious apparent limitation of the present research lies in the relatively small numbers of subjects. How, one might ask, can we make the rather strong claims we seem to have made on the basis of working with only 13 subjects during the training, only seven of whom were able to actually complete the training in the time available? The answer we would give to this question has little to do with statistical considerations. The samples we chose had primarily to do with what was both feasible and sufficient for our purposes. (More detailed design considerations are addressed in the body of the study.)

The main purpose of our study was to demonstrate the viability of the rule-based analysis as a means both of characterizing the behavior of conservers and of guiding the training of nonconservers (who are nonetheless already capable of making needed concept comparisons). In this regard, our main concerns have been with the individual child and with how that child performs on particular problems.* (Normative data involving group comparisons were of

*To be sure, the analysis did not account perfectly for the children's behavior down to the last data point. For one thing, the
strictly secondary interest.) Consequently, it was essential to work intensively with individual subjects in carefully prescribed ways over relatively long periods of time. This type of involvement simply would not have been feasible in working with the kinds of larger groups characteristic of traditional normative studies. Even so, the overall pattern of obtained results would be statistically significant under any reasonable set of assumptions. Almost all of the critical normative differences were large in magnitude, for example, and most were statistically significant despite the relatively small numbers involved.

A second major limitation of the research has to do with the nature of the concept comparison rules introduced during training. Specifically, there is no guarantee that these particular rules are equivalent to the CC rules children naturally acquire as they grow and develop. Correspondingly, the CC rules taught may to some extent have been artificial in the sense of not being directly relatable to children's existing cognitive structure (that is, to what they already know). This comment, of course, in no way bears on the validity of the present analysis and results as they pertain to the acquisition of conservation behavior in children who enter with the requisite CC rules. However, it does limit the applicability of the results in the sense that details of the acquisition and performance capabilities of other children may be expected to differ from those of our experimental subjects just to the extent that the CC rules introduced in this study differ from those that might occur naturally.

One might be tempted to fashion a similar argument regarding the anticipatory higher-order t rule that was taught. This rule was based solely on adding or taking away and was only partially adequate, given the range of conservation problems introduced. (It generated, for example, incorrect answers when applied to height conservation and the balance beam.) There can be no guarantee that such a rule actually is acquired by children during the course of natural development. (As a minimum, they almost surely would learn other, partially overlapping rules as well.) On the other hand, a strong case can be made for the proposition that many children who conserve

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analysis was admittedly incomplete, so that behavior on some tasks was neutral with respect to the analysis. Definitive predictions could be made in most cases, however, and here the vast majority of data points were consistent. Moreover, even where this was not the case, the deviations were well within the range of tolerance one might realistically expect for any training program. Elsewhere, such deviations have been referred to as "deviations from the ideal" (Scandura, 1977b, esp. chap. 1).
on some concepts and not on others (something that seems to be characteristic of all children at one time or another) may learn precisely such a higher-order rule, or at least some behavioral equivalent thereof. (Note that one rule is behaviorally equivalent to another if both generate identical behavior.) Among other things, the close parallel observed between the behavior of the criterion subjects and the natural conservers would argue in this direction.

In general, the present research served solely to evaluate the identified rules as a basis for promoting the training of young children operating at what Genevans would consider a pretransitional stage. Although this is an important accomplishment, the study addressed only indirectly the question of compatibility of the identified rules with those rules children normally acquire as they develop cognitively. Put differently, it is not entirely clear, on the basis of our research, whether the identified rules would serve as useful prototypes for the operational definition of individual knowledge (see the section on Structural Learning Theory in Chapter 1). True, evidence that suggests equivalence after training was cited above (for example, the trained experimental-conserver comparison). From a structural learning point of view, however, such evidence would not be deemed definitive (Scandura, 1977b, Appendix A). To determine diagnostic adequacy, it would be essential that rules of competence be used to devise explicit tests for measuring individual differences at arbitrary stages (in relation to the population in question) and that the reliability of these tests be determined empirically. (For an introductory discussion of the process, see Scandura, 1977a. A more detailed treatment may be found in Scandura, 1973; Durnin and Scandura, 1973; Scandura, 1977b; and Scandura and Durnin, 1978.)

IMPLICATIONS AND FUTURE DIRECTIONS

Although there have been numerous successful attempts at training specific conservation performance (Brainerd, 1978; Gelman, 1969; Murray, 1978), none of these, we think, has fully reflected the course of natural development (beginning at a preoperational stage). Among other things, the training programs that have been used have resulted in highly restrictive learning of limited generality and/or have been relatively successful but only with children who appear to be operating at transitional stages (Inhelder, Sinclair, and Bovet, 1974).

In spite of the extensive effort that has gone into the task, and although real progress has been made (Gelman, 1977; Cooper at al., 1978; Lawton and Hooper, 1978), we are still a long way from having identified exactly what preoperational children need to learn (in order
to operate at a concrete operational level) and even further from understanding precisely how such learning comes about over relatively long periods of time.

The present analysis and results constitute one step in this direction and help to clarify a number of important issues pertaining to the acquisition of concrete operations, at least insofar as this involves the conservation domain. More than this, however, the study illustrates the sheer complexity of the problem. According to our analysis, the acquisition of concrete operations, as defined in Piagetian theory, necessarily involves a relatively large amount of highly interrelated learning. Global cognitive changes associated with child development will be reduced to more specific and operational terms only gradually and as the result of considerable effort. Toward this end, we view the present analysis and results as constituting only a temporary way station but, it is hoped, one that points in a new and more promising direction.

Specifically, the present study was conceived on the assumption that empirical research alone would be insufficient to achieve real understanding. To guide research in the area, we felt, a comprehensive, yet operational, theoretical superstructure would be critical. (Without some such theory, we feared, it would not only become increasingly difficult to assimilate the growing amount of data demonstrating the content- and context-dependent nature of many Piagetian phenomena but also to understand, predict, and, it is hoped, manipulate cognitive growth.) Unlike most contemporary theories in cognitive psychology, Piagetian theory well satisfies the first requirement; it clearly is suitably comprehensive. The theory, however, is not operational in terms of individual behavior. We suggested initially that the Structural Learning Theory, or more accurately, the class of structural learning theories (Scandura, Appendix A), might better satisfy both requirements. More importantly, a specific structural learning theory was devised (through structural analysis) and was evaluated empirically in order to illustrate the potential of this class of theories as a means of clarifying and making operational Piagetian phenomena.

A major goal of future research will be to build on the present foundation. There is much to be done. For one thing, the structural analysis itself is not complete, even with regard to conservation per se. Thus, only one preautomatized higher-order t rule was specified. During the course of natural development, children may learn other higher-order t rules as well. Which ones are most prototypic of children at various points in the course of their development? This needs to be determined.

Second, restriction of the analysis to the conservation domain raises the obvious question of whether the present rule set might be
extended to encompass other kinds of tasks associated with concrete operations; seriation and class inclusion are just two of the more obvious ones. If structural analysis could be used to identify rules of competence underlying these other subdomains, and there is no reason to suppose that it could not, then there is still the question of integration with the results of the existing analysis. Piaget's insistence that concrete operations involve operating on operations is suggestive of such parallels (see the discussion of seriation in Chapter 1). What needs to be done, however, is to represent such parallels concretely and specifically in the form of higher-order rules.

Despite its generality, the present research appears to be as precise and operational as any we have seen in the area of cognitive development (that is, short of computer implementation of the theory, which would have been superfluous for our purposes). Nonetheless, the present analysis has glossed over several aspects of the competence associated with conservation. The truly important roles played by analogy, generalization, and automatization were discussed, but these roles were not specified in operational detail. A third question arises as to what form these processes might take during the course of child development. Specifically, can one determine prototypic higher-order rules that provide useful bases for explicit training and/or for characterizing the knowledge of individual children—as we think we have done in the case of higher-order t rules?

Our observations also suggest that children are better able to learn/generalize new higher-order t rules (that is, to pass from a preoperational to a concrete operational stage) after corresponding CC rules have been not only mastered but automated. Before automatization, many of the experimental children appeared to have difficulty concentrating on newly introduced conservation tasks, even after they had achieved minimal criterion on the corresponding concept comparison rules. Apparently, merely knowing how to do something does not guarantee that the underlying knowledge may be used effectively as a basis for further learning.

According to a more generalized form of the Structural Learning Theory (Scandura, 1977b, pp. 81-135, esp. pp. 102-110), people have a fixed, limited capacity for processing information. In this view, a newly acquired concept comparison rule may be expected to impose a relatively heavy load on the processor (because it normally will consist of a number of distinct cognitive units, each requiring a separate decision or operation/action). After automation, which is a natural result of practice, the concept comparison rule acts more as a single unit. On subsequent conservation tasks, then, such automation would have the effect of reducing memory load and thereby of freeing more space for learning higher-order t rules.
In effect, once any learned rule has been automated, it can more readily be used as a basis for future learning. Specifically, as in the case of conservation learning, automated rules (that is, CC rules) provide an efficient basis for evaluating potential responses. More generally, in the case of cognitive development, automation may constitute a crucial prerequisite for passage from one stage of development to the next. Thus, it is reasonable to suppose that automation of higher-order rules may be a crucial precursor to passage from the concrete operational stage to the stage of formal operations.*

Fourth, there is no need, of course, to limit structural analysis to concrete operations. Why not use structural analysis as a means of identifying the prototypic rules of competence associated with preoperational stages? Or the stage of formal operations?

In the former case, one major task would be to identify prototypic rules that children might use in making static comparisons on primarily perceptual grounds. What leads preoperational children, for example, to say that a tall, thin glass contains more water than a short, wide one when, in fact, they both contain the same amount? In addition to perceptual comparison rules per se, structural analysis, iteratively applied, might be expected to lead to the more basic prerequisite rules that still younger children use in acquiring preoperational (perceptual) comparison rules.

Once having identified preoperational comparison rules, as they naturally occur, one also would be in a position to extend the present conservation analysis in a more definitive and naturalistic way. In particular, in the natural state, we believe that such rules would act, as our CC rules did, to effectively define conservation tasks for the children. Thus, for example, in the natural state, perceptual comparison rules sometimes provide the basis for defining (and evaluating) anticipatory conservation problems (as well as comparison problems per se).

Moreover, we believe that conservation learning in the natural environment progresses gradually as children observe and learn to anticipate the effects of various transformations on given kinds of comparisons. This assertion, if true, indicates why preoperational, perceptually oriented children have never been trained to conserve in a manner that Genevans could accept. The perceptual rules such

*The postulated course of development is quite compatible with one proposed by Case (1978). Rather than assuming that reduction in memory load is a result of automation, however, Case postulates a generalized increase in processing capacity as children grow older.
children use in making comparisons frequently yield results that are not consistent with the facts as adults know them. Consequently, there is no way that the children could possibly discover reliable bases (rules) for anticipating such comparisons—let alone come to prefer such anticipations in the face of subsequent perceptual illusions. Nonetheless, it is important to observe that even perceptual rules are adequate for making certain types of comparisons.

In general, however, the reliability with which children may learn to anticipate transformational effects cannot be expected to exceed the reliability of the rules they use to evaluate the resulting comparisons. This was readily apparent in the case of the extended liquid conservation problem. In the natural environment, presumably, children may gradually be expected to learn what changes (transformations) lead to what comparisons by a discovery process. Early experiences team up with subsequent ones and eventually lead to the acquisition of anticipatory higher-order transformation rules of the sort specified in the present study. The discovery process, in turn, might well involve higher-order generalization rules acting on various instances, involving transformations and comparisons, to generate more general rules (for examples of higher-order generalization rules, see Scandura, 1974b, 1977b).

In the present study, no attempt was made to parallel the interactive nature of learning as it occurs in the natural untutored environment—largely because we knew that the CC rules introduced in the present study were at best approximations of the knowledge that five-year-old children might normally be expected to bring to a conservation task. One would hope, in the future, that it might be possible to detail some of the interactive nature of the natural acquisition process, as well as to duplicate its effects, as was done here.

One might expect something very similar to occur in the case of formal operations. Rather than increasingly adequate CC rules leading gradually to the acquisition of generalized higher-order transformation rules, however, the same line of thinking might lead one to expect preliminary forms of the latter to lead to the acquisition of still higher-order types of rules.

Consider the case of the balance beam, for example. It is not sufficient in this case that the child at the formal operational level be able to correctly predict which way the balance beam will turn given a particular configuration of weights and distances. In addition, such a child must be able to envisage the results of any possible hypothetical configuration, whether seen or unseen.

Attempting to draw the analogy more precisely, given the results of applying a given higher-order transformation rule to a situation involving a given transformation (compare the results of given the results of an initial
comparison), and some (possible) change in that transformation (compare changing a given comparison through some transformation \( t \)), the child at the formal operational stage should be able to anticipate the outcome of applying the higher-order \( t \) rule to the hypothetical situation (compare anticipating the outcome of applying a CC rule to the results of a transformation). In effect, the child at the formal operational level must be able to anticipate post-transformation concept comparisons associated with arbitrary hypothetical situations.

Moreover, as in characterizing competence associated with concrete operations, the search in any complete analysis of formal operations would be for generalized, concept-independent capabilities. Ideally, the person at the formal operational level must be able to anticipate hypothetical effects on a wide variety of tasks (including conservation of number, height, and so forth) and, more importantly, on analogues of higher-order \( t \) rules associated with seriation, class inclusion, and so forth.

In line with the above, a fifth area for future research involves the development of comprehensive training systems that are fully adaptive to individual needs. In particular, it would be a major theoretical and practical accomplishment to develop explicit instructional "blueprints" that (1) specify precisely, in the form of operational rules, what must be learned, or more accurately, what is learned, by children operating at some particular, given level of development; (2) represent the prototypic competence associated with children operating at some entering developmental stage, presumably lower than that in (1); (3) utilize the results of (1) and (2) to specify adaptive test sequences that make it possible to adequately characterize individual knowledge at various points in the indicated developmental sequence; (4) provide a basis for administering instructions tailored directly to individual needs at each point in the child's development; and (5) allow, in both the instruction and the testing, for the dynamic changes in knowledge that take place as a result of training.

Clearly, this sounds like a lot to accomplish, and it would be. The theoretical rationale for doing so, however, already exists, and various aspects of the implied process have been tested empirically—with respect to nondevelopmental task domains (Scandura, 1977b). Whether it would be feasible to devise a comprehensive system of this sort in the case of children's cognitive development (for example, restricted to Piagetian tasks) is a challenging question that can only be answered by trying. We hope that some of our readers will be motivated to so.
BIBLIOGRAPHY


131


Kuhn, D. Inducing development experimentally: Comments on a research paradigm. *Developmental Psychology, 1974, 10*, 590-600.


Scandura, J. M. On higher-order rules. *Educational Psychologist*, 1974, 10, 159-160. (a)


Scandura, J. M. Measurement sequences, Piagetian structures, and higher-order rules. *Brain and Behavioral Sciences*, 1978. (b)

Scandura, J. M. Theoretical foundations of instruction: A systems alternative to cognitive psychology. Appendix A to this volume.


APPENDIX A

THEORETICAL FOUNDATIONS OF INSTRUCTION: A SYSTEMS ALTERNATIVE TO COGNITIVE PSYCHOLOGY

During the reign of E. L. Thorndike, educational psychology was a natural extension of academic psychology. To be sure, Gestaltists saw to it that Thorndike's law of effect constituted only one major approach, but the bonds of marriage were strong. In the period shortly after World War II, on the other hand, what had been viewed as a continuum from laboratory to educational practice had become a chasm—a situation well summarized by a remark concerning educational researchers often attributed to the late Kenneth Spence: "Give them all pensions. Let them do anything. But don't let them do research." In fact, Spence perhaps was one of the more honest practitioners of academic psychology. He was under few delusions regarding applicability. Studying the white rat might lead to an understanding of the white rat but with respect to education—who knows!

Nonetheless, much was heard during the late 1950s and early 1960s about the science of learning and the technology of teaching. Unfortunately, many educational psychologists accepted this view uncritically. The kinds of questions we asked, the paradigms we used, and our experimental methodologies, all were carbon copies—albeit ten to 20 years removed—of those we adopted from our academic brethren.

Few of us take seriously anymore the contention that instruction is simply a technology that might reasonably be based on S-R learning theory of the early 1960s, no matter how elaborate and updated. Yet, today, with varying degrees of conviction, many academic psychologists, including a high proportion of converted S-R associationists, are trying to convince us that cognitive psychology is the new deity, all the while proclaiming originality and largely ignoring the fact that the contemporary motivation for studying cognition came earlier and from elsewhere—including from educational psychology.

This appendix is based on invited addresses given by J. M. Scandura at the American Psychological Association Meeting in Toronto on August 29, 1978 and at the American Educational Research Association Meeting in San Francisco on April 10, 1979. It is based on research that was supported, in part, by N.I.C.H.D. Grant 9185 to J. M. Scandura.
Why am I being so hard on cognitive psychology—especially since my own work on rule and strategy learning in the 1960s antedates most of the contemporary empirical work in the area? In fact, I do believe, strongly, that people have minds, that they do think, and that a proper understanding of cognitive processes may be of great benefit in improving understanding of the instructional process. My concern lies at a deeper level.

Among other things, I am concerned about basic expectations. In experimental psychology, the traditional tendency has been to devise special purpose theories, to study particular problems individually as if they could be understood in isolation, independently of other aspects of human functioning. Thus, we have had theories of serial learning and paired-associate learning, of short- and long-term memory, and of all the rest. More recently, we have seen the rise of more comprehensive theories of cognitive functioning, most of which have emphasized the role of memory or of language. Nonetheless, when viewed from an instructional perspective, the basic strategy has been one of "divide and conquer." The implicit belief has been and still is, I think, that the whole of human functioning will become transparent once enough different subareas have been understood: phonics, working memory, semantic memory, problem solving, and so forth. This step-by-step approach to behavioral science might aptly be characterized as one of "bricklaying."

In my opinion, any attempt to understand anything as complex as the instructional process by studying its pieces would be at best presumptuous. Without equal attention to the overall architecture, to interrelationships among the parts, piecemeal accumulation is more apt to result in a pile of bricks than in a functioning structure. It is not, then, that I love cognitive psychology less. It is rather that I love instructional systems more. We need to consider not only cognition and other components of the instructional process but also the interrelationships among these components in the context of dynamic interactional systems.

To see why I feel this way, consider the parallel field of medical research. Specifically, let me relay some of the essentials of an ongoing dialogue I have been having with a friend who is trying to find the cause of emphysema, a debilitating lung disease. Toward this end, he is committed heavily to piecemeal and painstaking experimental research. Oh, he knows about Watson and the double helix and other important and integrating theoretical contributions to biology, but he strongly believes that the answers he is seeking will come from highly directed empirical research rather than from more global theoretical insight.

With regard to instruction, my own views are quite the opposite. We have had literally generations of piecemeal empirical
research in education, but progress has been at best marginal. All too frequently, the findings either have tended toward vacuous truisms or have been nonreplicable.

In any case, this friend and I were discussing the relative merits of theory and research one day when the question arose as to why he felt so confident of his present empirical course. He answered, "Because when we find the cause we will know it." Think about that for a moment! How often is it that we can say on the basis of our empirical research, "Now I know why, precisely why, Johnny can't add, or spell, or read"? Of course, the literature is filled with research that purports to provide answers to such questions, but all is partial and very little definitive. More important, the answers proposed rarely allow us to do much of anything about overcoming these cognitive deficits, much less do it reliably and with assurance.

Consider some of the reasons for this difference. First, a good deal is understood about proteins, human tissue, and sundry other biological details. The basic form of the theory is well established. What needs to be done in the case of emphysema is to devise a specific realization of this basic theory—to show specifically how emphysema may be explained in terms of existing principles and constructs. My friend is banking on the assumption that no basically new theoretical principles or constructs will be needed. His task in this respect is made easier by the fact that many key theoretical constructs are realized in concrete biological materials: cells, proteins, and the like.

Second, emphysema and many other well-known diseases apparently have specific causes that, when determined, can be treated rather independently of other aspects of the human biological system. (At best "side effects" are considered unavoidable annoyances that are necessary to effect desired cures.)

This situation simply does not obtain with respect to instruction. There is little agreement among educational psychologists as to what are the basic principles and constructs, much less as to what might constitute an adequate basic theory. Equally important, many of the most important instructional problems cannot be dealt with statically and in isolation. Overcoming educational deficits may involve both testing and teaching in complex dynamic interactions. (Incidentally, it may be worth noting that medical researchers also are now beginning to appreciate how little they know about how complex biological systems operate to maintain health. Such awareness has come about gradually as hoped for breakthroughs failed to materialize, for example, in the well-publicized "war on cancer.")
My main point is that what we need in educational psychology is not just more empirical research and is not just warmed-over versions of theories developed by our brethren in cognitive psychology—or, for that matter, theories in any specialized academic discipline. For too long, educational psychology has looked to others for its intellectual nourishment. If educational psychology is to become a viable discipline, if we are to have viable instructional theories—practical theories that will elucidate—then we would do better to build them from within than to wait for others to lead the way. Only when we have accomplished this, I think, will other scientists, and lay people as well, begin to take educational research seriously. The alternative, I think, is to remain, paraphrasing Jackson's (1977) remarks, "a vast and varied domain of stagnant waters, with a long past and no future."

In the following pages, I shall describe, first, the essentials of what I consider to be a scientifically and practically viable approach to instructional theory. (In the process, I will briefly summarize some of the theoretical and empirical progress that has been made to date based on this approach.) Then, I will contrast some basic features of this approach with those more traditionally associated with cognitive psychology.

**ESSENTIALS OF INSTRUCTIONAL THEORY**

Any viable theory of teaching and learning must include, first of all, some way of specifying what must be learned, that is, some way to represent knowledge. Second, any viable theory must elucidate the processes by which people use, acquire, and modify existing knowledge. Third, there must be some way to find out what individuals know at any given stage of learning, including a way to determine their initial knowledge. In addition, a fully adequate theory of teaching and learning must allow for the growth of knowledge over time as learners interact dynamically with a changing teaching environment.

During the past two decades considerable progress has been made in the above direction. Thus, considerable progress relating to various aspects of the above has been made in such fields as artificial intelligence (Minsky and Papert, 1972; Bobrow and Collins, 1975), individual differences measurement (Glaser, 1963; Hively, Patterson, and Page, 1968; Cronbach and Snow, 1977), and cognitive psychology (Kintsch, 1974; Anderson, 1976), as well as in educational psychology per se (Gagne, 1962; Merrill and Boutwell, 1973; Merrill, 1978; Glaser and Resnick, 1972; Rothkopf, 1972; Tennyson, 1977).
There also have been important developments in dealing with the instructional process as a whole (Pask, 1976; Landa, 1976) and with relationships to general systems theory (especially Pask, 1976). Specifically, significant progress has been made in understanding the interrelationships among content, cognition, and individual differences and in the way they interact over time as a result of instruction.

Global considerations, of course, necessarily play some role, even in the most prescribed research, as does actual human behavior in global, systems-oriented theories. Nonetheless, the extent to which "top-down" considerations have influenced the former and the extent to which "hard data" have influenced theorizing about instructional systems have generally been quite limited.

Somewhat orthogonal to the above dichotomy has been the widely sensed gap between theory and practice (Scandura, Frase, Gagne, Green, and Stolurow, 1978). Typically, theories associated with the various academic disciplines have been perceived as having at best peripheral relevance to instruction. On the other hand, pragmatically generated teaching techniques and/or design principles have been largely devoid of theory.

To date, I am aware of only one theory that has seriously probed the "no-man's-land" between these alternative views and concerns. It may come as no surprise, in this regard, that I am referring to my own Structural Learning Theory (Scandura, 1971a, 1973, 1977b). This theory and a rather large body of supportive empirical research have been well documented in the literature, most recently and comprehensively in my books on structural learning and problem solving. I shall not attempt to survey this literature here.* Rather, in order to provide a basis for comparison

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*It is impossible within the space of a few pages to summarize adequately even the main features of the Structural Learning Theory. Early beginnings, followed by a large body of related research on rule learning, appeared in print as early as 1962 (Scandura, 1962, 1964). The earliest systematic presentation of the theory (Scandura, 1971a), although somewhat outdated, still provides perhaps the best introduction—although Scandura (1977c) also provides a useful survey. The first volume on structural learning (Scandura, 1973) provides a relatively formal treatment, but the more recent book on problem solving (Scandura, 1977b) provides perhaps the clearest version along with important refinements, extensions, and applications to education. The most recent book, edited by Scandura and Brainerd (1978), and the Journal of Structural Learning (vol. 6, no. 4) include instructive commentaries on problem solving.
with contemporary theorizing in cognitive psychology, I shall emphasize essential historical, global, and methodological considerations, with special attention to relationships to the instructional process.

Historically, development of the Structural Learning Theory was motivated by instructional considerations. Specifically, the goal of my very first piece of serious research (Scandura, 1962, 1964) was to help clarify the roles of expository and discovery modes of problem-solving instruction. What I found was that it is essentially impossible to obtain reliable results no matter how precisely one attempts to specify instructional treatments. More critical than how information was imparted was when that information was given in relationship to what learners knew at the time. If presented too early, pupils not only were unable to use the information, but also they gradually learned not to attend when presented with subsequent information. This research, incidentally, employed a combination of informal observation and analysis, supported by precise operational definitions and experimental methodology not unlike that which characterizes much contemporary cognitive psychology.

 Nonetheless, although certain analytical tools were used (for example, the use of algorithms to represent what was to be learned), a major problem with this research was the inability to make operational individual knowledge. Specifically, it was difficult to tie in the phenomena being studied with the S-R and concept-learning studies, or with the computer simulation studies, of the day. Given what seemed to me to be an inadequate S-R language and unnecessarily cumbersome computer programs, I turned my attention in the early and mid-1960s toward the development of a simple, but suitably general, scientific language for theorizing about such phenomena.

 Others during that period, most notably Gagne (1965), also were concerned with clarifying relationships between simple S-R and more complex kinds of learning. Rather than attempting to represent rule, problem solving, and other complex forms of learning as complications of S-R learning, however, it seemed to me both more parsimonious and more useful to take the rule as basic and to explain simpler types as special cases (Scandura, 1967a, 1970b). (Equally important, this type of formulation appeared to be considerably more precise, thereby making it possible to avoid certain problems that arise, for example, in attempting to represent rules or principles in terms of concepts or associations [Scandura, 1967a, esp. p. 339; 1970b, pp. 517-521].)

 In the set-function language developed as a result of this work, the emphasis was on sets of observable input-output (stimulus-
response) pairs and on rule (function) constructs needed to explain how outputs were to be generated from the inputs. Specifically, rules were characterized as triples, each rule having a domain, or set of conditions to be satisfied by inputs; a range, or set of anticipatory conditions characteristic of the outputs the knower expects the rule to produce; and an operation or procedure (algorithm), which, when applied to inputs in the domain, generates a unique output (Scandura, 1970b).

My students and I used rules, so defined, during the 1960s in the analysis and empirical study of a wide variety of rule-based phenomena, ranging from simple to complex. (Many of these studies are summarized in Scandura, 1969, 1976.) This characterization was subsequently adopted in research by a number of influential educational psychologists (Merrill, 1978; Merrill and Boutwell, 1973; Schmid and Gerlach, personal communication) and apparently is now widely accepted. *

THE STRUCTURAL LEARNING THEORY

The Structural Learning Theory is a natural extension of this early work and provides a unifying theoretical framework within which to view the teaching-learning process. In fact, the theory is not really a specific theory at all but rather defines a class of theories, much as is the case, for example, with the stimulus sampling theory of S-R behaviorism (Estes, 1959).

The Structural Learning Theory, however, is not simply a scientific language. As we shall see below, very definite assumptions are made about how and why people behave as they do. Furthermore, numerous specific realizations of the theory have been detailed and empirically tested to good effect (Scandura, 1977b).

*Since its early development, this characterization has undergone a number of important refinements. For example, although rules are similar to "productions," as originally conceived by the logician Post (Minsky, 1967) and later utilized for psychological purposes by Newell and Simon (1972) and other members of the Carnegie school, they are not identical. Specifically, the operations/procedures in rules, although restricted as to form, are more general than those in productions. Also, productions do not distinguish ranges apart from what one gets when an operation is applied. Both differences, while seemingly technical, are crucial in converting the rule construct into an operational scientific theory sufficiently broad to encompass the instructional process.
As shown in Figure 9, the theory is concerned with the specification of what must be learned (in order to perform as desired on the given educational goals), the characterization of cognitive essentials of individual learners, and the ongoing and goal-directed interactional process between teacher and learner.

What Must Be Learned

In the theory, "content" is effectively characterized in terms of the tasks, or problem situations, that the teacher wants the learner to master (or to deal with effectively) and is referred to as a problem domain.* The prototypic processes that collectively make it possible to solve problems in a problem domain are referred to as rules of competence. (Rules of competence are defined as indicated above.) Collectively, the set of competence rules is called a competence account of the problem domain and constitutes what the learner must learn in order to master the "content." (Note that what is referred to here as "content" corresponds to what others sometimes call educational goals.)

It is important here not to confuse "problem domain" with sets of "behavioral objectives." While the latter may constitute a problem domain, the converse is not necessarily true. As observed in an earlier exposition (Scandura, 1971c, pp. 28-29), the behavioral objectives approach has a major disadvantage.

Because the [solution] rules [for each objective] are discrete, they cannot account for behaviors which go beyond the given corpus [i.e., for tasks not associated with one of the given behavioral objectives]. . . . For example, suppose [a set of behavioral objectives in arithmetic] only involved rules for adding, subtracting, multiplying, and dividing. In this case, the subject would be unable to even generate the addition fact corresponding to a given subtraction fact, although one might reasonably expect this type of behavior from a person . . . well versed in arithmetic. One might

*In structural learning theories, problems are formally characterized (represented) in terms of finite sets of elements, relations and operations defined on the elements, and higher-order relations and operations. (Unlike standard mathematical systems, the relations and operations need not be defined on the same domains and ranges.)
### Figure 9. Schematic representation of the Structural Learning Theory.

**Target Population**: (with assumed encoding/decoding abilities)

<table>
<thead>
<tr>
<th>Encoding</th>
<th>Decoding</th>
</tr>
</thead>
</table>

**Observable Data**

**Content Domain**: Underlying structures include description language, \( L_1 \), and object language, \( L_p \)

<table>
<thead>
<tr>
<th>Analyst</th>
<th>Idealized Teacher</th>
<th>Learner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite selection of structures (problems/terminal objectives) representative of content/population: ( (S_1, S_2, \ldots, S_p) ) with assigned value/cost f'ns, v and c</td>
<td>Perfect knowledge of relevant prototypic competence and of how to teach and test. No processing constraints.</td>
<td>Specific (Individual) Knowledge ( (r_1^l, r_1^t, \ldots, r_p^l) )</td>
</tr>
<tr>
<td>Corresponding set solution rules associated with content/population: ( (r_1, r_2, \ldots, r_p) )</td>
<td></td>
<td>Feedback</td>
</tr>
<tr>
<td>Prerequisite rule set (includes higher-order rules) and associated prerequisite structures (objectives/problems): ( (r_1^l, r_1^t, \ldots, r_n^l) ) ( (S_1^l, \ldots, S_n^l) )</td>
<td></td>
<td>Universal Constraints</td>
</tr>
<tr>
<td>Entry level basic rule set and associated entry structures (objectives/problems): ( (r_1^q, r_2^q, \ldots, r_n^q) ) ( (S_1^q, \ldots, S_n^q) )</td>
<td></td>
<td>Goal switching control</td>
</tr>
<tr>
<td>Etc.</td>
<td></td>
<td>Processing capacity</td>
</tr>
</tbody>
</table>

143
counter, of course, that it would be a simple matter to add a new rule to the original list. [Such a rule might map 5 - 3 = 2 into 2 + 3 = 5, for example.] In effect, when confronted with the criticism that their objectives do not contribute a . . . viable curriculum, [curriculum constructors] would simply say we can add more objectives.

. . . This sort of argument . . . misses the point entirely. Not only would such an approach be ad hoc—which really says nothing in itself except to convey some ill-defined dissatisfaction—but it would be completely inceasible where one is striving for completeness. To see this, it is sufficient to note that a new rule would have to be introduced for every conceivable interrelationship, and that the number of such interrelationships is indefinitely large. One could easily envision a number of rules so large that no human being could possibly learn all of them. . . .

The sum total of all mathematical knowledge, for example, is so vast that no one has or could possibly acquire all of it. As vast as this knowledge is, however, a really good mathematician is capable of generating any amount of new mathematics which does not appear in print anywhere. That is he can create. Much of the new mathematics might be utterly trivial, of course, but the very fact that it exists at all strongly suggests that any [behavioral objectives] characterization would almost certainly miss much that is important.

In the theory, then, the term "problem domain" is used in a broad sense and, in principle, may encompass anything from arithmetic to language or moral behavior. Orthogonally, problem domains may be narrow in scope (for example, two-digit subtraction problems) or comprehensive (for example, an elementary school mathematics curriculum).

A central problem in all scientifically viable cognitive theories is that of how to represent competence, and the Structural Learning Theory is no exception (Scandura, 1971a, 1973). A variety of constructs have been proposed for this purpose, ranging from relational networks (Quillian, 1968) and frames (Minsky, 1975), which tend to emphasize static considerations, to productions (Newell and Simon, 1972) and procedures (Minsky and Papert, 1972), which emphasize cognitive operations.
Rules (Scandura, 1970b) fall in the latter category.* More specifically, it is assumed in the Structural Learning Theory that

*In general, inferences drawn about cognitive processes from observable behavior necessarily involve both encoding/decoding and internal processes—in a combination that cannot be determined through observation alone. Specifically, a theorist may absorb equally well most of the explanation into the encoding/decoding (of static structures) or into internal cognitive operations. (See Anderson, 1976, for a good discussion of the issues involved.)

Hence, technically speaking, rules in the Structural Learning Theory do not operate on observable inputs but on (static) cognitive representations (structures) of such inputs. Similarly, rules do not generate observable outputs but internal structures representing such outputs. (These distinctions between observables and representations of these observables are schematized at the top of Figure 9.)

To avoid the ambiguity mentioned above, the Structural Learning Theory does not attempt to explain encoding and decoding processes. Rather, it requires that all of the targeted students (and the teacher) agree on what are the effectively operating inputs and outputs (that is, on what are the encoded and to-be-decoded structures representing the inputs and outputs). In this regard, it is important to emphasize two things.

First, assuming the uniform availability of certain minimal encoding and decoding capabilities is not a limitation, in practice or in principle. Not only have psychologists long made such assumptions implicitly, but it is always possible to reduce the amount assumed enough in order not to exceed the capabilities of students in the target population. In effect, assumed encoding and/or decoding processes can always be detailed in terms of (internal) rules, together with simpler forms of encoding or decoding. Given such a problem as $45 + 34 = ?$, for example, it is normally assumed that all students will encode the digits in the standard way. While such an assumption might be quite realistic with most second graders, this is not necessarily the case with children who cannot yet read or write. In the latter case, for example, the assumed decoding capabilities could be reduced by absorbing into the rules the processes by which simple line segments, curves, and corners are combined to form the digits. (Correspondingly, notice that one could analyze the writing process in this way.)

Second, by assuming minimal encoding/decoding capabilities in this way, it is possible to make unambiguous inferences concerning
the competence underlying any given problem domain can be represented in terms of finite sets of rules (Scandura, 1971c), each of which may be represented in terms of elementary or atomic components (Scandura, 1970b, 1976).

Theoretically speaking, the problems associated with any given problem domain can be solved in any number of ways (for example, through any number of rule sets). In practice, however, only a small number of alternatives normally will be compatible with how a teacher wants students to go about solving them. It would make a big difference, for example, whether the teacher simply wants the students to be able to perform successfully on a given class of tasks (for example, subtraction problems) or whether the teacher also wants the students to do so with "understanding" (for example, be able to relate the process to concrete reality). The underlying rules of competence necessarily reflect these preferences. In a similar vein, for example, German children are taught the equal additions method of subtraction, whereas U.S. children are taught borrowing. (Note that in the unrestricted Structural Learning Theory [that is, not limited to instruction], rules of competence are more generally viewed as prototypic of some subject population—for example, prototypic of how concrete operational children are assumed to solve conservation tasks (this volume).

Whereas teacher expectations place constraints on the form of what is to be learned, the entering capabilities of the student population determine the level of detail with which competence rules must be specified. Thus, for example, whereas reading may be assumed to be an elementary or atomic operation for most college students, this certainly is not true of third graders. In general, underlying rules must be represented in sufficient detail, in order that all of the specified components make direct contact with assumed minimal capabilities of all students in the target population. Specifically, these components must be either uniformly available or atomic in the sense that they are so (relatively) simple that the students in question cannot master part of such a component without mastering it all. (Basic mathematical considerations guarantee some such level of representation [Scandura, 1970a, 1976; Suppes, 1969].)

cognitive processes. The rules (and certain cognitive universals mentioned below) contain all that is important insofar as explaining, predicting, and controlling behavior with respect to the given problem domain is concerned.
In solving problems (that is, in generating outputs associated with given inputs), it is not necessary that this be accomplished directly by applying rules individually. Rather, in the Structural Learning Theory, solutions may be generated indirectly, since rules are allowed to operate in higher-order fashion on other rules to generate new rules. (More accurately, higher-order rules operate on data structures that contain rules. See previous explanatory note and Scandura, 1977c.) The new rules, in turn, may generate the solutions.

"Higher-order rule," as the term is used here, should not be confused with other common uses of the term, most especially with rules that are associated with higher levels in learning hierarchies (compare Gagne, 1965, and Ehrenpreis and Scandura, 1974). Whereas the latter involve combinations of lower-order rules, our higher-order rules include, for example, the processes by which lower-order rules, associated with any number of different hierarchies, are combined to form correspondingly more complex rules (that is, "higher-order rules" in Gagne's terminology).

This is not to say that the only thing higher-order rules are good for is to combine lower-order rules in learning hierarchies. A wide variety of behavioral phenomena can be accounted for in this way, namely, learning (rule derivation/"invention"), breaking problems into subproblems (including constructing hierarchies of subproblems), assigning meanings, motivation or rule selection, problem definition, storage and/or retrieval from memory, automatization, and so forth (Scandura, 1977b, 1978a). The basic differences in each case reside in the general types of higher-order rules required. "Invention," for example, may involve higher-order analogy rules, which construct new rules having the same form as given ones, as well as higher-order composition rules, which serve to integrate component rules into more comprehensive wholes.

The explicit introduction of higher-order rules has a number of important general advantages. First, higher-order rules represent interrelationships in a way that appears to allow for "creative potential" (that is, unanticipated outcome). In addition, the introduction of higher-order rules, as well as lower-order ones, often makes it possible to account for relatively complex domains in an unusually efficient manner (Scandura and Durnin, 1977). Second, as indicated in the section on the learner, higher-order rules (along with lower-order rules) appear to provide a general, viable means for representing individual knowledge (that is, actual human behavior potential). Moreover, their introduction for this purpose is highly consistent with what is known (or at least what may safely be assumed) about how humans function as information processors. Third, as we shall see in discussing individual differences measure-
ment, higher-order rules, just as rules in general, are fully operational. It is possible by testing to determine which parts of which higher-order rules have and have not been acquired by individual learners at any given stage of learning. Finally, the introduction of higher-order rules appears to facilitate the relatively difficult task of specifying the competence underlying complex problem domains. The quasi-systematic form of analysis that has been used for this purpose is called structural analysis.

The Process of Structural Analysis

Detailed structural analyses have been undertaken of several rather comprehensive problem domains, including geometry construction problems (Scandura, Durnin, and Wulfeck, 1974), an entire mathematics curriculum for elementary school teachers (Scandura et al., 1971), algebraic proofs (Scandura and Durnin, 1977), arithmetical skills (Scandura, 1972), and, most recently, the domain of Piagetian conservation problems (this volume). Empirical evaluations strongly supportive of the analyses, involving geometry construction problems (Scandura et al., 1977), the mathematics curriculum (Ehrenpreis and Scandura, 1974), arithmetical skills (Scandura, 1972), and conservation, have been completed.

The point to emphasize here, perhaps, is that the constraints imposed on (the representation of) competence in structural learning theories go beyond those normally associated with a scientific language. As we shall see, these constraints play an important role in satisfying the needs of instructional theory. Specifically, unlike most cognitive theorizing, considerable attention has been given to the crucially important problem of how to identify the rules of competence underlying given problem domains. In the case of instruction, for example, it is one thing for a teacher to be able to give examples of the kinds of problems the students must solve or to illustrate the kinds of things that must be learned. It is quite something else to be able to identify precisely and comprehensively what it is that the students need to learn in order to solve all or most of the problems. Clearly, being able to represent needed competence involves much more than simply being able to solve problems by oneself or agreeing on some form of representation.

In general, structural analysis involves specification of the problem domain, including both the individual problems and the extent of the domain, and specification of the rules needed to solve the problems. In the case of relatively simple domains, for example, the domain of subtraction problems, both the problems and the underlying solution rules can be specified relatively easily.
Sample subtraction problems and a subtraction rule (algorithm) based on "borrowing" are shown in Figure 10. In addition to mastery of the content, the major prerequisites for reliable analysis in simple cases, such as this one, would appear to be some facility in constructing flow diagrams and representing them at a level of detail (that is, in terms of atomic components) that is appropriate for the students in question.*

The situation with more complex domains is far less obvious. For one thing, it is not always easy to identify the effectively operating problems, or the extent of the given problem domain. Both factors, for example, had to be dealt with in our recent analysis of the Piagetian stage of concrete operations. As presently practiced, the constraints on structural analysis in this regard reside primarily in the required form of representation (for example, of problems as a type of structure) and in the need for, if not an analytic description of the domain, then the existence of some oracle (for example, teacher) who, given an arbitrary problem, can determine whether or not it belongs to the domain.

Considerably more work has been done in identifying the rules of competence underlying complex domains. Specifically, in addition to rules of competence per se, progress has been made toward the development of systematic and relatively efficient methods of structural analysis (designed to identify rules of competence).

Unlike simple domains, complex domains are not easily reduced to single rules of competence. This may be the case for some domains in principle as well as in practice. To some extent, this difficulty is circumvented in structural analysis by adopting a modular approach.

In schematic form, structural analysis begins with some given domain of problems and involves the following steps. (1) Select a representative sample of problems. (2) Identify a solution rule for solving each of the sampled tasks. (These solution rules are designed to reflect the way in which successful subjects in a given target population might solve the sampled problems. The initial set

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*In recent work, we have discovered what appears to be a close relationship between devising prototypic solution rules and "top-down" programming, a general method often used in computer programming to construct what are called "structured programs." (See Haskell, 1978, for a highly readable introduction.) Use of this method to supplement existing forms of structural analysis appears to have considerable promise and could lead to the development of reliable, systematic, and efficient methods of analysis. Research in this direction is currently underway.
FIGURE 10. Subtraction rule (algorithm) and sample subtraction problems.
of solution rules is denoted R.) (3) Identify higher-order rules that reflect parallels among the initial solution rules and that operate on lower-order rules. (4) Eliminate lower-order rules made unnecessary by the higher-order rules. (5) Test and refine the resulting rule set on new problems from the problem domain. And, (6) extend the rule set where necessary so that it accounts for both familiar and novel problems in the domain. Collectively, the higher- and lower-order rules of step 3 constitute a more basic set of rules from which the initial solution rules, among others, may be derived.

Consider, for example, step 1, two sample problems from the domain of geometry construction problems, and step 2, their corresponding solution rules.

Sample Problem 1 Using only a straightedge and compass, construct a point X at a given distance d from two given points A and B.

![Diagram](image1)

Solution Rule 1 [Set (the radius of) the compass to distance d, put the point of the compass on point A, and draw a circular arc (that is, the "locus" of points at distance d from point A).] [Place the compass on point B and draw another circular arc.] [Label the point(s) of intersection of the two circles X.]

Sample Problem 2 Given a point A, a line 1 and a distance d, construct a circle with radius d that goes through point A and is tangent to line 1.

![Diagram](image2)
Solution Rule 2 [Construct a circle with center at A and radius d.]
[Construct a locus of points at distance d from line 1 (that is, a parallel line at distance d from line 1).] [Construct a circle with center X (the intersection of the circle and the parallel line) and radius d.]*

Step 3 involves noticing that the two solution rules have the same general structure (set off by brackets). Although the component rules of these solution rules differ substantially, each solution rule involves two independent "locus" constructions, with the intersection X of the two loci playing a critical role. In the first problem, X is the solution. In the second problem, it is the center of the desired goal circle.

In general, each type of structural parallel can be realized concretely in the form of higher-order rules. (The above type of parallel is only one of several basic kinds that may be shared by two or more rules.) In the present illustration, for example, both solution rules can be derived by applying the higher-order "two-locus" rule of Figure 11 to the respective component rules. This higher-order two-locus rule operates on simple locus rules (for example, for constructing circular arcs and parallel lines) and generates solution rules (that is, combinations of the simpler locus rules). It is important to emphasize that the two-locus higher-order rule can be used to derive solution rules for a wide (potentially infinite) range of problems, not just for the two sampled problems.

Incidentally, notice also that the higher-order rule is only represented schematically. For example, the notion of a "locus condition" in the first decision would almost certainly not be atomic (that is, sufficiently elementary) for most populations of learners. For this purpose, "locus condition" must be detailed in terms of the more basic conditions shown in Figure 12—for example, the "picture" contains a point X that is a given distance from two given points or lines or is equidistant from two pairs of given points or lines.

In step 4, given the higher-order two-locus rule and the lower-order component rules, the solution rules themselves may be eliminated as redundant since they can be derived from the former rules.

*Each step in solution rule 2 can be detailed in terms of the more molecular operations of setting a compass, using a fixed compass to construct a circular arc, and using a straightedge to construct a line segment.
Construct a "Picture" of Problem as Solved.

Does "picture" contain point X that satisfies two locus conditions? If so, is there a rule, \( r^* \), that operates on X and generates goal figure?

yes

Store: Rule \( r_g \)

Are there rules \( r_1 \) and \( r_1' \), which apply to given information and generate loci which contain X?

yes

Construct Solution Rule:

\[
\begin{align*}
& r_1 \\
& r_1' \\
& r_g
\end{align*}
\]

STOP

no

STOP fail

FIGURE 11. High-level description of higher-order two-loci rule.
acting collectively. Illustrating steps 5 and 6 of structural analysis would require more space than is available, but the general intent is clear. (For details of the analysis, see Scandura, Durnin, and Wulfeck, 1974.)

Does "picture" contain a point X that satisfies two of the following conditions:
- X is given distance from given point or line,
- X is equidistant from given pair of points or lines,
- X is vertex of angle of given measure subtending given segment.

If so, is there a rule, r_q, that operates on X and generates goal figure?

FIGURE 12. Partial specification of first decision of higher-order two-loci rule.

It is important to notice that structural analysis may be applied iteratively (repeatedly). Given an initial set of solution rules, one need not stop by deriving a more basic rule set (for example, a set including both higher- and lower-order rules). The derived rule set, in turn, can be subjected to precisely the same type of analysis with a resulting rule set that is still more basic. In general, structural analysis may be reapplied as many times as desired, each time yielding a rule set that is more basic in two senses: individual rules are simpler and the new rule set as a whole has greater generating power—that is, it provides a basis for solving a greater variety of tasks (Scandura, 1973, pp. 114-117; 1977b).

Empirical Research Using Structural Analysis

To date, my collaborators and I have completed several structural analyses. Our first attempt in this direction was a very prac-
tical endeavor. I had just completed a book entitled *Mathematics: Concrete Behavioral Foundations* (1971b), and the publisher wanted a workbook to parallel the text. Rather than use "seat-of-the-pants" methods entirely, Scandura et al. (1971) decided to attack the task (more or less) systematically, in the process incorporating some of our then new ideas concerning higher-order rules. The book itself provided the basis for analysis. First, we went through the text, page by page, identifying the tasks (behavioral objectives) that seemed to be inherent in the material. Next, for each class of tasks we constructed a solution rule that paralleled the way in which the book would have the students solve them. Third, we looked for parallels among the solution rules. These parallels were used as a basis for identifying what turned out to be a small, but potentially powerful, set of higher-order rules. These higher-order rules made it possible to eliminate almost half of the lower-order solution rules as redundant; that is, using one of the higher-order rules, it was possible to derive each of the eliminated solution rules by applying the higher-order rule to some other solution rule.

As it turned out, not only was it possible to identify the basic solution rules and higher-order rules underlying the text, but an empirical investigation provided strong support for the analysis (Ehrenpreis and Scandura, 1974). Two groups of elementary school teachers were trained as part of a regularly scheduled summer school course. One group (D) was trained on 304 discrete solution rules. The other (H) was trained on the reduced set of 164 solution rules plus five higher-order rules. The results showed that all of the students learned the solution rules that they had been taught to a high degree of proficiency, that those H subjects who were trained on the higher-order rules performed just as well on transfer tasks requiring new solution rules (which they had to derive) as did those D subjects who were trained on the solution rules directly, and that the higher-order rules (H) group performed better on transfer tasks on which neither group had been trained. The results, without serious distortion, can be summarized by saying that "the higher-order rules students were taught less but learned more."

In another project, Scandura, Durnin, and Wulfeck (1974) undertook a more intensive analysis of geometry (straightedge and compass) construction tasks. Among other things, this study demonstrated that the heuristics identified by Polya (1962) can be sufficiently detailed so that children can be taught to use them both successfully and reliably. (Also, they were programmed on a computer.)

A subsequent empirical analysis (Scandura et al., 1977) demonstrated that the identified higher-order rules provided an
adequate basis both for diagnosing individual strengths and weaknesses in problem-solving ability and for overcoming these weaknesses. In particular, instruction on the higher-order rules made it possible for students to solve new problems that they had never seen before. Moreover, they got progressively better at learning new higher-order rules. In this study, predictions could be made with far greater precision than in the Ehrenpreis and Scandura (1974) study cited above. In particular, the predictions referred to individual performance on specific problems rather than to group averages.

Nonetheless, the analytical results of the Scandura, Durnin, and Wulfeck (1974) study were limited in a number of important ways. No attempt was made to include logical inference. All of the higher-order rules had the effect of composing rules—no other kinds of higher-order rules were considered. And, no distinctions were made between deriving solution rules and breaking problems into subproblems. The analysis did not, for example, allow for the widely used technique of breaking problems into hierarchies of subproblems and attacking the subproblems in turn.

A subsequent study by Scandura and Durnin (1977) dealt specifically with these limitations. In particular, a total of 24 lower-order and higher-order derivation and problem definition rules were shown to be adequate for proving more than 130 theorems and proof exercises (along with an undetermined number of others) in an experimental high school text on number systems (Brumfiel, Eicholz, and Shanks, 1961). In addition to dealing directly with logical inference, the higher-order rules identified in this study involved analogy, generalization, restriction, and subproblems, as well as simple combination of lower-order component rules.

Another study (Lowerre and Scandura, 1973) dealt with logical inference in the context of verbal discourse. In this case, it was possible to both identify and teach rules dealing with logical interrelationships among sentences.

The study by Scandura, Durnin, and Wulfeck (1974) was limited in another important respect. The rules identified, including the higher-order rules, were of varying degrees of complexity. At first, we were puzzled as to why this had not been a problem in the other analyses. It later occurred that there was a very good reason. Unlike the earlier analysis of my mathematics text, and the subsequent analysis of the algebra text, the geometry problems were not organized in the form of a curriculum. Rather, the problems were randomly selected from Chapter 1 of Polya's (1962) mathematical discovery. The problems, in effect, were not ordered according to ease of learning but to make the points Polya felt essential to his discussion of heuristics.
This recognition opened up to us a whole new area of concern, fortunately one that could be dealt with in a surprisingly simple way. As noted above, once one has identified a set of lower- and higher-order rules underlying a problem domain (more accurately a set of rules derived from a finite sample of problems taken from the domain), there is nothing to prevent one from repeating the analysis. Thus, by seeking out relations and parallels among lower- and higher-order rules identified on the first go-round, one can frequently identify a second generation of higher- and lower-order rules from which the original ones may be derived.

In fact, this is precisely what Wulfeck and Scandura (1977) did in extending the geometry analysis of Scandura, Durnin, and Wulfeck (1974). Structural analysis was repeated iteratively until all of the rules, higher- and lower-order alike, consisted of such molecular operations as setting a compass, using a fixed compass to construct a circular arc, and combining pairs of simple operations. Not only were the individual rules simpler but, collectively, they had qualitatively more generating power.

Moreover, introducing limitations on processing capacity (see discussion below on the learner) made it possible to generate optimal learning sequences (that is, to sequence any given set of problems in a learnable order). Comparison of these theoretically derived sequences with learner-controlled and random varieties indicated an approximately two-to-one advantage in favor of the former. This difference held even though remedial training was provided on needed solution rules immediately after a subject failed a problem, thereby helping to insure that the controls would not get further and further behind. Success on problems that came later in the instructional sequence, however, required new and more complex higher-order rules as well as new solution rules. According to the theory, learners would gradually acquire the needed higher-order (as well as lower-order) rules as learning progressed.

Hence, it appears that the obtained difference was due to the fact that the experimental subjects (and others who succeeded in solving the problems by themselves) gradually learned the needed higher-order rules, whereas the unsuccessful controls did not.

In this volume, we further demonstrated the applicability of structural analysis to what are normally thought of as "unstructured" domains. Specifically, they systematically analyzed the domain of Piagetian conservation tasks. Beginning with a rule-based characterization of the prototypic child at the concrete operational level, successive reapplication of the method of analysis resulted in the identification of increasingly more primitive sets of rules—leading ultimately to rules characteristic of the child at the preoperational stage. The identification of preautomatized versions
of the rules used by conservers played a key role in the analysis. In effect, the structural analysis not only provided a (partial) characterization of competence at two different stages of cognitive development but also of the transition between them. This analysis, in turn, provided an explicit basis for training children at the pre-operational stage to conserve in a manner that was difficult to distinguish from that of natural conservers. Perhaps more important, it was possible to explicitly manipulate "horizontal decalage" (that is, uneven transfer across conservation concepts), something which has been difficult to explain, let alone to predict or control, within Piagetian theory. (Feibel [1978] made similar use of the Structural Learning Theory in designing a comparative training study involving formal operations. Essentially, what he found was that explicit rule-based training leads to more rapid growth than does less directive incongruity training. The results of the study further demonstrate the need for more detailed and operational analysis of formal operations along the lines indicated above.)

The Learner

Prototypic competence is not the same as rules of knowledge that characterize individual behavior potential. It is assumed in the Structural Learning Theory that what an individual does and can learn depends directly and inextricably on what he already knows. More particularly, as shown in Figure 9, it is assumed that human cognition may be adequately characterized in terms of universal characteristics of the human information processor and individual knowledge. As we shall see, the latter is operationally defined in terms of rules (of both higher- and lower-order) in relation to prototypic competence (prototypic competence is associated with given problem domains and learner populations).

Clearly, instruction is concerned primarily with individual knowledge. From an instructional point of view, universal characteristics are best thought of as those aspects of human cognitive functioning that are inherent to man generally. They need not, and indeed in some cases cannot, be taught.

Universal Characteristics of Human Cognition

Control Mechanism

Control mechanisms are among the most important universal characteristics. They are essential in all information-processing systems, whether man or machine, and serve to tell the learner
which rules to use and when to use them. Whereas all complete information-processing theories make a distinction between processes (rules) and control, control in most cases either plays a subordinate role (Newell and Simon, 1972) or is distributed among a variety of different control mechanisms whose coordination, in turn, is often left unspecified (Pascual-Leone, 1970).*

*Equally important, the amount of information processing that is designated as control and the amount that is designated as process is largely arbitrary. As with encoding/decoding and internal processing, the decision as to how much of a theory to assign to control and how much to other internal processes (for example, rules) depends on factors other than simply accounting for a specific domain of behavioral phenomena.

In the Structural Learning Theory, a primary consideration has been to insure that (rule) constructs reflecting individual knowledge are operational (that is, definable in terms of observable behavior). Furthermore, I wanted the division between control and other internal processes to be generalizable and, if possible, to be completely independent of specific content and subject populations. Put differently, I did not want to assign more to control than could reasonably be assumed of all human processors. If one were to assume too much, then there surely would be some populations of subjects who would be unable to perform as expected—even where they had mastered all of the requisite internal processes (for example, rules and higher-order rules). On the other hand, I wanted to assume as much as possible. Doing so simplifies the identification of competence associated with specific content.

I cannot prove in any formal sense that the division of labor (between control and rules) proposed in the Structural Learning Theory is the only one that would satisfy the above considerations. This division, however, does appear to meet these constraints more completely and precisely than any other cognitive theory with which I am familiar (or for that matter with any other alternatives that I have been able to devise). Indeed, many well-known cognitive theories appear not to have even attended to these considerations. In some cases, as mentioned in the main text, they have been demonstrated empirically to be in error.

Thus, for example, although our research demonstrates that all rules, including higher-order rules, are teachable, it also demonstrates that one may not safely assume the uniform availability of even the simplest higher-order rules, as is frequently the case in other comprehensive cognitive theories. In the General Problem Solver (Ernst and Newell, 1969), for example, it is
In contrast, in structural learning theories, control mechanisms have been subjected to direct empirical study (Scandura, 1971a, 1973, 1974b, 1977b). It is well known, for example, that people are not always able to solve problems, even when they know all the necessary components. What has not been so clear, however, is why this is so. Are successful persons somehow more capable than others? Or, do they simply know something that unsuccessful people do not?

The explicit introduction of higher-order rules helps provide answers to these and a wide variety of other questions. As observed above, for example, higher-order rules may provide an explicit basis for explaining, predicting, and/or controlling behavior involving analogy, generalization, problem definition, rule selection, and so forth. (For details, see Scandura, 1973, 1977b.)

As noted above, however, even introducing higher-order rules does not provide a sufficient basis for explaining individual behavior. A complete theory must include (control) mechanisms that explain how and why various rules are used in particular situations. In this regard, the Structural Learning Theory postulates a simple, goal-switching control mechanism that makes minimal assumptions about the processor, assumptions that appear to be generalizable to all people. This mechanism simply makes precise what has been implicitly assumed for many years. If a person does not know how to solve a given problem, but still wants to solve it, then he will automatically turn his attention to finding some way to do it.

More specifically, given a problem, the human information processor is assumed to first check to see if a solution is directly available. If not, the processor is assumed to search through (the ranges of) his available rules to see which, if any, might solve the problem. (A rule is a potential solution rule if its range "matches" the problem goal and its domain includes the problem given.) If a unique rule is found, then the rule is applied, and the output is tested to see if it solves the problem. If there are no potential solution rules, then the search takes place at a still deeper level. In this case, control directs the search for (higher-order) rules that generate potential solution rules.

assumed that means-ends analysis is characteristic of all problem solving. Similarly, cyclical, stack-type control has been commonly assumed in simulation theories based on production systems (Newell and Simon, 1972). These types of control, while perhaps common, implicitly make assumptions about how people utilize lower-order knowledge that are not universal.
If such a higher-order rule is found, then it is applied. The newly generated rule is added to the set of available rules, and the search reverts to the next lower level. The augmented rule set is then checked as before—only this time the newly derived (potential) solution rule is available. In general, whenever there are no rules (or more than one rule) that apply at a given level of search, control moves to a still deeper level. Conversely, whenever a match is achieved (so that a rule is applied), control reverts to the preceding level.

This simple "goal-switching" mechanism* is hypothesized to be common to all humans and to govern all cognition, irrespective of the specific knowledge involved. Consider, for example, the problem of rule derivation, of how individuals derive new solution rules for solving new problems they have never before seen. According to the Structural Learning Theory, rule derivation takes place as a result of applying various higher-order rules to other rules. These higher-order rules may serve to combine component rules, to generate analogous rules, to generalize given rules, and so forth.

To make things concrete, suppose a child knows rules for converting yards into feet (multiply by three) and for converting feet into inches (multiply by 12) and that he is asked, "How many inches are there in two yards?" Clearly, this problem can be solved by combining the two available rules, the rule for converting yards into feet and the rule for converting feet into inches. But how does

*In earlier formulations (Scandura, 1971a, 1973, 1977b), emphasis was given to switching between higher- and lower-level goals, and, correspondingly, the control mechanism was referred to as the "goal-switching mechanism." The above description is behaviorally equivalent to these earlier formulations. However, although it is beyond the scope of this article to discuss the reasons, recent theoretical advances and computer implementations of the mechanism convince me that the formulation sketched above is preferable. Among other things, it makes it possible to program the control mechanism in a way that is completely independent of content (that is, specific rules of competence). The latter can be removed, replaced, and/or modified without requiring any change in control. Strictly speaking, implementation of the goal-switching mechanism, as originally conceived, is not possible. In this case, it is necessary either to restrict oneself to incomplete approximations (Wulfeck and Scandura, 1977) or to introduce natural, but nonetheless ad hoc, assumptions concerning specific sets of competence rules.
the child know how to combine the given rules? Knowing component rules is surely not logically equivalent to knowing when and how to use them.

A basic assumption in the Structural Learning Theory is that new rules are derived by application of certain rules to other rules. In the present case, a child might be expected to succeed if he knows a higher-order rule that operates on pairs of rules of the form $A \rightarrow B$, $B \rightarrow C$ (that is, like but not limited to those above) and combines them to form composite rules of the form $A \rightarrow B \rightarrow C$, in which the components are performed in sequence.

While knowing both requisite higher-order and lower-order rules is a necessary condition for solving problems, this is not sufficient. In order to effectively use available rules to derive solution rules and to solve problems, some type of control mechanism is needed to determine when each rule is to be used and how. The question here is whether the above control mechanism is sufficient for this purpose and, if so, whether this mechanism is available to all human beings as postulated. Although the former question is strictly analytic in nature and, hence, has an analytic answer (Scandura, 1977b, chap. 2), it seems best here to consider both questions in the context of empirical evidence.

Specifically, in a study reported by Scandura (1973, 1974b), 30 children, ranging in age from seven to nine, were trained on simple $A \rightarrow B$ and $B \rightarrow C$ conversion rules and tested on an $A \rightarrow ? C$ problem. In most information-processing theories based on production systems (Newell and Simon, 1972), it is (sometimes implicitly) assumed that people automatically combine rules. If all humans can actually do this, then all of our subjects would be expected to succeed on the problem. The fact that 24 of 30 subjects failed strongly suggests that many people do not automatically combine rules, at least not young children.

With the above control mechanism in mind, the 24 subjects who failed on the $A \rightarrow ? C$ problem were randomly divided into two groups of 12. One group was trained on the higher-order composition rule identified above. The other group served as a control. After the higher-order rule training, all subjects were trained on a new pair of $A' \rightarrow B'$, $B' \rightarrow C'$ rules, which the subjects had never seen before. Then, they were tested on the corresponding $A' \rightarrow ? C'$ problem (which was also new). This time essentially all of the experimental subjects succeeded, whereas all of the control subjects again failed.

These results are perfectly in accord with the hypothesized control mechanism. Given the $A \rightarrow ? C$ pretest problem, for example, the 24 failure subjects did not have a solution rule immediately available, nor apparently did they know an appropriate higher-order
rule. They only knew an \( A \rightarrow B \) rule and a \( B \rightarrow C \) rule (for example, rules for converting yards into feet and feet into inches). Under these conditions, they failed uniformly.

After training on the higher-order composition rule, the experimental subjects fared uniformly well. Presumably, according to the goal-switching control mechanism, the subjects first checked to see if they knew the solution (to the \( A' \rightarrow ?C' \) problem). Not finding one, they again searched their available knowledge, this time looking for a rule that generates potential solutions (for example, numbers of inches) and that applies to the given input (for example, two yards). Again, no such rules were available. Hence, another search was initiated, at this level for a (higher-order) rule that generates potential solution rules. According to hypothesis, this level of search resulted in the identification of the higher-order composition rule mentioned above. The range of this rule, recall, contains the \( A' \rightarrow B' \rightarrow C' \) solution rule. Its domain consists of pairs of rules of the form \( X \rightarrow Y, Y \rightarrow Z \) and, hence, clearly contains the given pair \( A' \rightarrow B', B' \rightarrow C' \). According to hypothesis, the higher-order rule is applied to the \( A' \rightarrow B', B' \rightarrow C' \) rules giving \( A' \rightarrow B' \rightarrow C' \). The latter, composite rule is added to the set of available rules (that is, it is learned), and control reverts to the previous level. The subsequent search, again, is for a solution rule, only this time the \( A' \rightarrow B' \rightarrow C' \) rule is available. Once identified, this rule is applied, and the problem is solved (that is, the potential solution obtained by applying the composite rule is tested to see if it satisfies the original goal).

I must caution that this simple control mechanism is an idealization and applies only in situations where processing capacity is not a factor and, specifically, where all of the requisite higher- and lower-order rules are learned perfectly and are active in "working" memory. (See Scandura, 1971a, 1973, 1977b.) Perhaps surprisingly, however, this limitation has not proved to be as critical as one might expect. Empirical support has been strong, although not deterministic, even under "real-world" conditions. Ehrenpreis and Scandura (1974), for example, found that higher-order (as well as lower-order) rules underlying a college course for teachers could be identified in a systematic manner and that instruction on such rules had a highly positive effect on prespecified kinds of "far transfer." Furthermore, the degree of transfer was directly related to the degree to which the test conditions approximated the ideal (Scandura, 1977b, chap. 11).

When used in conjunction with appropriate kinds of higher- and lower-order rules, the goal-switching control mechanism provides an adequate basis for explaining, predicting, and controlling a wide variety of behavior. This includes solving analogy problems,
generalizing given rules, motivation (rule selection), problem definition (subgoal formation), automatization, and rule retrieval. For details and related empirical support, the interested reader is referred to Scandura (1973, 1977b).

**Processing Capacity and Processing Speed**

Processing capacity is the second general characteristic of the theory that has been empirically tested (Scandura, 1973; Voorhies and Scandura, 1977). In one form or another, almost all contemporary information-processing theories assume that "working memory" has a limited capacity. In contrast to most theories, however, working memory in the Structural Learning Theory is assumed to hold not only data (the stuff on which rules operate) but rules (processes) themselves. While capacity per se is assumed to be fixed (although it may vary over individuals), the memory load associated with any given task depends directly on the process (rules) used in attacking it. The "chunks" (Miller, 1956) that are effectively operating at each point in time are defined dynamically by the rules in question as they undergo execution (see Scandura, 1973, chap. 10). Thus, for example, whereas it may be impossible to multiply large numbers in one's head using the standard algorithm, many people know shortcut processes that enable them to perform successfully. Rather than applying to all multiplication problems, shortcut processes typically work only with special types (for example, numerals ending in 5 or 0).

In the theory, the concepts of memory load and processing speed are closely related (Scandura, 1977b, chaps. 2 and 7). Specifically, in order to account for processing speed (that is, response latency minus encoding and decoding times), as well as the nature of the response itself, the underlying rules must be represented in more detail (than in just accounting for responses per se). For example, predicting the time it takes to generate the sum of two numbers requires that the rules not only be represented in terms of components that are atomic in a behavioral sense but also that these components require equal processing times. (Rules that are represented in terms of such components are called "process atomic" [Scandura, 1977b, pp. 111-135]. In contrast, predicting responses per se requires only that rules be represented in terms of behaviorally atomic components.)

In general, the desired level of behavioral precision determines which universal characteristics and which level of (knowledge) representation is required in a structural learning theoretical account of the phenomena. Specifically, two subclasses of structural learning theories are readily distinguished. In one subclass, the
theories involve only the control mechanism and rules represented in terms of behaviorally atomic components. Theories of this class make it possible to predict individual responses, but they are silent on the issue of processing time. As noted above, they are idealizations that fit reality only to the extent that processing capacity is not a factor. Thus, deterministic predictions may be expected to hold only in situations that satisfy appropriate boundary conditions. To the extent that processing capacity is involved, for example, theoretical predictions may be expected to deviate from obtained results.

In many ways, such theories may be likened to theories in classical physics. The law of the inclined plane, for example, allows one to calculate the force needed to move a given cart up an inclined plane but only where the inclined plane is perfectly smooth and the wheels on the cart are frictionless. Deviations from prediction may be expected just to the extent that the inclined plane is bumpy and/or that friction otherwise plays a role. Correspondingly, structural learning theories can only be tested under appropriate idealized conditions in the same sense that the law of the inclined plane must be tested using smooth inclined planes and frictionless wheels.

The second subclass of structural learning theories involves the control mechanism, processing capacity/speed, and process atomic rules. Theories of this type obviously provide more detailed accounts of behavioral phenomena and can validly be tested under less stringent conditions. For example, the experimenter need not insure that processing requirements lie within each subject's processing capacity. Encoding/decoding assumptions still must be met, however.

Increasing the range of applicability does not come without a price. Theories of the second type are correspondingly more difficult and exacting to construct. Thus, for example, although traditional cognitive psychology is replete with theories that predict latencies, few satisfy the aforementioned structural learning constraints. Among those that appear to come closest are the chronometric theories of simple arithmetic by Groen and his associates (Suppes and Groen, 1967) and, especially, the list-processing and computational theories of Voorhies and Scandura (Scandura, 1973, 1977b).

Assessing Individual Knowledge

In contrast to universal cognitive constraints, specific knowledge is assumed to vary over individuals. The theory shows how
prototypic competence (that is, competence prototypic of given populations) may be used to operationally define the knowledge available to actual individual members of such populations. Specifically, the theory tells how, through a finite testing procedure, one can identify which parts of to-be-taught rules individual subjects know. The rules in a very real sense serve as rulers of measurement and provide a sufficient basis for the operational definition of human knowledge (Scandura, 1977b, chap. 2).

Clearly, this is not the place to detail how this may be accomplished in general. For one thing, discussing the way process atomic rules are made operational gets one deeper into issues of representation than would be desirable here. (For details, see Scandura, 1977b.)

For present purposes, it will be sufficient to consider the process of assessing individual behavior potential with respect to rules of competence represented in terms of behaviorally atomic components. Even here, we consider assessment only with respect to single rules, rather than sets of competence rules considered collectively. The flow diagram in Figure 10 depicts a rule (procedure/algorithm) for subtracting numbers. This rule is broken down into atomic components (that is, steps that are so simple that each individual in the target population may be assumed able to perform each step either perfectly or not at all). In line with our previous discussion, it is worth emphasizing that what are atomic units in relation to one population may not be atomic units in relation to another (for example, less sophisticated) population.

Because success on any path of a rule depends on success on all atomic components, each path through the rule also acts in atomic fashion. Furthermore, there are only a finite number of behaviorally distinct paths. We do not distinguish paths according to the number of repetitions of loops, because the same cognitive operations and decisions are required regardless of how many times a given loop is traversed in carrying out a given "cognitive computation."

Collectively, the paths of the subtraction rule impose a partition on the domain of column subtraction problems; that is, they define a set of distinct, exhaustive, and homogeneous equivalence classes of subtraction problems such that each problem in a given equivalence class can be solved through exactly one of the paths.

One path through the aforementioned subtraction algorithm is represented schematically at the bottom of Figure 10, along with two-column subtraction problems to which that path applies. The first node designates "START." Operation (arrow) 1 says to go to the right-most column. The second node, then, asks whether the top number is greater than the bottom number. Since the answer
is "yes," operation 2 is applied (that is, the bottom number is subtracted from the top number). Next, in the third node, we ask if there are any more columns. If there are, we proceed to the next column (operation 3) and repeat. Otherwise, we "STOP."

The fact that each path is associated with a unique subclass of column subtraction problems makes it possible to pinpoint, through a finite testing procedure, exactly what it is that each subject knows in relation to the initial rule. It is sufficient for this purpose to test each subject on one randomly selected item from each equivalence class. Success on each item, according to our automaticity assumptions, implies potential success on all other items from the same equivalence class, and conversely for failure.

Individual knowledge (or behavior potential), in effect, may also be represented in terms of rules—specifically, in terms of subrules of given rules of competence. Notice in this regard that the knowledge attributed to different individuals may vary even where only one rule of competence is used to assess behavior potential. For example, if a person succeeds on only one path and fails on the others, then his knowledge would be represented by that path (which is a subrule of the initial one). Where two or more paths are involved, a combination of the paths would be used to represent individual knowledge.

Fortunately, the above discussion is not just a theoretical exercise. A significant amount of supporting data has been collected over the past several years on a variety of problem domains, with subjects ranging from preschool children to Ph.D. candidates. Given a class of tasks, the general form of each study proceeded in the following manner. One or more prototypic rules were identified that were both adequate for generating solutions to each of the tasks and compatible with the way a knowledgeable or idealized (prototypic) member of the target population might be expected to solve them. These rules singly and/or collectively were used to partition the class of tasks into equivalence classes. Subjects in the target population were tested on two items (tasks) from each equivalence class. Performance on one item from each equivalence class was used as a basis for predicting success or failure on the (second) item.

With highly structured tasks run under carefully prescribed laboratory conditions, it has been possible, given performance on initial items, to predict performance on new (second) items with over 96 percent accuracy (Scandura, 1973; Scandura and Durnin, 1978). When testing took place under ordinary classroom conditions,
with the subjects run as a group, the predictions were accurate in about 84 percent of the cases (Durnin and Scandura, 1973).*

There is one further major advantage of the structural approach to assessing behavior potential. The approach makes it possible to identify precisely not only what individuals can and cannot do but also what the learner does and does not know in relation to the particular rules involved. A simple basis for instructional decision making follows directly. Assume the paths the learner already knows and gradually "build in" those that he or she does not.

In summary, it would appear that any viable theory of performance testing must take into account underlying competence. Not only do rules of competence (associated with populations of subjects) provide a basis for measuring individual knowledge and for providing remedial instruction but they also provide an explicit basis for selecting appropriate test items. Furthermore, because

*In the latter study, the equivalence classes determined through the structural/algorithmic approach were compared with item forms identified by Hively, Patterson, and Page (1968) and by Ferguson (1969). Whereas the levels of prediction on success items were approximately the same, the algorithmic/structural approach yielded significantly better results with respect to failures. Equally important, these levels of prediction were obtained with half as many test items—with even greater increase in efficiency possible through the use of conditional testing (Durnin and Scandura, 1973).

In addition to yielding poorer results, the use of item forms has been limited to paper-and-pencil tests. And, as is generally recognized, item forms have intrinsic limitations with regard to non-paper-and-pencil applications, such as job analysis. The structural approach is not limited in this way. The direct relationship between molarity of atomic rules and sophistication of population allows for broader applicability. In job analysis, for example, it would make little sense to attempt a molecular analysis of arithmetic skills in order to judge the ability of accountants, or of writing syntax in evaluating professorial capabilities. Although the impatient reader may have some doubts, minimal capabilities can reasonably be assumed of all bona fide professionals. Thus, all trained accountants presumably can add a column of figures, and all experienced Ph.D.s have at least minimal writing capabilities. Hence, it is sufficient to consider only those molar competencies (atomic rules) that distinguish among individuals in the target population—for example, the ability to set up and administer efficient accounting systems for companies of various types.
the appropriate level of rule representation varies directly with population sophistication (and desired level of behavioral detail), it is often practicable to analyze even complex task domains (at a level of analysis that is sufficient for assessing the behavior potential of individuals in the population).

The interested reader is referred to the literature for information regarding hierarchical relationships among paths and the conditional testing this makes possible (Durnin and Scandura, 1973; Scandura, 1973; Scandura and Durnin, 1978), the consolidation of knowledge (Scandura, 1977b), a possible basis for assessing sentence production capabilities (Carroll, 1975), the use of sets of rules for assessment purposes (Scandura, 1977b), and the assessment of skilled performance where response measures (for example, latencies) more refined than success/failure are required (Scandura, 1973, chap. 8; 1977b, chaps. 2 and 7).

INSTRUCTIONAL SYSTEMS

It is not easy to reduce complex and sometimes subtle interrelationships to linear form (as in writing). I have, nonetheless, tried in the above sections to stress some of the more crucial interrelationships that exist among content, cognition, and individual differences in the context of instruction.

In each area, we have seen that the choice of representation (that is, the theoretical constructs, the way they are characterized, and their mode of being made operational) is crucial. Specifically, the rule construct was defined as it was (that is, as a triple consisting of a domain, a range, and a restricted type of procedure) to meet certain very critical requirements that appear essential in any viable theory of instruction that deals with individual behavior in specific situations.

Thus, for example, the above method of operationalizing individual knowledge in terms of observable behavior (that is, for assessing individual behavior potential) depends crucially on both the restrictions placed on the procedures (operations) of rules and the aforementioned assumptions concerning encoding and decoding. (The latter issue has already been discussed; therefore, only the former is considered here.) Although this is not the place to dwell on technicalities, the procedures in question are restricted to those that can be represented concretely as structured programs (Haskell, 1978; Alagic and Arbib, 1978). Unrestricted procedures, for example, allow for the generation of new (sub-)procedures and their subsequent use in executing such procedures. In effect, "goal switching" in unrestricted procedures, rather than being distinguished from specific competence, is intertwined with it. This confounding
leads to unnecessarily complex procedures, and, moreover, procedures that seem to have little heuristic power insofar as individual differences measurement is concerned.

At the other extreme, the operations of simple productions (Minsky, 1967) are too restrictive. Productions disallow the possibility of internal decisions, for example, and effectively act as atomic rules. The domains and operations, consequently, may not act independently of one another—something that seems at variance with everyday observation. It is quite possible, for example, for a person (for example, a child) to be able to identify any given column addition problem without necessarily knowing how to find its sum.

While the necessary richness might be obtained by introducing (sets of) production systems (Newell and Simon, 1972) to represent competence, doing so would destroy much of the heuristic value gained by representing competence in terms of rules. Independent justification for this observation can be gleaned from the fact that, although Siegler and Klahr (Klahr, 1978), for example, have used production systems in much of their work, the (finite) decision trees they used for diagnostic purposes are, in fact, rules. When represented in terms of production systems, the underlying processes appear to lose their heuristic power almost entirely. In effect, although there are any number of scientific languages that are sufficiently rich to characterize competence/knowledge, formal equivalence is not the same as psychological equivalence, even less so when the needs of instruction are taken into account.

The above restriction on the procedures of rules is made possible by the cognitive separation between specific knowledge and control, which, in turn, is made feasible by strongly supporting data which suggest the universality of goal-switching control. This separation of specific knowledge and control has the effect of extracting goal-switching from unrestricted procedures. To summarize, restricting the procedures of rules, together with the encoding/decoding assumptions referred to previously, greatly simplifies the representation of competence without reducing generality. These constraints "force" competence into a form that is unambiguous (in relation to encoding/decoding assumptions) and that has considerable heuristic power insofar as operationalizing individual knowledge is concerned.

Other features of the rule representation are equally crucial. For example, attaching ranges (to rules), which are independent of the domains and the procedures, plays a crucial role in goal-switching control. Given the assumption that people may be characterized as goal-directed information processors, it stands to reason that the use of rules depends on what the knower expects rules to do and not just on how and where they can do it. What a
person expects a rule to do is not necessarily the same as what the rule actually will do. In tightening a joint, for example, a plumber might expect (hope?) to stop a leak; instead, he might cause the pipe to break.

Goal-switching control, in turn, not only has found strong and direct empirical support in its own right, but it provides a pragmatically useful basis for identifying what must be learned in instructional situations. Thus, the separation of specific knowledge/competence from control greatly simplifies the task of dealing with high-level interrelationships that are frequently involved in complex domains. Specifically, taking the control mechanism as given not only makes it possible to represent competence in a modular fashion (in terms of independent rules) but also makes it easier to identify this competence.

In the latter regard, I have been only partially concerned in my theoretical work with specifying constraints on the representation of competence (that is, what must be learned). I have been equally concerned with the problem of how to identify the competence underlying given problem domains. Clearly, knowing the way in which such competence is to be represented (that is, the type of representation) is not the same as being able to construct such representations in the first place. As a result of carrying out an increasing number and diversity of research activities, I believe, nonetheless, that it may be possible to construct competence theories that satisfy structural learning constraints in a relatively efficient, systematic, and objective manner.* This is especially so in the case of competence theories that are intended to be used for instructional purposes.

Our understanding of how to analyze content has increased dramatically over the past several years, and the future looks even more promising. To see this, one has only to mention the important contributions made by traditional task analysis in this direction (Gagne, 1962, 1970) and to point out that this method of analysis is

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*I am not implying the possibility of automatic theory construction, however. I very much doubt that we will ever have a comprehensive or complete method of analysis. (Related questions of possibility and impossibility have been considered in the theory of computation [Rogers, 1967], but such questions are hard to prove in the context of human behavior because they require making explicit assumptions that one might not be willing to make about [real] people. To my knowledge, the possibility of systematic construction of structural learning theories remains an open question.)
a special case of structural analysis.* The main advantages of the latter are that it is more precise and that it enables one to analyze more complex domains, where higher-order relationships play an important role.

In many ways, the problem of how to devise specific competence theories is more basic than any of the others. Prior structural analysis of a given body of content, recall, is the essential prerequisite for realizing particular structural learning theories. Given an analyzed (and evaluated) problem domain, the rest of the theory follows directly: the assessment of individual knowledge, the specification of what individual learners will do and/or learn in particular problem situations, and detailed plans for guiding (teaching) the learner. It is worth emphasizing in this regard that the assessment method proposed, the assumed universals used to (partially) characterize human cognition and instructional methods refer to the entire class of structural learning theories (that is, to the Structural Learning Theory) and not just to particular realizations.

RELATIONSHIPS TO TRADITIONAL COGNITIVE THEORIES

Few behavioral scientists would argue with the general thesis that any viable theory of instruction must deal with the questions of what is to be learned, what the learner already knows that might be relevant (and by implication what is not known), and how to get the learner from where he is to where one wants him to be (Atkinson, 1972; Wulfek and Scandura, 1977). There are, however, differences of opinion as to how to achieve these ends.

The field of cognitive psychology, for example, obviously overlaps with structural learning. Nonetheless, contemporary research in the former area rests on assumptions that are quite different from those that have guided developments in structural learning. As we have seen, perhaps most basic is the fact that structural learning has been motivated largely (but not exclusively) by instructional considerations.

*Although going into the matter here would detract from present concerns, it would be easy to show that other common and pragmatically based methods of analysis, used in instructional planning, also are special cases. The "elaboration" theory proposed by Merrill (1978) provides a case in point. It corresponds directly to starting with the paths a learner knows and progressively adding more elaborate ones.
In this section, let us consider some of the more basic points of difference between structural learning theories and most contemporary theorizing in cognitive psychology.

First, consider the basic experimental paradigm used to test traditional normative theories in cognitive psychology. To begin with, given a class of tasks, one or more alternative cognitive theories is proposed as to how subjects perform on these tasks. Specifically, the theorist attempts to characterize the processes people go through in solving the tasks.* For example, cognitive psychologists interested in instructional problems have asked how people solve problems in arithmetic, how they read, and so forth. Next, based on the alternative theories, predictions are made as to how subjects should behave on the tasks. Then, groups of subjects are given the tasks and their behavior is recorded. Group means (and other statistics) are computed and analyzed statistically. The results are then compared with the predictions. Finally, depending on the respective fits between theoretical predictions and the empirically determined group means, inferences are made about how people perform on the tasks (that is, about the validity of the alternative theories).

Now, it is perfectly proper to do this. It is not, however, proper to infer that individual subjects perform in any given way just because a theory to that effect has been shown to be consistent with how groups of subjects perform—on the average. If we have learned anything over the past millennium, it is that individuals do not always, perhaps rarely, do things in the same ways or with the same effectiveness. Moreover, the more complex the tasks (that is, the more complex the theory needed to account for behavior on the tasks), the greater the deviation to be expected between normative expectation and individual reality. Equally important here, the aforementioned approach to science simply does not provide the kinds of information needed to make instructional decisions concerning individuals.

Consider another well-known approach to cognitive science—that which goes under the rubric of "computer simulation." Unlike cognitive theories devised by experimentalists, computer simulation

*In cognitive psychology generally, heavy emphasis also has been given to such either/or questions as Is information processed serially or in parallel? Is language (symbolism) necessary for thought? Do people use imagery? For an excellent discussion of such research and its limitations, see Newell (1973). Also see Cohen (1977) for a more comprehensive and evenhanded discussion of traditional approaches.
theories do deal with individual processes, both at the theoretical level and in empirical testing. (Studies that involve intensive observation and/or prediction of behavior as individuals solve problems are methodologically similar.) Thus, for example, predictions are typically matched with verbal protocols obtained as subjects solve the tasks at hand.

There also is a difficulty in this regard, however. Specifically, this approach gives too much attention to individual processes. A different cognitive theory may be required for each individual.

In effect, contemporary cognitive theorizing is posed with something of a dilemma insofar as instruction is concerned. In order to devise theories that deal with individual behavior, and, hence, that might have basic educational relevance, one is forced to devise a separate cognitive theory for each individual. On the other hand, if one wants generalizable theories, then one is restricted to normative behavior—-with inferences to the individual speculative at best.

As we have seen, structural learning theories do deal with individual behavior (that is, the behavior of individual learners in specific situations), but they do so in a generalizable way. In order to achieve this duality, however, it has been necessary to reject both the normative and simulation approaches as inadequate and to adopt a relativistic view of behavioral theorizing. In its essentials, this approach involves the identification of prototypic competence associated with given bodies of content (tasks) and given target populations (rather than either normative or individual competence) and the empirical evaluation of prototypic competence in terms of its reliability as a base for determining the behavior potential of individuals.

This approach explicitly rejects the implicit assumption of many cognitive psychologists that we can, in fact, find out how people actually do things. This hope, I fear, is forlorn. We can never know exactly the cognitive basis for an individual’s behavior. (In this sense, I strongly agree with Snow’s [1978] contention that instructional theory is necessarily task and situation dependent.) If we are to avoid the dilemma posed above, the best we can hope to do is to characterize individual knowledge in relativistic terms—in relation to predetermined rules of competence. (The latter, in turn, depend on the given problem domain and subject population.)

The precision with which individual knowledge can be specified depends on the extent and detail inherent in the problem domain (that is, in what is being observed). Beyond a certain point, further discriminations become immaterial and cannot, in any case, be detected within the domain (that is, with respect to the observables) in question. Thus, for example, competence need not be
specified at a fixed level of detail. For some instructional purposes, identified competence might need only to distinguish right answers from wrong. At a deeper level, competence might have to account for degree of skill (for example, latency) as well.

Similarly, if one is interested only in success or failure on a given class of tasks (for example, subtraction problems), then it makes no difference whether a subject applies a well-known algorithm or attempts to devise a new solution method. Such possibilities cannot be distinguished within the domain in question. This can only be determined through performance on extra-domain tasks—or, equivalently, by redefining the domain of interest.

In addition to our work and that of others consciously working in structural learning (see Journal of Structural Learning, vols. 1-6), the potential of structural learning theories is evidenced by the fact that several investigators in the United States (for example, Siegler, 1978; Klahr, 1978) have recently used such theories in their diagnostic studies of children's learning. In addition, "hypothesis theory," as developed by Levine (1966) to include probes for diagnostic testing, also is a structural learning theory restricted to particular kinds of simple discrimination learning tasks. Overseas, Landa (1976) developed a more general diagnostic method in the sense that it generalizes over content, as does that referred to here. As with others, however, his method is limited to situations where the underlying rules of competence have a simple (finite) tree structure. In effect, all of these applications are restrictions of that outlined above (where unrestricted "looping" is allowed)—all employ special cases of the general diagnostic theory, albeit sometimes without apparent awareness (compare Siegler, 1978; Wulfeck, 1978).*

Second, from our discussion of the Structural Learning Theory (that is, the class of structural learning theories), we have seen

*During the last few years, several related attempts have been made in artificial intelligence to develop more or less comprehensive instructional systems (Brown and Burton, 1978; Collins, 1978). In addition to instructional theory, however, this work relies heavily on the "brute force" processing capabilities of sophisticated computers. Nonetheless, to the extent that the work incorporates serious theory (apart from pragmatic programming), this theory tends to be highly consistent with the structural learning formulation. This fact is especially apparent in what Brown and Burton call "debugging."
that those aspects of the theory involving content, cognition, and individual differences are all interrelated. For example, the representation of competence—or of what must be taught—was dealt with in a way that was consciously sensitive to the requirements for testing and to what is known about how people use their available knowledge. Specifically, the representation of knowledge in terms of rules (and the structures on which they operate) was shown to be fully operational (that is, to lend itself to the assessment of individual knowledge, or behavior potential, in relation to given competence—as opposed to norms) and to be consistent with general constraints on cognitive processes imposed by the nature of the human information processor.

In effect, it is not sufficient, especially insofar as instruction is concerned, that competence be represented in terms of some more or less precise information-processing language, as is commonly assumed in contemporary cognitive theorizing. This lack of attention to broader requirements of instructional systems, I think, is a consequence of the "bricklaying" syndrome. One common approach is to start with some scientific language* and to show how it can be used to represent various types of cognitive processes. The important point here is that, if such representation can be effected in one sufficiently rich language, then it also can be accomplished with any number of others. Mathematical sufficiency (generative sufficiency in the case of computational/computer-based theories) does not guarantee behavioral adequacy, especially with respect to instruction.

For one thing, assumed constructs in the most commonly used languages are not easily or naturally tied to observables. Thus, for example, relational network theories are, at best, only partially operational (Cohen, 1977). Typically, one attempts to infer cognitive structure indirectly from latency data. This procedure is imprecise (as well as normative) and is almost totally inadequate for many instructional purposes.

Furthermore, such a procedure typically (although often implicitly) assumes a fixed, static cognitive representation (for example, a fixed relational net). Otherwise, it would be impractical, if not impossible, to infer structure from behavior (that is, from a variety of independent measures). The problem is that cognitive

*All too frequently, the language is selected by cognitive theorists on the say-so of computer scientists, whose choice, in turn, reflects formal mathematical properties unrelated to the needs of behavioral science.
structure is not static; it is dynamic and may change at each stage of processing (Scandura, 1973, 1978a).

Moreover, as noted previously, cognitive structure cannot be determined (that is, defined) independently of process. Rather, structures are best viewed either as assumed internal encodings/decodings* or as intermediate states generated and/or operated on by processes (that is, rules). This raises the question of why one should attempt to represent static knowledge at all; such knowledge is implicit in the cognitive processes themselves (Scandura, 1973, 1977b, 1978a). (As we have seen, rules may be operationally defined in terms of observable behavior.) Reasoning thus brings us full circle back to rules as constituting the basic, operational unit of cognitive functioning.

Moreover, learning and other basic phenomena have posed, and continue to pose, major problems for other contemporary theories in cognitive psychology—this holds for theories of both the normative and the computer varieties. The way rules have been defined in the Structural Learning Theory, on the other hand, provides a natural, apparently general, and precise way to conceptualize the learning process (Scandura, 1971b, 1973, 1974b, 1977b).

Third, the relevance of existing cognitive theory becomes even more problematical when it comes to questions of what is to be learned and problems of instruction per se. In the former regard, the distinction between what is to be learned and what the learner knows is typically confused in existing cognitive theorizing.

This lack of distinction, ironically, may partially derive from the traditional distinction that originated in psycholinguistics between competence and performance. Recall, in this regard, that

*The writings of a growing number of investigators suggest a gradual, but growing, awareness of the essential role played by superordinate constraints, such as this one. Kintsch and Van Dijk (1978, p. 364, also see p. 392), for example, found it convenient to construct a model of text comprehension and production that "operates at the level of assumed underlying semantic structures." Rather than being viewed as a serious limitation in principle, however, it might have been reassuring to recognize that some such assumption (for example, regarding encoding) is an absolute necessity if one wants to ensure unambiguous and operational cognitive theory that deals with individual behavior. (Incidentally, Kintsch and Van Dijk did not attempt to deal with individual behavior in their article. For a theory of text production that might lend itself to the assessment of individual potential, see Carroll, 1975.)
competence was equated with idealized grammar (for example, with theory for generating sentences in a language). Performance, on the other hand, refers to what human subjects are actually capable of.

Aside from issues pertaining to type of representation, the traditional notion of competence is quite compatible with the characterization of what is to be taught. The situation is not so direct with performance, however. For one thing, recall the complications due to the individual/normative considerations I have already raised. (See also Scandura, 1977b.)

In addition, and more importantly here, a key distinction in structural learning theories is confounded in the traditional view. Specifically, performance depends on both specific individual knowledge and cognitive universals. It is important, I think, to maintain this distinction in any viable theory of instruction. While instruction is and can properly be directed toward individual knowledge, this is not the case for cognitive universals.

Fourth, closely related to this confounding is the question of methodology. Clearly, the methods used in experimental psychology may be very useful and, indeed, may be indispensable for some purposes—such as finding normative theories of average behavior. They are, however, neither exhaustive, indispensable, nor perhaps even desirable for purposes of studying the instructional process. While the traditional experimental approach may provide reliable information in the laboratory, this has far less frequently been the case in instructional settings. Furthermore, the normative information provided may bear little relationship to the prototypic competence that one might want to teach. But suppose that such an approach did yield reliable and instructionally valid information. The approach would be so inefficient, in view of the time and expense required to conduct such studies and the large variety of content that might be taught, as to be almost useless for instructional purposes.

For instructional purposes, we need to develop systematic and efficient methods for the identification and evaluation of the prototypic competence characteristic of various populations (for example, culture types) and underlying arbitrarily given bodies of content. Insofar as the evaluation phase is concerned, the basic structural learning methodology has been reasonably well established (Scandura, 1973, 1977b; Durnin and Scandura, 1973; Scandura and Durnin, 1978).

The situation as regards structural analysis (that is, the identification of prototypic competence) is more open. The feasibility of such analyses has been established in a number of areas (Scandura, 1977b; Scandura, Durnin, and Wulfeck, 1974; Scandura and Durnin,
1977; Scandura and Scandura, 1978), but the method itself is still only partially systematic and objective (Scandura, 1977b, chap. 2). More efficient and reliable methods of analysis will be essential to insure its widespread use.*

As noted previously, different methods also are needed for the study of cognitive universals (Scandura, 1971b, 1973, 1977b). In this regard, Resnick and Glaser (1976) appear to have followed our lead in their work on the "invention" problem. Among other things, for example, they attempted in their studies to approximate "memory-free" conditions in the training and have utilized a variant of "goal switching" in guiding subjects' problem solving. In these studies, however, the problems all could be solved through simple composition of lower-order rules (much as in our "A→B, B→C, then A→C" paradigm), and no attempt was made either to identify or to teach the higher-order rules involved. (For a discussion of the relationships between their work and ours, see Scandura, 1977b, pp. 504-514.)

Fifth, few cognitive psychologists, of course, would imply that their theories deal directly with educational values, instructional costs, or teaching strategies, and perhaps at best only indirectly with respect to different types and modes of instruction. What is perhaps not fully appreciated, however, is that instructional strategies can and eventually must constitute a serious theoretical subject. It is not necessary or sufficient to rely solely on intuition, mathematical considerations, or empirical trial and error. While discussion of instructional strategies in the context of structural learning theories is beyond the scope of the present study, and while complete solutions are not yet available, some promising beginnings have been made in this direction (Scandura, 1977a, 1977c; Wulfeck and Scandura, 1977). These advances draw heavily on the foundations outlined above.

*For some years, research in this direction has been carried out in concert with, but largely incidental to, specific structural analyses. As a result of the experience gained from this work, we are now attacking the problem more directly and in a way that may generalize over diverse bodies of content. Progress has been relatively rapid during the past year, and I expect that before long we will be in a position to report on our work in the literature.
CONCLUSIONS

Generations of insignificant and/or nonreplicable research on instruction make it clear that global reference to expository, discovery, and other instructional methods will never provide a sufficient basis for instructional decision making (Scandura, 1962, 1964). Attempts to "match" global instructional methods with generalized learner capabilities appear to have met the same fate (Cronbach and Snow, 1977). It is now widely agreed that reliable decision making in the instructional arena will require direct reference to underlying cognitive operations (Snow, 1977; Merrill, 1978).

The preceding analyses make clear, however, that not just any cognitive theory will serve instructional needs. Close interrelationships exist among the constructs and assumptions used to characterize what is to be learned, the learner, and individual knowledge. These interrelationships place severe constraints on the form of any viable theory—a form that conforms to the Structural Learning Theory (or, equivalently, to the class of content-specific structural learning theories). Stated more boldly, any cognitive-based and operational instructional theory that deals with individual behavior in a generalizable way will necessarily be a structural learning theory. Moreover, such theory will be complete just to the extent that it satisfies the constraints associated with the Structural Learning Theory. In general, these constraints pertain to the way competence is represented, the way individual knowledge is operationalized, and cognitive universals.

While strong, these assertions are meant to be taken seriously. Indeed, irrespective of the validity of my arguments, the Structural Learning Theory is illustrative of the type of theory needed if behavioral science is to provide stable and reliable understandings. As stated by Booth (1978):

Mature science is characterized by its inclusion of the style of theory that specifies a system of processes which computably behaves as a whole like the real system, but in addition the individual processes and relationships between them are formally equivalent to processes independently observed to be operative in the real system. Such a systems analysis is genuinely explanatory, and resolves the polarity between reduction and wholism.

It should be emphasized, however, that the Structural Learning Theory, as it stands, is neutral regarding many specific phenomena. The point is that specific theory, if it is to meet the
above conditions, may not be inconsistent with established tenets of
the Structural Learning Theory. As noted previously, the recent
literature includes a growing number of such theories, both our own
and those of others. Those mentioned in the course of our discus-
sion represent only a small sampling. (There is an equally impor-
tant matter that has not been given nearly the space it warrants in
the present discussion. As described, the Structural Learning
Theory is restricted to tutorial situations and does not accommodate
unrestricted conversations between two or more participants. A
theory that accomplishes the latter goal has recently been proposed
by Pask [1976]. This theory treats phenomena at a relatively holis-
tic level, however, and does not make direct contact with the sort
of phenomena treated in cognitive psychology. A challenge for the
future will be to see if the Structural Learning Theory can be ex-
tended to encompass unrestricted conversations while retaining its
operational character and degree of specificity [see Pask, in press].)

Theory, however, does not tell the whole story. Equally, if
not more important, the aforementioned needs of instructional theory
call for different research methodologies. In effect, it is not cog-
nitive psychology per se that I am concerned about. We need to
know a lot more about cognitive processes and must support good
research directed toward that end. What I am concerned about is
the uncritical acceptance and maintenance in cognitive research of
a research methodology that was originally motivated by S-R em-
piricism*—even more so by the attempt to foist that methodology on
the scientific study of instruction.

In this regard, I would propose that the paradigm shift we have
heard so much about in cognitive psychology is largely a myth.
True, in our theorizing, we have begun to ask how and why—and
not only what a person will do. Rather than developing method-
oologies to fit the problem, however, I think we unfortunately have
taken the easy, but less profitable, route of applying a methodology
developed for other purposes.

What I have argued here, and at length elsewhere (Scandura,
1971b, 1973, 1977b, 1978a), is that we need new research method-
oologies as well as new problems and new theories. Specifically,
in instruction, we need to look at competence more in terms of
prototypes than as bases for direct prediction of averaged human
behavior. If we can adopt this shift in outlook, then I believe that

*As noted above, I also am concerned (for somewhat differ-
ent reasons) about the methodologies commonly used in computer
simulation studies (Scandura, 1977b, 1978a).
as instructional scientists we will be in a position not only to progress more rapidly in instructionally valid directions but also to say more about individual behavior in specific situations, and most importantly, to be able to do something about that behavior.

REFERENCES


Jackson, P. Address to Division 15 at the American Psychological Association, San Francisco, September 1977.


Miller, G. A. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review,* 1956, 63, 81-90.


Scandura, J. M. *The basic unit in meaningful learning—association or principle?* *School Review*, 1967, 75, 329-341. (a)


APPENDIX B
SELECTIVE REVIEW OF PIAGETIAN RESEARCH AND SCHOOL PROGRAMS

CONCRETE OPERATION TRAINING STUDIES

In the mid-1960s, interest in Piaget's theory brought concurrent interest in the possibility of training children to attain higher cognitive levels of functioning more rapidly. Piaget felt that the observations from which he developed his theory were a purely developmental phenomenon and not subject to training. The question of whether children could be moved along at a faster rate was always considered to be the "American Question." Not surprisingly, most of the early training research occurred in the United States, although much had gone on since that time in various countries in Europe, including the Center in Geneva.

Brainerd (1976) gives a good outline of the general procedures of the studies performed by researchers who hope to train children to attain the stage of concrete operations.

In research published to date, the prediction that stage and learning are positively correlated has generally been tested as follows. After a concept and a treatment are selected, a sample of subjects is drawn from the age range during which the concept usually appears. Pretests for the target concept (and perhaps others as well) are administered to the subjects. Pretest performance is used to assign subjects to their appropriate Piagetian stages. Only subjects whose diagnosed stage is below the one at which the concept spontaneously appears are retained for training. The treatment is administered to these subjects. A series of posttests is administered after training. Finally, the subjects' pre- and posttest performances are compared to determine whether their pretraining stage predicts how much they learned. (1976, p. 6)

Much of this research deals with narrowly defined empirical issues such as whether Piagetian concepts are in fact trainable, whether trained concepts can be extinguished, whether some training procedures are more effective than others, whether transfer of training occurs, and so forth (Gelman, 1969; Bearison, 1969). The available literature provides reasonably clear answers to these and similar questions.
One of the earliest studies was done by Beilin (1965). He tried four methods of training: nonverbal reinforcement, verbal orientation reinforcement, verbal rule instruction, and equilibration. One of the treatments, verbal rule instruction, appeared to work better than the others. Beilin's main conclusion tended to reaffirm Piaget's stage sequence theory. "The group most likely to profit from training is that group whose members have at least some (but not the fullest) conservation ability, i.e., the group who passed one pretest as compared to the groups that passed no or two pretests" (1965, p. 335). These children would be described by Piaget as being in the transitional stage.

Subsequent to Beilin's work, researchers became interested not only in the training of transitional children, but also in the training of pretransitional children as well. Additionally, some researchers became particularly concerned with the training of concepts closely related to school work. Could such concepts as conservation of number be trained and would the training benefit school performance over a long period of time?

Following Beilin's work, which confirmed that conservation did appear to be trainable, Gelman (1969), among others, confirmed the possibility of the training of number conservation to pretransitional children. She administered pretests of conservation to five-year-olds in four concept areas: number, length, liquid quantity, and mass. Those children failing all pretests were trained on number and length with a procedure designed to reduce their dependence on spatial clues. She felt that children tended to pay attention to irrelevant attributes of the concepts they were evaluating. By gradually introducing slight transformations and giving feedback, she set up a situation that Piaget would describe as a state of "disequilibrium." She then focused the children's attention on those aspects of the concept that had not changed. Two to three weeks after the training the pretests were readministered and virtually perfect results were observed on the number and length conservation posttests.

Bearison (1969) was interested in the question of whether the decalage question could be answered as a function of the differences between those concepts that are discrete (number) and those that are continuous (liquid). Using a rather interesting training procedure of "pouring" glass beads (which could subsequently be counted after each transformation) he was also able to train for conservation in some kindergarten children. However, in a follow-up study (1975), he was disappointed to find that his trained children could be distinguished from the "natural" conservers, although they were still ahead of those who had no training (and were not "natural" conservers at the time they were originally tested). Clearly, there were some other things that the natural conservers had attained.
Members of the Genevan school have been less than convinced by such research. According to Piaget (1965), an understanding of the concept of number, for example, results from an operational synthesis of classification and seriation. In his view, the number 5, for example, includes the numbers 4, 3, 2, and 1 as well. In particular, the individual numbers may be represented individually in set-theoretic terms. These numbers are related to one another by the ordering relation involving the classes of sets denoting the individual numbers—in the sense that the class of sets denoting the number 4 is strictly contained in the class of sets containing the number 5, and so forth. As it was put by Inhelder and Piaget (1964), "the reasoning involved in the conservation of number presupposes the ability to organize the objects of experience into qualitatively distinct groups or classes and to handle the essential relationships of class membership, namely seriation." Piaget argues that number is a synthesis of the logic of classes and the logic of relations, and he emphasizes that (in his view) children master all three modes of reasoning at the same time. Piaget believes that it is the "irreversibility in the child's thought during the preoperational stage that prevents him from acquiring the power of decomposition that is necessary for understanding inclusions and relations" (Vaidya, 1977). Children eventually discover the correct inclusion simultaneously with their acquisition of reversibility. (Note: The reversibility of operations refers to an operation defined on operations and, hence, can be thought of as what has been called a higher-order rule in the sense of Scandura [1973]. More accurately, reversibility, as used by neo-Piagetians, actually refers to a class of higher-order rules. In fact, the well-known phenomenon of "horizontal decalage" may be attributed to the fact that the domains of applicability of rules, including those of "reversibility higher-order rules," may vary widely over individual children.)

Clearly, Piaget does not believe that such learning, or more generally that associated with the stage of concrete operations, perse, can be acquired over the short periods of time associated with most contemporary training studies. In making this point, Piaget (1970, p. 31) raises the following concern: "Is it good to accelerate the learning of these concepts? Acceleration is certainly possible but first we must find out whether it is desirable or harmful. Take the concept of object permanency. . . . A kitten develops this concept at four months, a human baby at nine months; but the kitten stops right there while the baby goes on to learn more advanced concepts."

In attempting to clarify this important issue, we are faced with an unhappy prospect. On the one hand, contemporary training studies seem inadequate to resolve the problem (Bearison, 1969,
1975). According to Brainerd (1978), training studies tend to fall into several identifiable categories (for example, simple correction, observational learning, conformity training, and rule learning. See the following: verbal training, Braine and Shanks, 1965; perceptual, Gelman, 1969; reinforcement, Wohlwill and Lowe, 1962; self-discovery, Smedslund, 1961; cognitive conflict, Piaget, 1971; Inhelder et al., 1966.) Although many of the earlier studies (Smedslund, 1961; Wohlwill, 1959) have been unsuccessful, many of the more recent training studies have been successful, in many cases impressively so (Gelman, 1969). According to Brainerd (1978), this has been particularly true in training studies based on rule learning.

Nonetheless, essentially all existing training studies have at least three major kinds of limitations.

1. Although most training studies are based on some general conception as to how learning takes place, these conceptualizations leave to the experimenter the critically important task of specifying what it is that is to be taught (and learned). In the case of rule-based training studies, for example, the question arises as to exactly what rules to teach (Scandura, 1970, 1977a, 1977b). As noted above, it is not sufficient to say simply that the rules ought to involve reversibility, and so forth. Because there are any number of kinds of reversibility rules, one must specify not only what the operation should do, but also the elements on which such rules operate. (Note: In the case of higher-order reversibility rules, the domain elements are themselves operations.)

None of the existing training studies appear to have considered higher-order rules explicitly (although the metacognitive processes, of Flavell 1977, for example, deal with this concept informally). Perhaps more important, very little, if any, attention has been given to the processes by which underlying competence (including rules and higher-order rules) may be identified. Both of these concerns (that is, higher-order rules and the processes by which they may be identified) play a central role in the Structural Learning Theory.

2. The methods used to determine the stages at which children enter training have equally important limitations. Where the tests used are precise and reliable they typically fail to satisfy Genevans (that is, such tests, in their opinion, do not tap essential characteristics of the child's thought) (perceptual screening, Bruner, 1966; cognitive operations, Wallach and Sprott, 1964; Sonstroem, 1966). And where they are based directly on Piagetian methods, they frequently lack the objectivity one might prefer in a scientific theory.
3. Although many of the training studies are based on more or less specific theories of learning, very few of them involve both higher-order rules and specific (that is, hypothesized) mechanisms by which such rules interact to produce new knowledge. (A major exception is the study by Klahr and Wallace, 1973, which is based on Newell and Simon's, 1972, production systems. In this case, however, the postulated theory is highly restricted in scope.) Without a general and explicit theory of this sort, which tells exactly what we learn and when, it is hard to see how such issues as the important one raised (above) by Piaget can ever be resolved (compare Kuhn, 1974; Uzgiris, 1964).

PIAGETIAN SCHOOL PROGRAMS

Recently, much interest has switched from the training studies to the application of Piagetian theory to the classroom. Because the training studies answered in the affirmative the "American question" of whether the attainment of cognitive stages can be accelerated, investigators have begun to explore the problem of what type of environment is necessary for such attainment. Lawton and Hooper refer to a list by Ginsburg and Opper (1969) of "six very general implications" from Piaget's theory. They have summarized them briefly as follows:

1. The language and thought of the child are qualitatively different from that of the adult, and the child may find it difficult to assimilate some sequences of ideas.
2. The young child learns more easily from concrete experiences.
3. Tasks should take into account the child's readiness for, and interest in, learning.
4. The development of curricula must take into account the invariant sequence of cognitive development.
5. Social interaction is a crucial factor in the child's development.
6. Traditional methods of instruction—that is, group lessons with a given sequence of material, transmission of materials via lecture or other verbal explanation—have grave deficiencies. (1977, p. 175)

Even these somewhat explicitly stated assumptions, however, are subject to interpretation when actually applied in the classroom. They also represent the state of the art in 1969, since subsequent
experimental classrooms have tended to specifically reject the sixth implication above (Lawton and Hooper, 1978).

One of the earliest attempts to implement Piagetian theory was the Early Childhood Curriculum: A Piaget Program (ECC) done by Lavatelli (1970). She organized her materials around three themes: classification; number, space, and measurement; and seriation, the objective being to lay the foundation for the emergence of concrete operations. A language component, in addition to manipulable concrete materials (for example, small toys) is prescribed for the teacher, who organizes short training sessions several times a week with groups of five to six children. Other than the sequence of materials used, Lawton and Hooper (1977) explain that Lavatelli stresses the importance of the teacher's understanding of the educational objectives of her program in terms of mental operations and language development. She feels that there is no one right way to conduct training sessions. She reports pilot programs where "significant gains on Binet scores and on Piaget-type tests" were found (1970, p. 4). However, Lawton and Hooper disagree with some of her conclusions.

Although we agree that similar learning may result from various kinds of instructional techniques, nevertheless it is important to realize that there are underlying principles that guide mental functioning. The concepts of, for example, hierarchical classification or seriation have critical attributes, and the teacher must know what these attributes are. The establishment of critical attributes of process concepts should provide guidelines for the selection of materials that can be organized into hierarchical classes or into series quantifiable on some scale. Guided learning should make some provision for the identification and organization of critical attributes that lead to the acquisition of the concept. In that sense, there is a right way of selecting materials and conducting learning activities. (p. 179)

Another program, the Cognitively Oriented Curriculum: A Framework for Preschool Teachers (Weikart, 1973), based on Piagetian concepts, has as a focus a single theme: classification. The curriculum revolves around separate exercises of 11 different skills. The program is relatively formal, although not as formal as Lavatelli's. Weikart stresses an important relationship between the learning initiated by the teacher and the course of intellectual development. He also feels that the child will show a desire for self-directed activity. However, it is not always clear exactly how
the teacher generates the activity in the way she desires, or how the
self-directed child's activity results in the activities the teacher
feels have the most promise.

One of those originally involved in the Weikart program was
Constance Kamii. She eventually disagreed with the program's
focus on classification and felt that this focus was a fundamental
misunderstanding of the basic concepts of Piaget. Like Piaget in
Geneva, she did not feel that the cognitive skills (such as classifi-
cation) could be taught directly. She was also directly opposed to
the idea of moving children "prematurely" from one stage to the
next through specific training activities. Kamii set up the pre-
school program called Piaget for Early Education at the University
of Illinois, Chicago Circle. Subsequently, she spent a year working
with Piaget in Geneva and has just published her curriculum as a
source book, Physical Knowledge in Preschool Education: Implica-
tions of Piaget's Theory. Interestingly, Piaget feels that Kamii,
among American researchers, best represents his views regarding
the educational applicability of his theories (Karmeloff-Smith, 1977,
personal communication). While the main body of the book deals
with examples of activities done with children in the Circle Chil-
dren's Center, it is perhaps more valuable to list the cognitive ob-
jectives as Kamii and DeVries (1978) see them:

For the child to come up with a variety of ideas, prob-
lems, and questions . . . an objective in opposition to
recitation of the right answer the adult wants to hear
. . . teachers who use these principles of teaching are
frequently astonished at the difficult problems that chil-
dren set for themselves, problems that they would
never think of suggesting.

Putting objects and events into relationships and no-
ticing similarities and differences, is a natural out-
growth of the first. When children invent their own
problems, these invariably lead to the construction
of relationships. (pp. 44, 45)

The focus of Kamii and DeVries's program is to provide an atmos-
phere conducive to interaction among the children and the acceptance
of right and wrong conclusions in response to activities and interests
that may change from moment to moment.

Because the approach used at Chicago Circle is so informal
and because it has been in existence for such a relatively short
period of time, it is rather difficult to evaluate. In addition, Kamii
and DeVries feel that it may not be possible to evaluate the approach
scientifically in a long-term study, as some might feel necessary, "as long as children have to go to repressive, traditional schools from ages six to sixteen" (1974, p. 74) [following their preschool experience].

The Piagetian Preschool Educational Program (PPEP), a preschool program with a similar, orthodox Piagetian approach, was given a three-year field evaluation at the University of Wisconsin Early Childhood Study Center. The basic question was whether such a program affected the cognitive developmental process. Here, also, the emphasis was on spontaneously developed concepts, in an informal setting, without specific, "correct" answers being taught. The act of looking for a solution was deemed as valuable as the act of finding one, if done from internal motivation. Subsequent to this three-year program, a revised program, similar to it but emphasizing more structured, teacher-directed, small group instruction, was begun in 1975 (Bingham-Newman, Saunders, and Hooper, 1976). The evaluation of the PPEP program had shown no significant differences in performance between it and a conventional preschool nursery school program (Lawton and Hooper, 1977).

As a reaction to these relatively unstructured approaches to instruction, the Ausubelian Preschool Program was initiated at the University of Wisconsin Preschool Laboratory (Lawton, Lewis, and Deibert, 1976). Here the ideas of Ausubel, and what he called "advanced organizers," were used. That is, an attempt was made to proceed from general concepts, corresponding to Ausubel's (1969) advanced organizers, to specific exemplars. However, although Lawton and Hooper (1978) elsewhere suggest that cognitive structure is composed of concepts and propositions (learned from various subject matter areas) arranged in hierarchical structures, Ausubel has not made a distinction between subject-matter and process concepts. The approach used in this experimental program has recently been modified to include this distinction. Moreover, as Lawton and Hooper also note, Scandura emphasizes the point that "Knowing subject matter content . . . is not equivalent to specifying the relevant competence" (Scandura, 1977a, p. 38).

The basic procedure used in the Lawton et al. program involves working with small groups of children and introducing high-level subject-matter concepts, high-level process concepts, or a combination of both. These ideas are presented both verbally and in the form of concrete exemplars. The children are actively involved in the manipulation of these concrete learning materials. Follow-up teaching sequences involve the relation of these concepts to more particular learning materials. The basic hierarchical arrangement of content and process concepts has been established and learning experiences are sequenced in reference to this structure.
However, the structure is subject to modification in reaction to student feedback. Although learning activities are directed by the teacher, a major emphasis is also placed on encouraging child-directed learning.

The important distinction between this approach to preschool education and those previously mentioned is the formality of the presentation of lessons and the explicit analysis of the content being taught. Special attention is given to hierarchical relations within and between concept areas. In the most recent evaluation of the Ausubelian Preschool Program, the children "achieved significantly better performances on certain classification, relations and conservation tasks than their counterparts in the traditional preschool. . . . Delayed posttests administered after an interim period of almost six months indicated a continued improvement on almost all tasks for the Ausubelian group, whereas the children in the traditional program showed little improvement."

CONCLUSIONS

In appraising Jean Piaget's theories of cognitive development, their interpretation by Genevans and Americans, their subjectiveness to training intervention, and their appropriateness as a basis for preschool curriculum, one is struck by a number of inconsistencies. Some of the difficulties rest within the theory itself and Piaget has been subject to much criticism on basic methodological grounds.

1. Piagetian theory is not sufficiently detailed to provide an exact account of horizontal (within stage) or vertical (between stage) decalage. In particular, "the literature seems to raise serious doubts about the theory's ability to explain the laboratory learning of Piagetian concepts" (Brainerd, 1976b, p. 2). According to Piaget, successful training requires spontaneous development; forced training will only be successful when children are at an appropriate transitional stage. While this contention may be true based on present, and incomplete, knowledge, it hardly constitutes a scientific ideal for a thoroughly developed theory. One would wish for a more exact theory that tells how and why decalage occurs and, moreover, how one might intervene to overcome unwanted decalage. (Note that Piaget does not say such intervention is impossible, but only that the kinds of learning theories associated with American psychology are inadequate to that end.)

2. According to Brainerd (1976b), for one, Piaget is open to the criticism that his definitions of groupings (in the case of concrete operations, for example) are circular (1976b, p. 37). Thus,
here, Piaget is well within his rights to maintain that contemporary training studies do not tap what he feels to be essential in characterizing the stage of concrete operations. There remains, however, the task of specifying exactly what does constitute such attainment. It is not sufficient, on the one hand, to use the usual Piagetian tests to determine which children are at the stage of concrete operations before training, and then, after training, to deny that those who have attained this same level of performance on the same tasks have not really attained this level of functioning. In order to avoid this dilemma, it would seem that the grouping corresponding to concrete operations would need to be operationalized independently of the conservation tasks usually used to determine their presence.

3. Another concern closely related to the first is that while Piaget talks about the general characteristics of various kinds of groupings (structures), his theory deals only with commonalities among subjects. A more detailed assessment of the current state of individual structures, and more detailed accountings of the sequences through which individuals progress, will require a (behaviorally valid) theory that deals with individual processes as well (as epistemic ones). Moreover, it must be possible to specify these processes and/or structures operationally in terms of observable behavior.

Although one shares Piaget's concern for general readiness to learn, one can also (partially) agree with Brainerd's (1976b) contention that Piaget's particular approach is inadequate as it stands. In this regard a more adequate conceptualization should deal, at the least, with knowledge specific to various individuals, as well as to commonalities (epistemic knowledge), as in Piagetian formulation. Among other things, such a theory should provide a more detailed accounting of horizontal decalage, as well as some basis for facilitating the kinds of cumulative learning that are so characteristic of much of school learning.

The aforementioned inadequacies are inherent in Piagetian theory and have led to similar inadequacies in training studies conducted in both laboratories and schools. In a recent review Murray (1978) concludes that "there is an overwhelming and somewhat pessimistic result that training—even highly individualized training—is only successful [by whatever criteria] with about half the children in the sample, or more precisely, that the children make about half the gains in conservation performance that could be made, although the gains are stable for as long as a month and are significantly different from pretest or control group subjects' performance" (p. 421).

Although orthodox Piagetians would disagree with the way the results of many training studies have been interpreted, even they concede that it is often possible to move some (transitional) children
to the next stage via certain kinds of training (especially, incongruity training in a social situation). Therefore even Genevans see value in general enrichment of the preschool environment. Their view is best expressed by Kamii and DeVries (1978):

The use of Piaget's theory beyond the preschool level implies drastic changes in the teacher's conception of the educative process. When the focus of teaching shifts from what the teacher does to how the child constructs his own knowledge, the center of the classroom will no longer be the subject matter or the method of teaching, and the teacher's thinking will undergo a revolution similar to the Copernican revolution.

The educational implications of Piaget's theory are not just a different way of reaching the same goal as traditional education. In order to foster autonomy schools will have to give up trying to fit the individual into a mold. . . . We firmly believe . . . that those schools which take autonomy seriously will produce more adults with inquisitive, critical, and inventive minds. We also believe that the graduates of such schools are more likely to go on learning and developing the rest of their lives. (p. 310)

However, Lawton and Hooper (1978), after a thorough review of many preschool educational programs, come to a very different conclusion. "In summary, the analysis of certain informal and formal preschool programs has led to the observation that most orthodox Piagetian [informal] instructional methods fail to show significant differences compared to other traditional teaching methods. . . . Improvements in performance, which have been shown to be both durable and generalizable, have occurred in formal programs" (1978, p. 15). They seem to feel that the measure of performance is specifically attuned to "the degree of structure in teaching methodology, organization of learning materials, and sequencing of learning situations and associated learning materials." We find it difficult not to agree with their conclusion:

We [they] would agree with Scandura (1977a) that, whatever view of development, learning and instruction we take, and of the relationships amongst this triad, what we need is to produce a "broad, fundamental theory tailored to the needs of education. . . . This means a theory that integrates content, cognition and individual differences into a unified system." (Scandura, 1977a, p. 52)
Obviously the question of the correctness of Piaget's theory as a true representation of the evolution of cognitive development and its use as a structure for the design of curriculum are still open questions to be answered as new research results are found.

REFERENCES


Kuhn, D. Inducing development experimentally: Comments on a research paradigm. Developmental Psychology, 1974, 10, 590-600.


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